HAESE MATHEMATICS

Specialists in mathematics education

Mathematics

7 MYP 2

third edition

Michael Haese

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for use with IB Middle Years Programme

MATHEMATICS 7 MYP 2 third edition

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FOREWORD

Mathematics 7 MYP 2 third edition has been designed and written for the International Baccalaureate Middle Years Programme (IB MYP) Mathematics framework, providing complete coverage of the content and expectations outlined.

Discussions, Activities, Investigations, and Research exercises are used throughout the chapters to develop conceptual understanding. Material is presented in a clear, easy-to-follow style to aid comprehension and retention, especially for English Language Learners. Each chapter ends with extensive review sets and an online multiple choice quiz.

The associated digital Snowflake subscription supports the textbook content with interactive and engaging resources for students and educators.

The Global Context projects highlight the use of mathematics in understanding history, culture, science, society, and the environment. We have aimed to provide a diverse range of topics and styles to create interest for all students and illustrate the real-world application of mathematics.

We have developed this book in consultation with experienced teachers of IB Mathematics internationally but independent of the International Baccalaureate Organisation (IBO). It is not endorsed by the IBO.

We have endeavoured to publish a stimulating and thorough textbook and digital resource to develop and encourage student understanding and nurture an appreciation of mathematics.

Many thanks to Rob Colaiacovo and our other contributors for their recommendations and advice.

We welcome your feedback. Email: info@haesemathematics.com

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ONLINE FEATURES

Each textbook comes with a 12 month subscription to the online edition and its range of interactive features. This can be accessed through the **SNOWFLAKE** online learning platform via a web browser or our offline viewer.

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SELF TUTOR

Self Tutor is an engaging feature that supports students learning independently and in classrooms.

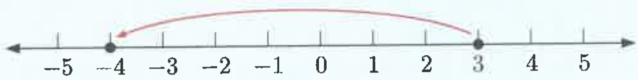
Click on any example box to access a step-by-step animation with teacher voice support providing explanation and understanding.

Example 2 **Self Tutor**

Use a number line to find:

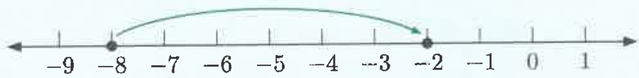
a $3-7$ **b** $-8+6$ **c** $2-5+8$

a We start at 3 and move 7 units to the left.



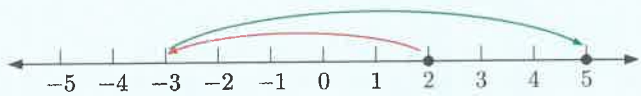
So, $3-7=-4$

b We start at -8 and move 6 units to the right.



So, $-8+6=-2$

c We start at 2, move 5 units to the left, and then 8 units to the right.



So, $2-5+8=5$

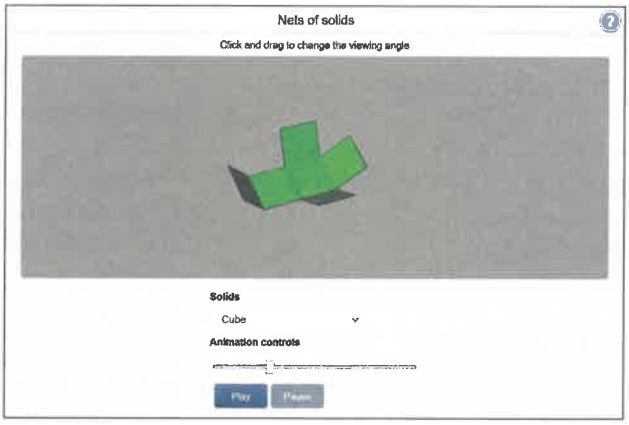
See **Chapter 5**, **Positive and negative numbers**, p. 91

INTERACTIVE LINKS

The **SNOWFLAKE** icons direct you to interactive tools to enhance learning and teaching.

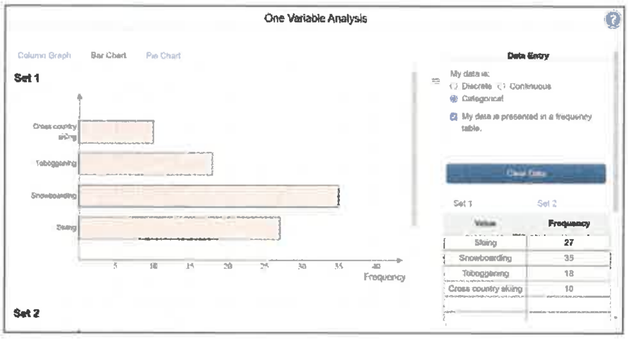
These features include:

- demonstrations to illustrate and animate concepts
- multiple choice quizzes to test understanding
- games to practise and build skills
- tools for graphing and statistics
- printable pages for use in class.



DEMO

GAME



TOOL

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GLOBAL CONTEXTS

The International Baccalaureate Middle Years Programme focuses teaching and learning through six Global Contexts:

- Identities and relationships
- Orientation in space and time
- Personal and cultural expression
- Scientific and technical innovation
- Globalisation and sustainability
- Fairness and development

The Global Contexts help students to develop connections between different subject areas in the curriculum.

GLOBAL CONTEXT — GREAT EMPIRES

Global context: Orientation in space and time
Statement of inquiry: Mathematics can help give us an understanding of people, history, and culture.
Criterion: Applying mathematics in real-life contexts

GLOBAL CONTEXT

Click on the icon to access the online link.

Each project contains a series of questions, divided into:

- Factual questions (in green)
- Conceptual questions (in blue)
- Debatable questions (in red).

The projects are also accompanied by the general descriptor and a task-specific descriptor for one of the four assessment criteria.

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Chapter 1

Whole numbers

Contents:

OPENING PROBLEM

A **gigabyte** of data is $2^{30} = 1\,073\,741\,824$ bytes.

Things to think about:

a What does 2^{30} actually *mean*?

b What is the value of the 3 in 1 073 741 824?

c **i** How would you *approximate* the number of bytes in a gigabyte?
ii Can you describe what you are doing to the number when making this approximation?

Over the ages, different people have created their own **number systems** to help them count. The Ancient Egyptians, Romans, and Greeks all used different symbols for their numbers.

The number system we use today is called the **Hindu-Arabic** number system.

In this system, the **numerals** which represent numbers are formed using symbols called **digits**. The ten digits are 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0.

In this Chapter we study **whole numbers**. These include:

- the **counting numbers** 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ...
- the **natural numbers** 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ...

The dots "...." tell us that these sets go on forever. We say they are **infinite**.

A PLACE VALUE

One of the features of the Hindu-Arabic system which makes it efficient to use, is its **place value system**.

The **place** of a digit in a number determines its value.

Notice how the pattern of units, tens, and hundreds repeats in the thousands, millions, billions, and trillions.

units	1
tens	10
hundreds	100
thousands	1000
ten thousands	10 000
hundred thousands	100 000
millions	1 000 000
ten millions	10 000 000
hundred millions	100 000 000
billions	1 000 000 000
ten billions	10 000 000 000
hundred billions	100 000 000 000
trillions	1 000 000 000 000

⋮

Using the place value system, we can write a number in:

- numeral form 5 038 402
- expanded form 5 000 000 + 30 000 + 8000 + 400 + 2
- words five million, thirty eight thousand, four hundred and two
- a place value chart

millions			thousands			units		
H	T	U	H	T	U	H	T	U
		5	0	3	8	4	0	2

The digit **zero** or 0 is used to show an empty place value.

EXERCISE 1A

1 Write down the value of the digit 7 in:

a 47 **b** 76 **c** 217 **d** 475
e 1731 **f** 7200 **g** 3867 **h** 786 000
i 271 043 **j** 7 105 839 **k** 76 000 000 **l** 146 070

2 Write in numeral form:

a $60 + 5$ **b** $700 + 20 + 1$ **c** $6000 + 800 + 5$
d $20\,000 + 400 + 30$ **e** $100\,000 + 9000 + 80 + 4$
f $500\,000 + 6000 + 90 + 8$ **g** $7\,000\,000 + 2000 + 60 + 3$

3 Write in expanded form:

a 734 **b** 3928 **c** 21 080 **d** 638 409

4 Write in words:

a 36 **b** 405 **c** 6501 **d** 11 085
e 54 760 **f** 285 400 **g** 13 000 000 **h** 8 700 000 000

5 Write in numeral form:

a seventeen **b** one hundred and sixty four
c three hundred and twenty eight **d** eight hundred and ten
e two thousand, nine hundred and one
f five million, four hundred and two thousand, three hundred and ninety
g three hundred and five million **h** twelve billion, eight hundred million.

6 There are about one hundred trillion atoms in a human cell. Write this number in numeral form.

7 Write the quantities in ascending order:

a one hundred and three dollars, $130, $113
b Wendy 118 cm, Xiao 109 cm, Sarah 126 cm, Kylie 116 cm
c giraffe 674 kg, hippopotamus 1872 kg, elephant 3058 kg, rhinoceros 2156 kg
d Cologne 157 m, Rome 138 m, Milan 107 m, Salisbury 123 m
e £4100, fourteen pounds, four thousand pounds, forty thousand pounds, fourteen thousand pounds.

8 Write the smallest number you can using each of the digits 4, 2, 7, and 5 once only.

9 Write the largest number you can using each of the digits 6, 3, 0, 8, 2, and 5 once only.

ACTIVITY 1 THE ANCIENT GREEK OR ATTIC NUMBER SYSTEM

The Attic number system was used by the ancient Greeks from the 7th century BC to the 3rd century BC.

Like the Egyptian system before it, it used a tally system based on the number ten. However, there was an extra symbol for 5 which could be combined with the symbols for 10, 100, and 1000 to make 50, 500, and 5000.

By the 3rd century BC, these symbols were used:

1	5	10	50	100	500	1000	5000	10 000
		Γ	Δ	𐅄	H	𐅅	X	𐅆	M

The value of a number can be found by adding the values of the symbols used.

𐅆X 𐅅HH 𐅄ΔΔΔ | = 6781

6000 700 80 1

What to do:

1 Write down the number represented by:

a Γ||
b Δ|||
c ΔΓ
d ΔΔΓ|
e 𐅄Δ||||
f HHΔΓ
g MXXHHΔΔΔ
h 𐅅HHΔΔ|||
i 𐅆HH𐅄Γ||||

2 Write the Ancient Greek symbols for:

a 9 **b** 14 **c** 31 **d** 27 **e** 53 **f** 68
g 99 **h** 555 **i** 4082 **j** 5601 **k** 7264 **l** 20 628

3 Discuss the similarities and differences between the Attic number system and our number system. What features make our number system more efficient?

B ROUNDING NUMBERS

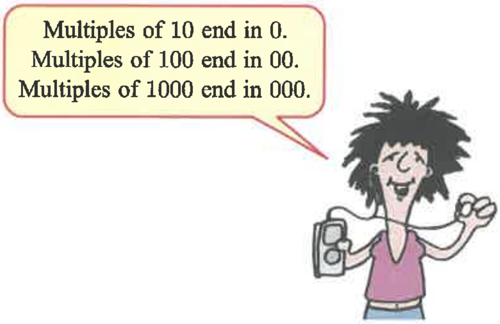
When a quantitiy is being described, we often do not need to know its *exact* value.

For example, the Salt Lake Stadium in India holds about 120 000 people. This *estimate* or *approximation* gives a good idea of the stadium's size, when the exact capacity is not known or not required.

To approximate a number of objects, we can **round** the number to a particular place value.

For example, to round a number to the nearest 100, we choose the multiple of 100 which is nearest to our number.

- 438 is nearer to 400 than to 500, so we round 438 *down* to 400.
- 467 is nearer to 500 than to 400, so we round 467 *up* to 500.
- 450 is midway between 400 and 500. We choose to round 450 *up* to 500.



To **round** to a particular place value, look at the digit in the place value to the right of it.

- If this digit is 0, 1, 2, 3, or 4, we round down.
- If this digit is 5, 6, 7, 8, or 9, we round up.

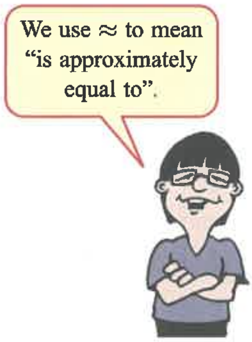
Example 1 | **Self Tutor**

Round:

a 769 to the nearest 10 **b** 6705 to the nearest 100.

a 769 is nearer to 770 than 760, so we round *up*.
$769 \approx 770$

b 6705 is nearer to 6700 than 6800, so we round *down*.
$6705 \approx 6700$



SIGNIFICANT FIGURES

We round to a number of **significant figures** if we believe this number of digits is important.

For example, the number 34 827 has 5 significant figures.

① ② ③ ④ ⑤
3 4 8 2 7

To round 34 827 to 2 significant figures, we notice that the digit in the second place represents *thousands*.

34 827 is nearer to 35 000 than 34 000, so we round *up*.

$34\,827 \approx 35\,000$ {to 2 significant figures}.

Example 2 | **Self Tutor**

Round:

a 3143 to 1 significant figure **b** 15 579 to 2 significant figures.

a 3143 is nearer to 3000 than 4000, so we round *down*.
$3143 \approx 3000$

b 15 579 is nearer to 16 000 than 15 000, so we round *up*.
$15\,579 \approx 16\,000$

EXERCISE 1B

1 Round to the nearest 10:

- **a** 62
- **b** 43
- **c** 68
- **d** 127
- **e** 99
- **f** 232
- **g** 305
- **h** 9995

2 Round to the nearest 100:

- **a** 412
- **b** 264
- **c** 91
- **d** 850
- **e** 905
- **f** 1952
- **g** 18 726
- **h** 25 870

3 Round to the nearest 1000:

- **a** 6218
- **b** 2324
- **c** 6587
- **d** 607
- **e** 13 500
- **f** 9866
- **g** 26 315
- **h** 254 430

4 Round to 1 significant figure:

- **a** 46
- **b** 205
- **c** 394
- **d** 467
- **e** 863
- **f** 1256
- **g** 8888
- **h** 49 580

5 Round to 2 significant figures:

- **a** 682
- **b** 206
- **c** 590
- **d** 173
- **e** 2019
- **f** 3862
- **g** 8973
- **h** 16 638

6 Round to 3 significant figures:

- **a** 1834
- **b** 26 765
- **c** 483 560
- **d** 3 620 715

7 Round to the accuracy given:

- **a** \$3165 (to the nearest \$100)
- **b** 349 g (to 1 significant figure)
- **c** 4621 m (to the nearest 100 m)
- **d** 67 891 people (to the nearest 1000)
- **e** \$26 990 (to 2 significant figures)
- **f** 695 flights (to the nearest 10).

8 In 2019, 46 983 runners participated in the Berlin Marathon.
Round this number to:

- **a** 2 significant figures
- **b** 3 significant figures
- **c** 4 significant figures.

DISCUSSION

Genevieve has noticed that:

- $6999 \approx 7000$ {to 1 significant figure}
- $6999 \approx 7000$ {to 2 significant figures}
- $6999 \approx 7000$ {to 3 significant figures}

Discuss whether each "0" is a significant figure, or whether it is only present as a place holder.

C OPERATIONS

The four basic **operations** are **addition**, **subtraction**, **multiplication**, and **division**.

ADDITION

To find the **sum** of two or more numbers, we **add** them together.

Zero is the **identity** for addition. When 0 is added to any number, the number remains the same.

For example, $38 + 0 = 0 + 38 = 38$.

SUBTRACTION

When we **subtract** a number, we take it away from what was there previously.

To find the **difference** between two numbers, we *subtract* the smaller number from the larger number.

When we subtract **zero** from a number, the number does not change.

For example, $38 - 0 = 38$.

MULTIPLICATION

To find the **product** of two or more numbers, we **multiply** them.

One is the **identity** for multiplication. When a number is multiplied by 1, the number remains the same.

When 0 is multiplied by any number, the product is 0.

For example:

- $17 \times 1 = 1 \times 17 = 17$
- $17 \times 0 = 0 \times 17 = 0$

DIVISION

To find the **quotient** of two numbers, we **divide** the first number by the second number.

The number being divided is called the **dividend**, and the number we are dividing by is called the **divisor**.

For example, the quotient of 84 and 12 is $84 \div 12$, which is 7.

$$\underset{\text{dividend}}{84} \div \underset{\text{divisor}}{12} = \underset{\text{quotient}}{7}$$

When a number is divided by 1, the number remains the same.

When 0 is divided by any non-zero number, the result is 0.

It is meaningless to divide a number by 0. We say the result is **undefined**.

For example:

- $17 \div 1 = 17$
- $0 \div 17 = 0$
- $17 \div 0$ is undefined.

EXERCISE 1C

1 Find the sum:

a $7+9$ **b** $11+6$ **c** $18+0$
d $6+3+5$ **e** $0+32$ **f** $7+9+4$

2 Find:

a $11-8$ **b** $27-0$ **c** $16-7$
d $28-12$ **e** $26-19-0$ **f** $73-20$

3 Find the sum of:

a 5, 8, and 3 **b** the first six natural numbers.

4 Find the difference between:

a 8 and 13 **b** 23 and 11 **c** 14 and 22.

5 Find the product:

a 5×9 **b** 8×3 **c** 6×1 **d** 4×12
e 11×0 **f** 3×14 **g** 0×157 **h** 8×8
i 4×11 **j** 11×4 **k** $9\times 1\times 6$ **l** $6\times 0\times 9$
m $2\times 5\times 4$ **n** $3\times 5\times 2$ **o** $4\times 6\times 3$ **p** $3\times 2\times 5\times 5$

6 Find the quotient:

a $21\div 7$ **b** $45\div 5$ **c** $36\div 4$ **d** $29\div 1$
e $0\div 5$ **f** $88\div 8$ **g** $0\div 1$ **h** $56\div 7$
i $0\div 93$ **j** $132\div 12$ **k** $7\div 0$ **l** $96\div 8$

7 Find the product of:

a the first four counting numbers **b** the first hundred natural numbers.

8 My mum gave me twelve \$5 notes for my birthday. How much did she give me in total?

9 Edward shared 56 lollies between 7 people. How many did each person receive?

D EXPONENT NOTATION

A convenient way to write a product of *identical numbers* is to use **exponent notation**.

For example, we can write $4\times 4\times 4$ as 4^3.

The 4 is called the **base**.

The 3 is called the **exponent**, **index**, or **power**. It is the number of times the base appears in the product.

$$4^3$$

exponent, index, or power

base

The following table demonstrates correct language when talking about exponent notation:

Natural number	*Expanded form*	*Exponent notation*	*Spoken form*
3	3	3^1	3 *or* 3 to the power 1
9	3×3	3^2	3 squared *or* 3 to the power 2
27	$3 \times 3 \times 3$	3^3	3 cubed *or* 3 to the power 3
81	$3 \times 3 \times 3 \times 3$	3^4	3 to the fourth *or* 3 to the power 4
243	$3 \times 3 \times 3 \times 3 \times 3$	3^5	3 to the fifth *or* 3 to the power 5

Example 3 Self Tutor

Write using exponent notation:

a $8 \times 8 \times 8 \times 8$ **b** $2 \times 2 \times 2 \times 3 \times 3 \times 5$

a $8 \times 8 \times 8 \times 8$
$= 8^4$

b $\underbrace{2 \times 2 \times 2}_{2^3} \times \underbrace{3 \times 3}_{3^2} \times \underbrace{5}_{5^1 \text{ or } 5}$
$= 2^3 \times 3^2 \times 5$

EXERCISE 1D

1 Match each number in exponent notation with the correct product:

a $5^3 \times 7$ **A** $5 \times 5 \times 7 \times 7 \times 7$
b 5^4 **B** $7 \times 7 \times 7 \times 7$
c $5^2 \times 7^3$ **C** $5 \times 5 \times 5 \times 7$
d $5^2 \times 7^2$ **D** $5 \times 5 \times 5 \times 5$
e 7^4 **E** $5 \times 5 \times 7 \times 7$

2 Write using exponent notation:

a 4×4 **b** $6 \times 6 \times 6$ **c** $11 \times 11 \times 11 \times 11$
d $2 \times 3 \times 3$ **e** $2 \times 2 \times 3 \times 5$ **f** $2 \times 5 \times 5 \times 5$
g $3 \times 3 \times 5 \times 5 \times 5$ **h** $2 \times 2 \times 2 \times 5 \times 7$ **i** $3 \times 3 \times 3 \times 7 \times 7$
j $3 \times 3 \times 3 \times 3 \times 5 \times 5$ **k** $7 \times 7 \times 7 \times 7 \times 7 \times 11 \times 11 \times 11$

3 Write in expanded form:

a 2^3 **b** 3^6 **c** $2^5 \times 3^2$ **d** $5^2 \times 7$ **e** 2×3^4 **f** $2^2 \times 3 \times 5^3$

Example 4 Self Tutor

Evaluate: **a** 10^4 **b** $2^3 \times 3^2$

a 10^4
$= 10 \times 10 \times 10 \times 10$
$= 10\,000$

b $2^3 \times 3^2$
$= \underbrace{2 \times 2 \times 2}_{8} \times \underbrace{3 \times 3}_{9}$
$= 8 \times 9$
$= 72$

4 Evaluate:

a 2^4 **b** 7^2 **c** 2^5
d 2×5^2 **e** 5×10^3 **f** $2^2 \times 10^2$
g $3^2 \times 5$ **h** $2^2 \times 3^2$ **i** $2 \times 3^2 \times 10$
j $2^3 \times 5 \times 7$ **k** $3^2 \times 7 \times 10^2$ **l** $2^2 \times 3 \times 10^3$

5 Use your calculator to evaluate:

a $2^4 \times 3^6$ **b** $2^2 \times 5^4 \times 7^5$ **c** $2^5 \times 3^3 \times 11^2$
d $2^3 \times 3^4 \times 5^2 \times 11$ **e** $3^4 \times 7^2 \times 11^3$ **f** $2^3 \times 5^5 \times 13^3$

Example 5 **Self Tutor**

Write in exponent form with 5 as a base:

a 5 **b** 25 **c** 125 **d** 625

a $5 = 5^1$

b $25 = 5 \times 5 = 5^2$

c $125 = 5 \times 5 \times 5 = 5^3$

d $625 = 5 \times 5 \times 5 \times 5 = 5^4$

6 Write in exponent form with 2 as a base:

a 2 **b** 4 **c** 16 **d** 64

7 Write in exponent form with 3 as a base:

a 3 **b** 27 **c** 81 **d** 729

8 Write in exponent form with 10 as a base:

a 100 **b** 1000 **c** 100 000 **d** 10 000 000

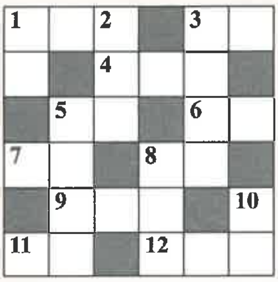
9 Write in exponent form with 11 as a base:

a 11 **b** 121 **c** 1331 **d** 14 641

10 A **googol** is the number 10^{100}. If you wrote a googol as a numeral, how many zeros would you use?

ACTIVITY 2 EXPONENT NOTATION CROSSWORD

Click on the icon to obtain a printable version of this crossword.



Across

1	19^2	8	9^2
3	2^4	9	22^2
4	4^4	11	4^3
5	5^2	12	13^2
6	3^4		
7	3^3		

Down

1	6^2
2	5^3
3	41^2
5	14^3
8	29^2
10	7^2

CROSSWORD

GLOBAL CONTEXT — THE BASE OF A NUMBER SYSTEM

Global context: Personal and cultural expression
Statement of inquiry: Knowing the base of a number system can help us understand how numbers are represented.
Criterion: Knowing and understanding

GLOBAL CONTEXT

QUICK QUIZ

MULTIPLE CHOICE QUIZ

REVIEW SET 1A

1 Write 9602 in words.

2 Write down the value of the digit 7 in 17 260.

3 Write the largest number you can using the digits 1, 5, 9, 6, 0, and 2 once only.

4 Round 49 552 to the nearest:

a 10 **b** 100 **c** 1000

5 A website has had 35 942 views in the past year. Round this number to 2 significant figures.

6 Find the difference between:

a 21 and 12 **b** 16 and 30.

7 Find the product:

a 9×0 **b** 7×11 **c** $2 \times 6 \times 8$

8 Sam shared 72 lollies equally between 9 friends. How many lollies did each friend receive?

9 Write using exponent notation:

a $2 \times 2 \times 2 \times 3$ **b** $3 \times 3 \times 5 \times 5 \times 5 \times 5$

10 Evaluate:

a 3×10^2 **b** $2^3 \times 5$ **c** $3^2 \times 5 \times 10^3$

11 Write in exponent form with 4 as a base:

a 4 **b** 16 **c** 64

12 **a** Use your calculator to evaluate:

i 9^2 **ii** 9^3 **iii** 9^4 **iv** 9^5 **v** 9^6

b Hence predict the last digit of:

i 9^{45} **ii** 9^{60}

REVIEW SET 1B

1 Write in expanded form:

a 384 **b** 7902 **c** 60 150 **d** 8 050 071

2 Write in numeral form:

a three thousand and sixty eight
b four million, seven hundred thousand, two hundred and nine
c fifty billion, seventy six million, three hundred thousand.

3 Write these quantities in ascending order:
$217, two hundred and seven dollars, $277, two hundred and seventy dollars.

4 Round to 2 significant figures:

a 573 **b** 37 193 **c** 4239 **d** 119 603

5 The distance from Perth to London is 14 473 km. Round this distance to:

a the nearest 1000 km **b** 3 significant figures.

6 Find the sum:

a $17 + 14$ **b** $0 + 28$ **c** $9 + 6 + 8$

7 Find the quotient:

a $64 \div 8$ **b** $67 \div 1$ **c** $0 \div 73$

8 Victoria ran 6 km each day for 8 consecutive days. How far did she run in total?

9 Write in expanded form:

a 3^4 **b** $2^5 \times 7^2$ **c** $2^4 \times 3^3 \times 5$

10 Use your calculator to evaluate:

a $2^6 \times 3^2$ **b** $3^2 \times 5 \times 7^3$ **c** $3^4 \times 5^2 \times 11^3$

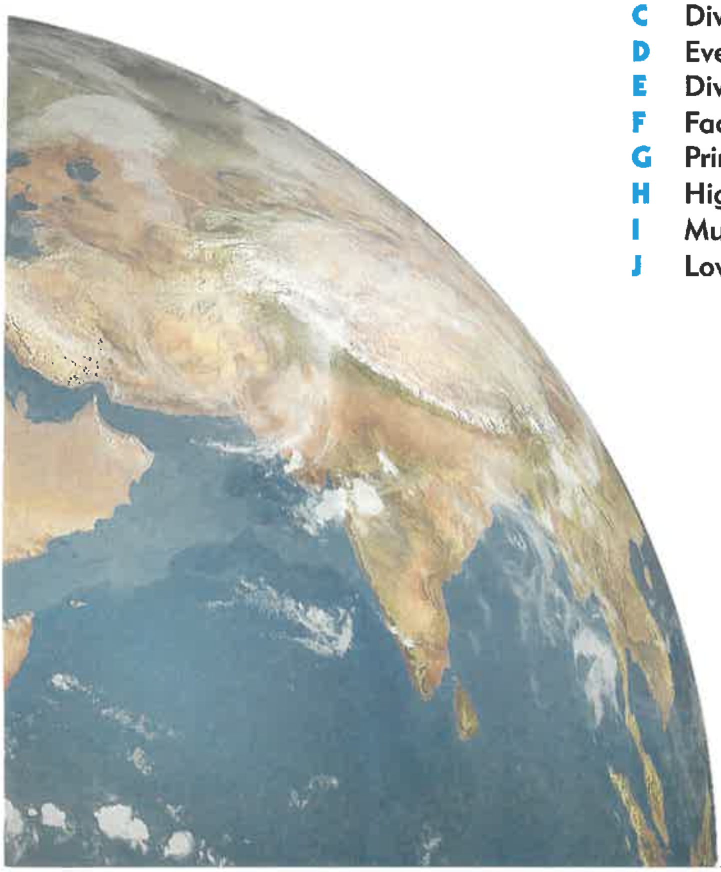
11 Answer the **Opening problem** on page **12**.

12 Kareem bought 7 packets of dog biscuits on Monday, and 5 packets on Tuesday.

a How many packets of dog biscuits did Kareem buy in total?
b Each packet contains 10 biscuits. How many biscuits did he buy?
c The biscuits are shared equally between Kareem's 4 dogs. How many biscuits did each dog receive?

Chapter 2

Number properties



Contents:

OPENING PROBLEM

A school building has 100 lockers. The lockers are lined up in a row, all closed.

Consider the following scenario:

- Eddie goes down the row and opens every locker.
- Terry then goes down the row and closes every *second* locker, starting with locker number 2.
- Nick then goes down the row and *changes the state* of every *third* locker, starting with locker number 3. So, if the locker is closed he opens it, and if the locker is open he closes it.

Things to think about:

a How can we *describe* the lockers that:

 i Terry closes **ii** Nick changes?

b How many lockers are touched by *both* Terry and Nick? Which lockers are they?

c After the three students have passed, how many lockers are left open?

d Imagine this scenario continues for 100 students. The fourth person changes the state of every fourth locker, starting with locker number 4, and so on.

 i Which lockers will be left *open*? **ii** What is special about these numbers?

A SQUARE NUMBERS

The product of two identical whole numbers is called a **square number** or **perfect square**.

We call it a square number because it can be represented by a square array of dots.

For example:

"one squared"

$1^2 = 1 \times 1 = 1$

"two squared"

$2^2 = 2 \times 2 = 4$

"three squared"

$3^2 = 3 \times 3 = 9$

"four squared"

$4^2 = 4 \times 4 = 16$

Since $3^2 = 9$, we say that 3 is the **square root** of 9.

We write square roots using the symbol $\sqrt{}$.

For example, $\sqrt{9} = 3$ and $\sqrt{36} = 6$.

Notice that $\sqrt{9} \times \sqrt{9} = 9$ and $\sqrt{36} \times \sqrt{36} = 36$.

To use the square root function on your calculator, you may need to first press a key such as 2nd or SHIFT .

EXERCISE 2A

1 Copy and complete:

Number	1	2	3	4	5	6	....	20
Perfect square	1	4						

Try to memorise these perfect squares.

2 Use a calculator to find the value of:

a 25^2 **b** 32^2 **c** 83^2

3 Write down the perfect squares between 50 and 150.

4 Use your calculator to find the:

a largest 3 digit square number **b** smallest 4 digit square number.

5 Find, using a calculator if necessary:

a $\sqrt{4}$ **b** $\sqrt{16}$ **c** $\sqrt{49}$ **d** $\sqrt{81}$

e $\sqrt{100}$ **f** $\sqrt{121}$ **g** $\sqrt{0}$ **h** $\sqrt{10\,000}$

i $\sqrt{289}$ **j** $\sqrt{625}$ **k** $\sqrt{1024}$ **l** $\sqrt{9801}$

6 Copy and complete:

a $\sqrt{25} = ...$
$\sqrt{25} \times \sqrt{25} = ...$

b $\sqrt{64} = ...$
$\sqrt{64} \times \sqrt{64} = ...$

c $\sqrt{144} = ...$
$\sqrt{144} \times \sqrt{144} = ...$

d $\sqrt{196} = ...$
$\sqrt{196} \times \sqrt{196} = ...$

7 **a** Use a calculator to complete the following:

$1^2 = ...$
$11^2 = ...$
$111^2 = ...$
$1111^2 = ...$

b By observing the pattern, find the following *without* using a calculator:

i $11\,111^2$ **ii** $111\,111^2$

8 Consider the pattern:

$1 \times 3 + 1 = ...$
$2 \times 4 + 1 = ...$
$3 \times 5 + 1 = ...$
$4 \times 6 + 1 = ...$

a Copy and complete the pattern, then add three more rows.

b Use the pattern to find:

i $19 \times 21 + 1$ **ii** 29×31

9 **a** If a number is doubled, what happens to its square?

b Can you explain *why* this happens?

10 Explain why 2 679 430 077 712 313 cannot be a perfect square.

B CUBIC NUMBERS

The product of three identical whole numbers is called a **cubic number** or **perfect cube**.

We call it a cubic number because it can be represented by a cubic array of blocks.

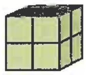
For example:

"1 cubed"

$1^3 = 1 \times 1 \times 1 = 1$

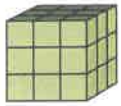
"2 cubed"

$2^3 = 2 \times 2 \times 2 = 8$



"3 cubed"

$3^3 = 3 \times 3 \times 3 = 27$



Since $3^3 = 27$, we say that 3 is the **cube root** of 27.

We write cube roots using the symbol $\sqrt[3]{\ }$.

For example, $\sqrt[3]{8} = 2$ and $\sqrt[3]{27} = 3$.

Notice that $\sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8} = 8$ and $\sqrt[3]{27} \times \sqrt[3]{27} \times \sqrt[3]{27} = 27$.

EXERCISE 2B

1 Copy and complete:

Number	1	2	3	4	5	6	7	8	9	10
Perfect cube	1	8								

2 Find:

a 13^3 **b** 20^3 **c** 100^3 **d** 1000^3

3 How many cubic numbers are less than 10 000?

4 Find two consecutive numbers such that one number is a perfect square and the other is a perfect cube.

5 Use your calculator to find the largest 5 digit cubic number.

6 Find, using a calculator if necessary:

a $\sqrt[3]{1}$ **b** $\sqrt[3]{64}$ **c** $\sqrt[3]{216}$ **d** $\sqrt[3]{729}$

7 Copy and complete:

a $\sqrt[3]{125} = ...$
$\sqrt[3]{125} \times \sqrt[3]{125} \times \sqrt[3]{125} = ...$

b $\sqrt[3]{343} = ...$
$\sqrt[3]{343} \times \sqrt[3]{343} \times \sqrt[3]{343} = ...$

8 **a** Copy and complete:

$$\begin{array}{rlll}
1^3 &= 1 & = 1 & = 1^2 \\
1^3 + 2^3 &= 1 + 8 & = 9 & = (1+2)^2 \\
1^3 + 2^3 + 3^3 &= & = & = \\
1^3 + 2^3 + 3^3 + 4^3 &= & = & =
\end{array}$$

b Predict the value of:

i $1^3 + 2^3 + 3^3 + 4^3 + 5^3$ **ii** $1^3 + 2^3 + 3^3 + ... + 10^3$

Check your answers using a calculator.

9 Find the smallest cubic number which can be written as the sum of three consecutive cubic numbers.

C DIVISIBILITY

One whole number is **divisible** by another if, when we divide, the quotient is a whole number.

For example: 16 is divisible by 2 because $16 \div 2 = 8$.
16 is not divisible by 3 because $16 \div 3 = 5$ with remainder 1.

EXERCISE 2C

1 Decide whether:

a 22 is divisible by 2 **b** 27 is divisible by 3 **c** 28 is divisible by 4
d 17 is divisible by 3 **e** 31 is divisible by 5 **f** 54 is divisible by 9
g 70 is divisible by 10 **h** 76 is divisible by 8 **i** 132 is divisible by 11.

2 Find the number between 90 and 100 which is divisible by 8.

3 Find the smallest number greater than 100 which is divisible by 7.

4 Decide whether each statement is true or false. Explain your answers.

a Every counting number is divisible by 1.
b Every perfect square is divisible by its square root.

D EVEN AND ODD NUMBERS

A natural number is **even** if it is divisible by 2.
A natural number is **odd** if it is not divisible by 2.

For example: 18 is even because $18 \div 2 = 9$.
23 is odd because $23 \div 2 = 11$ with remainder 1.

EXERCISE 2D

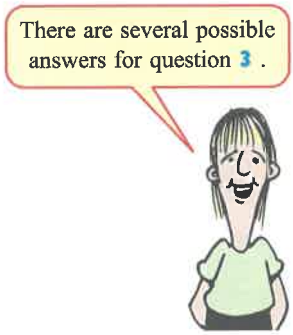
1 Decide whether each number is even or odd:

a 16 **b** 17 **c** 45 **d** 38
e 52 **f** 89 **g** 130 **h** 143

2 Write down the numbers between 20 and 40 which are:

a even and divisible by 3 **b** odd and divisible by 11
c even and not divisible by 4 **d** odd and not divisible by 3
e even and perfect squares **f** odd and perfect cubes.

3 Write the number 60 as:
- **a** the sum of two even numbers
- **b** the sum of two odd numbers
- **c** the product of two even numbers
- **d** the product of an even number and an odd number.



4 Decide whether the following are even or odd:
- **a** the sum of three even numbers
- **b** the sum of three odd numbers
- **c** the product of an even and two odd numbers
- **d** the sum of four odd numbers.

5 **a** Copy and complete:

$$1 = 1 = 1^2$$
$$1 + 3 = 4 = 2^2$$
$$1 + 3 + 5 = 9 = 3^2$$
$$1 + 3 + 5 + 7 =$$
$$1 + 3 + 5 + 7 + 9 =$$

b Use the pattern to find the sum of the first:
- **i** 6 odd numbers
- **ii** 10 odd numbers.

E DIVISIBILITY TESTS

There are tests we can use to determine whether one number is divisible by another, without actually doing the division.

For example, if the last digit of a number is even, then the number is divisible by 2.

INVESTIGATION 1 DIVISIBILITY BY 4 AND 9

In this Investigation you should discover tests for divisibility by 4 and by 9.

What to do:

1 **a** Copy and complete the table below. Start with the *third* column by writing down the last two digits of each number. Then use your calculator to check the numbers for divisibility by 4.

Number	*Divisible by 4?*	*Last 2 digits*	*Number formed by last 2 digits divisible by 4?*
81		81	
154		54	
774			
3624			
6957			
9908			

b Copy and complete: "A natural number is divisible by 4 if ......".

2 **a** Copy and complete this table:

Number	*Divisible by 9?*	*Sum of its digits*
81		$8+1=9$
154		
774		
3624		
6957		
9908		

b Copy and complete: "A natural number is divisible by 9 if ......".

Number	*Divisibility test*
2	If the last digit is even, then the number is divisible by 2.
3	If the sum of the digits is divisible by 3, then the number is divisible by 3.
4	If the number formed by the last *two* digits is divisible by 4, then the original number is divisible by 4.
5	If the last digit is 0 or 5, then the number is divisible by 5.
6	If the number is divisible by both 2 and 3, then it is divisible by 6.
9	If the sum of the digits is divisible by 9, then the number is divisible by 9.
10	If the last digit is 0, then the number is divisible by 10.
11	Add the digits in odd positions. Add the digits in the even positions. Find the difference between your two answers. If the difference is divisible by 11, the original number is divisible by 11.

Example 1 **Self Tutor**

Test for divisibility by 3 and 11:

a 846 **b** 2618

a The sum of the digits of 846 is $8+4+6=18$.
Since 18 is divisible by 3, so is 846.
In 846, the sum of the digits in the odd positions is $8+6=14$ and the sum of the digits in the even positions is 4.
The difference is $14-4=10$, which is not divisible by 11.
$\therefore$ 846 is not divisible by 11.

b The sum of the digits of 2618 is $2+6+1+8=17$.
Since 17 is not divisible by 3, 2618 is not divisible by 3.
In 2618, the sum of the digits in the odd positions is $2+1=3$ and the sum of the digits in the even positions is $6+8=14$.
The difference is $14-3=11$, which is divisible by 11.
$\therefore$ 2618 is divisible by 11.

$\therefore$ means "therefore".

EXERCISE 2E

1 Determine whether each number is divisible by 3:

a 87 **b** 512 **c** 977 **d** 1002 **e** 56 947 **f** 123 456 789

2 Determine whether each number is divisible by 4:

a 2250 **b** 1024 **c** 30 420 **d** 215 962

3 Determine whether each number is divisible by 9:

a 801 **b** 2763 **c** 3079 **d** 269 730

4 Determine whether each number is divisible by 11:

a 596 **b** 7282 **c** 10 837 **d** 908 281

5 Answer true or false for the following:

a 26 is divisible by 2 **b** 5221 is divisible by 5 **c** 127 is divisible by 3
d 1010 is divisible by 10 **e** 1900 is divisible by 4 **f** 1326 is divisible by 3
g 111 is divisible by 2 **h** 166 is divisible by 9 **i** 9288 is divisible by 9
j 247 is divisible by 11 **k** 5922 is divisible by 6 **l** 5071 is divisible by 11.

6 Test each number for divisibility by 2, 3, 4, 5, and 9:

a 250 **b** 3609 **c** 12 345 **d** 14 641

7 A four-digit number with digit form □5□1 is divisible by 3. What possible values could the *sum* of the unknown digits have?

8 Consider the five-digit number 82 51□. What digits could replace □ so that the number is divisible by:

a 3 **b** 4 **c** 5 **d** 6 **e** 9 **f** 11?

9 **a** Rearrange the digits 1, 4, 5, and 8 to form a number which is divisible by:

i 5 **ii** 4

b Explain why every rearrangement of the digits in **a** will form a number which is divisible by 9.

10 Simone's teacher told her that 1738 is divisible by 11. Simone noticed that reversing the digits of this number gives 8371, which is also divisible by 11.
Explain why reversing the digits of *any* number which is divisible by 11 gives a number which is also divisible by 11.

ACTIVITY 1 DELECTABLE NUMBERS

A number is called **delectable** if the number formed by its first n digits is always divisible by n.

For example, 4236 is a delectable number because
the number formed by the first digit (4) is divisible by 1,
the number formed by the first 2 digits (42) is divisible by 2,
the number formed by the first 3 digits (423) is divisible by 3,
and the number formed by the first 4 digits (4236) is divisible by 4.

2052 is *not* a delectable number because, although 2 is divisible by 1, and 20 is divisible by 2, 205 is *not* divisible by 3.

What to do:

1 Decide whether each number is delectable:

a 63 **b** 945 **c** 7236 **d** 5222 **e** 34 245

2 How many three-digit delectable numbers can you make using the digits 1, 2, and 3 once each?

3 Show that it is impossible to make a four-digit delectable number using the digits 1, 2, 3, and 4 once each.

4 There is only one nine-digit delectable number which can be made using the digits 1 to 9 once each. Can you find it?
Hint: The first digit is 3.

F FACTORS

The **factors** of a natural number are the natural numbers which divide exactly into it.

For example:

- $32 \div 4 = 8$, so 4 is a factor of 32.
- $32 \div 8 = 4$, so 8 is a factor of 32.
- $32 \div 7 = 4$ remainder 4, so 7 is *not* a factor of 32.

We can write 32 as 4×8 where 4 and 8 are both factors of 32. We say that 4 and 8 are a **factor pair**.

When a number is written as a product of factors, we say it is **factorised**.

Example 2 Self Tutor

Write down the factor pairs of 12.

Hence list the factors of 12.

We can write 12 as 1×12, 2×6, or 3×4.

$\therefore$ the factors of 12 are 1, 2, 3, 4, 6, and 12.

EXERCISE 2F

1 **a** Is 8 a factor of 24? **b** Is 5 a factor of 43?
c Is 7 a factor of 34? **d** Is 9 a factor of 72?

2 List the factors of:

a 8 **b** 16 **c** 20 **d** 22 **e** 25

3 Copy and complete each factor pair:

a $24 = 6 \times ......$ **b** $28 = 4 \times ......$ **c** $88 = 11 \times ......$
d $100 = 5 \times ......$ **e** $143 = 11 \times ......$ **f** $91 = 13 \times ......$

4 Write down the factor pairs of 36. Hence list the factors of 36.

5 List the factors of:

a 28 **b** 29 **c** 30 **d** 32 **e** 44 **f** 49
g 50 **h** 56 **i** 63 **j** 65 **k** 75 **l** 76

6 Write down the largest factor (other than itself) of:

a 14 **b** 27 **c** 55 **d** 70 **e** 81 **f** 90

7 Write down the smallest number which has factors:

a 1, 2, 3, and 4 **b** 2, 3, and 5 **c** 3, 5, and 7 **d** 2, 3, 5, and 7.

8 A number has six factors. Two of its factors are 9 and 21. Find the number.

9 **a** How many different factors does each number have?

i 4 **ii** 9 **iii** 25 **iv** 100

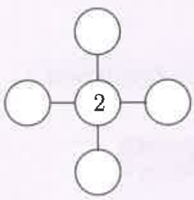
b What numbers have an odd number of factors?

c *Explain* why this happens.

PUZZLE — CROSS-PRODUCTS

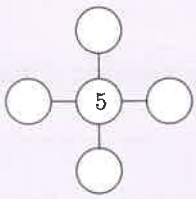
In each diagram you must put a different number from 1 to 10 in each circle. The product of the three numbers going across and the product of the three numbers going down must equal the product given below.

a



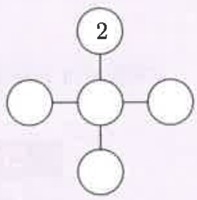
product 48

b



product 60

c



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G PRIME AND COMPOSITE NUMBERS

A **prime** number is a natural number which has exactly two different factors, 1 and itself.

A **composite** number is a natural number which has more than two different factors.

For example:

- 7 is a prime number because its only factors are 1 and 7.
- 12 is a composite number because it has six factors: 1, 2, 3, 4, 6, and 12.

The number 1 is special since its only factor is itself.

The number 1 is neither prime nor composite.

Every natural number greater than 1 is either prime or composite.

Apart from order, every composite number can be written as the **product of prime factors** in **one and only one way**.

When we write a number as the product of prime factors, we call this a **prime factorisation** and say the number is written in **prime factored form**.

For example, the prime factorisation of 12 is $2 \times 2 \times 3$ or $2^2 \times 3$.

There are two methods we can use to write a composite number as the product of prime factors:

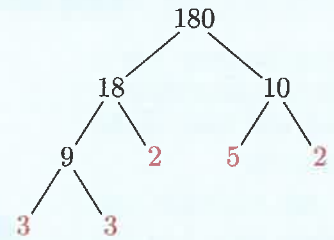
- In **repeated division**, we systematically divide the number by prime numbers which are its factors, starting with the smallest.
- In a **factor tree**, we find a factor pair for the number and use these factors as branches of the tree. We continue finding factor pairs for each branch until we are only left with prime numbers.

Example 3 Self Tutor

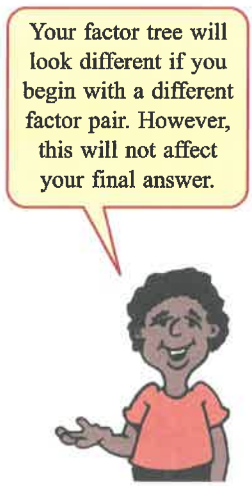
Write 180 in prime factored form.

Repeated division *or* **Factor tree**

$$\begin{array}{r|r} 2 & 180 \\ \hline 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$



$\therefore$ $180 = 3 \times 3 \times 2 \times 5 \times 2$
$= 2^2 \times 3^2 \times 5$



EXERCISE 2G

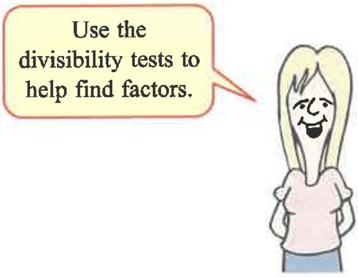
1. List the prime numbers less than 50.
2. Explain why:
   a. 1 is not a prime number
   b. there is only one even prime number.
3. Find:
   a. the smallest odd prime
   b. the only odd two-digit composite number less than 20
   c. a prime number which is a factor of 20, 30, and 105
   d. a number with two identical digits whose product is *not* composite.

4 Show that each number is composite:

a 6485 **b** 9320 **c** 2222 **d** 4279

5 Use repeated division to write as the product of prime factors in exponent form:

a 28 **b** 27 **c** 84
d 160 **e** 216 **f** 528
g 784 **h** 138 **i** 250
j 189 **k** 726 **l** 9625



6 Copy and complete these factor trees:

a 84 → 4, 21

b 120 → 10, ...

c 675 → 5, ...

7 Use a factor tree to write as the product of prime factors in exponent form:

a 24 **b** 70 **c** 63 **d** 72
e 225 **f** 88 **g** 480 **h** 1024

8 Benny thinks he has found *two* ways of writing the number 16 in prime factored form:

$$16 = 2^4 \qquad 16 = 4^2$$

Explain why Benny is wrong.

INVESTIGATION 2 THE SIEVE OF ERATOSTHENES

Eratosthenes (pronounced Er-ra-toss-tha-nees) was a Greek mathematician and geographer who lived between 275 BC and 194 BC. He is credited with many useful mathematical discoveries and calculations.

Eratosthenes found a method for "sieving" out composite numbers from the numbers from 1 to 100 to leave only the primes.

His method was:

- Cross out 1.
- Cross out all even numbers except 2.
- Cross out all multiples of 3 except 3.
- Cross out all multiples of 5 except 5.
- Cross out all multiples of 7 except 7.
- Continue this process, each time crossing out the multiples of the smallest uncrossed number except the number itself.

What to do:

PRINTABLE TABLE

1 Print the table of numbers from 1 to 100. Use Eratosthenes' method to find the prime numbers between 1 and 100.

2 Copy and complete this table:

Set of numbers	*Total number of prime numbers*
1 to 10	
11 to 20	
21 to 30	
⋮	
91 to 100	

3 Discuss whether there is any pattern in the appearance of prime numbers.

ACTIVITY 2 THE 1000 POINT WORD

Rachel decided to allocate points to different words. She first converted each letter into a number using $A \to 1$, $B \to 2$, ..., $Z \to 26$. The product of the numbers was her score for each word.

Using Rachel's method, H E L L O has the value
$8 \times 5 \times 12 \times 12 \times 15 = 86\,400$ points.

What to do:

1 Find the points value for the word:
a BED **b** MILK **c** JUMP

2 Find the points value for your name.

3 Explain why any word containing a "J" will have a points value ending in 0.

4 What can you say about a word containing a "D" and two "E"s?

5 Rachel is particularly interested in words whose value is exactly 1000 points.
a Explain why it is impossible for such a word to contain the letter "G".
b Make a list of the possible letters that a 1000 point word could contain.
c Rachel found that B E A D Y is a 1000 point word, since
$2 \times 5 \times 1 \times 4 \times 25 = 1000$.
Can you find another one?

ACTIVITY 3 GOLDBACH'S "GOLDEN RULES"?

In 1742, **Christian Goldbach** suggested two "golden rules":

- Every even number greater than 4 can be written as the sum of **two odd primes**.
- Every odd number greater than 8 can be written as the sum of **three odd primes**.

What to do:

1 Complete the following table to test Goldbach's first "rule".

$6 = 3 + 3$	$8 = 3 + 5$	$10 =$	$12 =$	$14 =$
$16 =$	$18 =$	$20 =$	$22 =$	$24 =$
$26 =$	$28 =$	$30 =$	$32 =$	$34 =$
$36 =$	$38 =$	$40 =$	$42 =$	$44 =$
$46 =$	$48 =$	$50 =$	$52 =$	$54 =$

2 Complete the following table to test Goldbach's second "rule".

$9 = 3 + 3 + 3$	$11 = 3 + 3 + 5$	$13 =$	$15 =$	$17 =$
$19 =$	$21 =$	$23 =$	$25 =$	$27 =$
$29 =$	$31 =$	$33 =$	$35 =$	$37 =$
$39 =$	$41 =$	$43 =$	$45 =$	$47 =$
$49 =$	$51 =$	$53 =$	$55 =$	$57 =$

3 Discuss your results.

H HIGHEST COMMON FACTOR

The **highest common factor** or **HCF** of two or more numbers is the largest factor which is common to all of them.

For small numbers, we can find the HCF mentally or by listing the factors.

For example: The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.

The factors of 40 are 1, 2, 4, 5, 8, 10, 20, 40.

So, the HCF of 24 and 40 is 8.

For larger numbers, we can find the HCF by first writing each number as a product of primes.

Example 4 Self Tutor

Find the HCF of 132 and 360.

2	132
2	66
3	33
11	11
	1

2	360
2	180
2	90
3	45
3	15
5	5
	1

$132 = 2 \times 2 \times 3 \times 11$

$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$2 \times 2 \times 3$ is common to the factorisations.

So, the HCF of 132 and 360 is $2 \times 2 \times 3 = 12$.

EXERCISE 2H

1 Find the HCF of:

a 12 and 15 **b** 9 and 12 **c** 21 and 35 **d** 16 and 56

e 20 and 36 **f** 18 and 45 **g** 40 and 46 **h** 44 and 110.

2 Find the HCF of:

a 4, 6, and 8 **b** 9, 12, and 18 **c** 24, 32, and 40.

3 Find the HCF of:

a 81 and 108 **b** 135 and 315 **c** 144 and 196 **d** 180 and 324.

4 A tiramisu is 56 cm long and 42 cm wide. It will be cut up into square pieces of equal size. Find the largest size that the pieces could be.

5 A hardware store sells nails in packets which all contain the same number of nails. Ron bought a total of 320 nails and Tess bought a total of 200 nails. Find the largest possible number of nails that could be in each packet.

6 Pradeep sells pappadums in boxes which all contain the same number of pappadums.
He sold 150 pappadums on Monday, 240 pappadums on Tuesday, and 195 pappadums on Wednesday. Find the largest number of pappadums that each box could contain.

I MULTIPLES

The **multiples** of any natural number have that number as a factor. They are obtained by multiplying it by 1, then 2, then 3, then 4, and so on.

The multiples of 10 are:	10,	20,	30,	40,	50,	....
	↑	↑	↑	↑	↑	
	10×1	10×2	10×3	10×4	10×5	
The multiples of 15 are:	15,	30,	45,	60,	75,	....
	↑	↑	↑	↑	↑	
	15×1	15×2	15×3	15×4	15×5	

The number 30 is a multiple of both 10 and 15, so we say 30 is a **common multiple** of 10 and 15.

EXERCISE 2I

1 List the first six multiples of:

a 6 **b** 8 **c** 13 **d** 15 **e** 25

2 Find the:

a seventh multiple of 6
b ninth multiple of 11
c eleventh multiple of 15
d hundredth multiple of 99.

3 Find the:

a smallest multiple of 7 that is greater than 500
b largest multiple of 12 that is less than 1000
c multiples of 9 between 60 and 110
d smallest multiple of 13 which is a perfect square.

4 List the numbers from 1 to 30.

a Draw a circle around each multiple of 3.
b Draw a square around each multiple of 4.
c List the common multiples of 3 and 4 which are less than 30.

5 Answer the following:

- **a** I am an odd multiple of 9. The product of my two digits is also a multiple of 9. Which two numbers could I be?
- **b** I am a square number which is a multiple of 6 and a factor of 252. What number am I?
- **c** I am a multiple of 7 and a factor of 210. The product of my two digits is odd. What number am I?

6 Find the common multiples of:

- **a** 6 and 10 which are less than 100
- **b** 8 and 12 which are between 50 and 100
- **c** 6 and 9 which are between 30 and 70.

J LOWEST COMMON MULTIPLE

The **lowest common multiple** or **LCM** of two or more numbers is the smallest multiple which is common to all of them.

Example 5 — Self Tutor

Find the LCM of 6 and 8.

The multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, ...

The multiples of 8 are 8, 16, 24, 32, 40, 48, 56, ...

So, the LCM of 6 and 8 is 24.

DISCUSSION

How do we observe a "lowest common multiple" on the face of an analogue clock?

EXERCISE 2J

1 Find the lowest common multiple of each pair of numbers:

a 4 and 10	**b** 5 and 15	**c** 8 and 12	**d** 12 and 15
e 6 and 10	**f** 4 and 7	**g** 8 and 9	**h** 6 and 14
i 6 and 11	**j** 5 and 13	**k** 15 and 25	**l** 27 and 36

2 Buses arrive at a sports stadium every 8 minutes, and trains arrive every 14 minutes. A bus and a train have just arrived simultaneously. How long will it be before this happens again?

3 A baker bakes the same number of buns each day. On weekdays he sells them in packs of 12, and on weekends he sells them in packs of 13. He always sells complete packs. What is the least possible number of buns he bakes?

4 I am thinking of two numbers. The smaller number is a factor of the larger number. Describe their HCF and LCM, explaining your answers.

INVESTIGATION 3 HCF AND LCM

Is there a relationship between the HCF and LCM of two natural numbers?

What to do:

1 Copy and complete:

Pair of numbers	*Product of numbers*	*HCF*	*LCM*	*HCF* $\times$ *LCM*
6 and 8	48		24	
6 and 9				
4 and 10				
15 and 25				

2 Discuss the relationship between the HCF and LCM of two natural numbers.

3 Find the HCF of 21 and 28, and *hence* find their LCM.

4 Find the LCM of two numbers whose product is 720 and whose HCF is 6.

5 What can you say about the LCM of two different prime numbers?

QUICK QUIZ

MULTIPLE CHOICE QUIZ

REVIEW SET 2A

1 **a** Use your calculator to find $\sqrt{169}$.

b Copy and complete: $\sqrt{169} \times \sqrt{169} = ...$

2 Find:

a 17^3 **b** 35^3 **c** $\sqrt[3]{27}$ **d** $\sqrt[3]{1000}$

3 Decide whether:

a 54 is divisible by 6 **b** 42 is divisible by 4.

4 Find all possible values of the missing digit □ if:

a 32□ is divisible by 4 **b** 9□4 is divisible by 3 **c** 2□7 is divisible by 9.

5 Write the number 36 as:

a the sum of two even numbers **b** the sum of two odd numbers

c the product of two even numbers

d the product of an even number and an odd number.

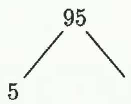
6 List the factors of:

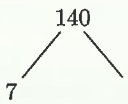
a 72 **b** 90 **c** 126

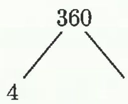
7 Show that each number is composite:

a 2950 **b** 1863

8 Complete these factor trees:

a


b


c


9 Use repeated division to write as the product of prime factors in exponent form:

a 44 **b** 504 **c** 693

10 Find the highest common factor of:

a 14 and 49 **b** 84 and 150.

11 List the multiples of 7 which lie between 40 and 60.

12 Find the lowest common multiple of:

a 4 and 6 **b** 8 and 10 **c** 12 and 20.

13 At a netball club fundraiser, Jim sells boxes of cookies for \$10 each. The club uses the proceeds to buy netballs for \$12 each, with no money left over.

a List the first 10 multiples of: **i** 10 **ii** 12

b Find the lowest common multiple of 10 and 12.

c What is the smallest number of cookie boxes Jim could have sold?

d What is the smallest number of netballs the club could have bought?

14 **a** List the factors of:

i 14 **ii** 20 **iii** 35 **iv** 98 **v** 120 **vi** 150

b Which of the numbers in **a** have the same number of even and odd factors?

c Write each number in **a** in prime factored form.

d Copy and complete:

A number has the same number of even and odd factors if, when written in prime factored form, its power of two is equal to ......

15 12 dots can be arranged into a complete rectangle in three different ways:

1×12 2×6 3×4

Note that a 4×3 rectangle is the same as a 3×4 one.

a How many different complete rectangles can we make with:

i 42 dots **ii** 7 dots **iii** 36 dots?

b Suppose we have 120 dots.

i What is the longest rectangle you can make?

ii Of the different complete rectangles we can make, which is closest in shape to a square?

c How many rectangles are possible with a prime number of dots?

REVIEW SET 2B

1 Write down the perfect squares between 200 and 300.

2 **a** Use your calculator to find $\sqrt[3]{512}$.

b Copy and complete: $\sqrt[3]{512} \times \sqrt[3]{512} \times \sqrt[3]{512} = ...$

3 Determine whether 4536 is divisible by:

a 3 **b** 5 **c** 6 **d** 11

4 Write down the:

a 47th even number **b** eighth multiple of 11.

5 Write down the factor pairs of 40. Hence list the factors of 40.

6 Is 3601 a prime number?

7 I am a two digit number. I am a multiple of 8. Both my digits are prime numbers. Which two numbers could I be?

8 Use a factor tree to write 564 as the product of prime factors in exponent form.

9 Find the largest number which divides exactly into both 63 and 84.

10 Find the lowest common multiple of 21 and 28.

11 **a** Write as the product of prime factors in exponent form:

i 9 **ii** 16 **iii** 25 **iv** 36 **v** 81

b What do you notice about the exponents of all the prime factors in **a**?

c Write 576 as the product of prime factors in exponent form.

d Find $\sqrt{576}$, and write this as the product of prime factors in exponent form.

e What do you notice from **c** and **d**?

f Is 180 a square number?

12 Suppose the garbage man visits your house once every 14 days, and the green waste truck visits your house once every 10 days. If they both visit your house today, how long will it be before they are both at your house on the same day again?

13 **a** List the factors of: **i** 54 **ii** 78

b Use a factor tree to write 54 as the product of prime factors in exponent form.

c Use repeated division to write 78 as the product of prime factors in exponent form.

d A wedding cake is 54 cm $\times$ 78 cm. The bride and groom want it cut into square pieces of equal size.

i What is the largest size the pieces can be?

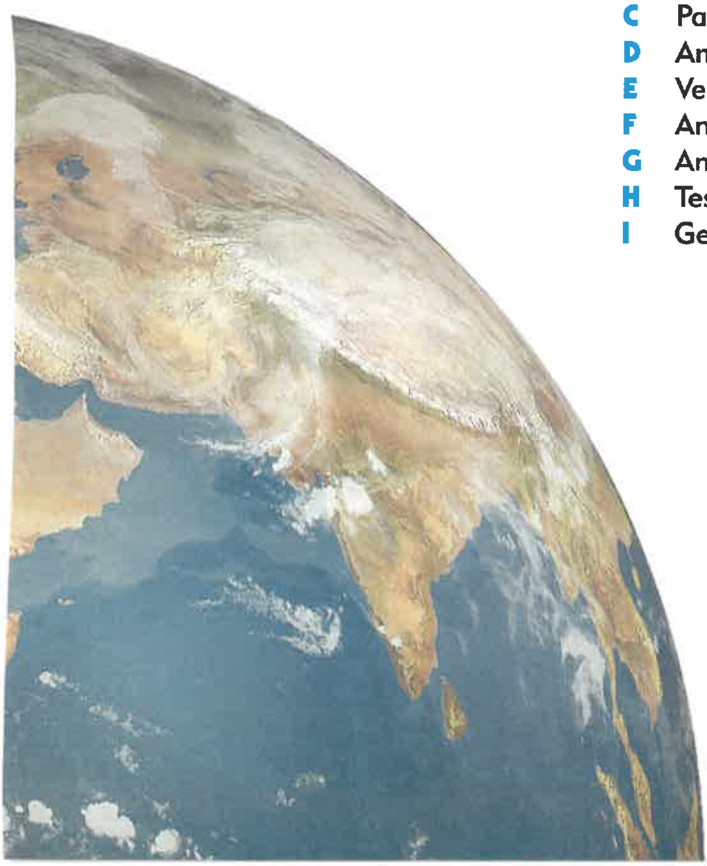
ii How many pieces of this size can be cut?

14 Answer the **Opening Problem** on page **24**.

Chapter 3

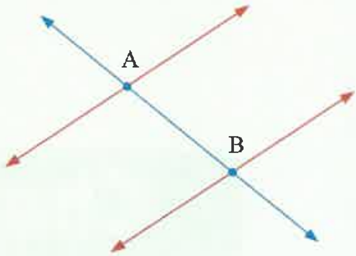
Lines and angles

Contents:



OPENING PROBLEM

This diagram shows three lines. We can see points of **intersection** where the blue line meets the two red lines.



Things to think about:

a How can we *describe*:
  i the blue line
  ii the section of the blue line between the red lines?

b The red lines do not meet in this diagram.
  i If we were to extend the lines, do you think they would meet eventually?
  ii By measuring angles in the diagram, can we test whether the lines will meet eventually?

If we look carefully, we can see **angles** all around us. We see them in the framework of buildings, the design of furniture and vehicles, and the positions of objects.

The measurement of angles dates back more than 2500 years, but it is still necessary today in architecture, building, surveying, engineering, navigation, and many other industries.

RESEARCH — DEGREE MEASURE

The Babylonian Empire was founded in the 18th century BC by **Hammurabi** in lower Mesopotamia, which is today in southern Iraq. It lasted over a thousand years, finally being absorbed into the Persian Empire of **Darius** in the 6th century BC.

1 How many degrees did the Babylonians decide should be in one full turn? Why did they choose this number?

2 The Babylonians invented the **astrolabe**.
  a What does an astrolabe measure?
  b Why were these measurements important?

POINTS

A **point** is used to mark a position or location.

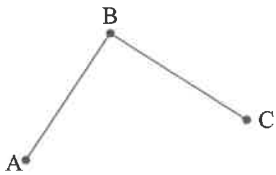
A point does not have any size. We say that a point is *infinitely small*.

However, in geometry we represent a point with a small dot so we can see it.

To help identify a point, we can label it with a capital letter.

We can then make statements like:

- "The distance from A to B is ...."
- "The angle at B measures ...."



A LINES

A **straight line**, usually just called a **line**, is a continuous infinite collection of points in a particular direction. A line has no beginning and no end.

The line alongside passes through points A and B. We use arrowheads to show that the line continues endlessly in both directions. We can call this line "line AB" or "line BA".

We use the following bracket notation to describe lines and parts of lines:

(AB) is the **line** which passes through A and B and continues endlessly in both directions.

[AB] is the **line segment** which joins the two points A and B. It is only a part of the line (AB).

The **length** of line segment [AB] is written AB.

[AB) is the **ray** which starts at A, passes through point B, and continues on endlessly.

COLLINEAR POINTS

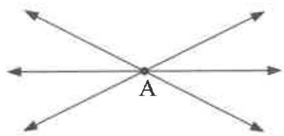
If three or more *points* lie on a single straight line, we say that the points are **collinear**.

For example, in the diagram the points A, B, C, and D are collinear.

CONCURRENT LINES

If three or more *lines* meet or intersect at the same point, we say that the lines are **concurrent**.

For example, in the diagram the lines are concurrent at point A.

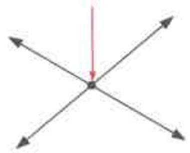


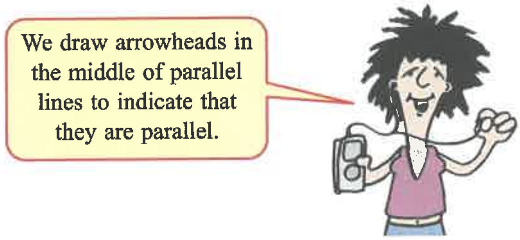
PARALLEL AND INTERSECTING LINES

Parallel lines are lines which are always a fixed distance apart and never meet.

In mathematics, a **plane** is a flat surface like a table top or a sheet of paper.

Two straight lines on a plane are either **parallel** or **intersecting**.





EXERCISE 3A

1 Describe, including a sketch, the meaning of:

a a line segment
b a ray
c a point of intersection
d parallel lines
e collinear points
f concurrent lines.

2 Give *all* ways of naming:

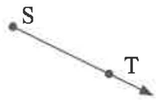
a

b

3 Use bracket notation to describe:

a

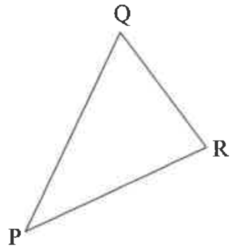
b



c

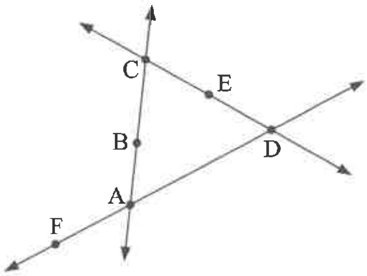
4 PQR is a triangle.

a Name the three sides of the triangle.
b Which sides intersect at point P?
c Are P, Q, and R collinear? Explain your answer.



5 State the intersection of:

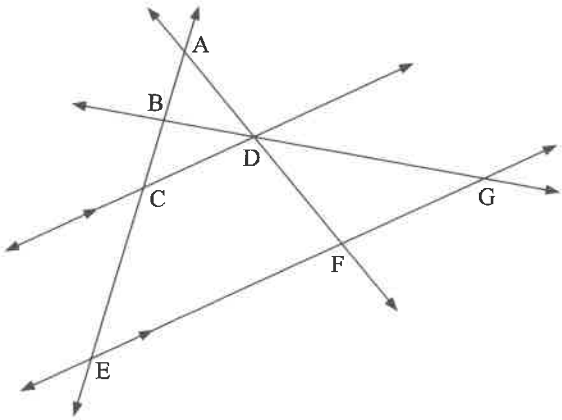
a (AC) and (AD)
b (DE) and (AB)
c [AC] and [DF]
d [DF] and [AD].



6 Draw a diagram for each statement:

a X is a point on [PQ].
b [EF) and (GH) meet at point M.
c S, T, U, and V are collinear.
d (JK) and (MN) are parallel.
e [AB], (CD), and (EF) are concurrent at G.

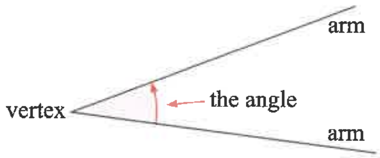
7 What can be said about:
- **a** lines (EF) and (AD)
- **b** points A, D, and F
- **c** lines (CD) and (EG)
- **d** lines (AF), (BG), and (CD)?



B ANGLES

An **angle** is formed where two straight lines meet.

The point where the lines meet is called the **vertex** of the angle, and the lines are called the **arms**.

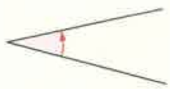
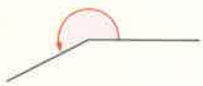


CLASSIFYING ANGLES

The **size** or **measure** of an angle is the amount of *turn* between its arms.

We measure the turn in **degrees**, and use the symbol °.

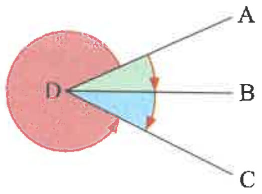
Revolution	**Straight angle**	**Right angle**
 One complete turn. 360°	 $\frac{1}{2}$ turn. 180°	 $\frac{1}{4}$ turn. 90°

Acute angle	**Obtuse angle**	**Reflex angle**
 Less than $\frac{1}{4}$ turn. Between 0° and 90°.	 Between $\frac{1}{4}$ turn and $\frac{1}{2}$ turn. Between 90° and 180°.	 Between $\frac{1}{2}$ turn and a complete turn. Between 180° and 360°.

THREE POINT NOTATION

In **three point notation**, we refer to angles using a point on each arm, and the vertex between them.

For example:



- the green angle is $A\hat{D}B$ or $B\hat{D}A$
- the blue angle is $B\hat{D}C$ or $C\hat{D}B$
- $A\hat{D}C$ is made up of the green angle and the blue angle
- the red angle is *reflex* $A\hat{D}C$, since its size is greater than 180°.

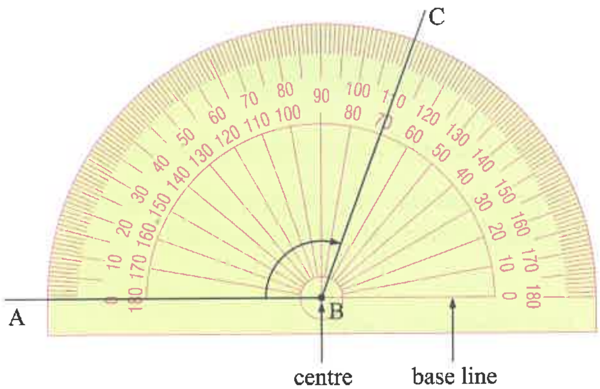
MEASURING ANGLES

We measure angles using a **protractor**.

We place the **centre** at the vertex of the angle, with the **base line** directly on top of one arm.

We then measure from 0° around to the other arm.

On this protractor, we see that $A\hat{B}C = 110°$.

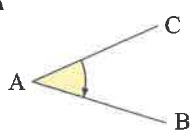


EXERCISE 3B

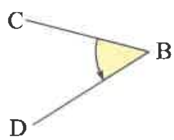
1 Match each name to the correct angle:

a $A\hat{B}C$ **b** $C\hat{A}B$ **c** $B\hat{C}A$ **d** $C\hat{B}D$

A



B



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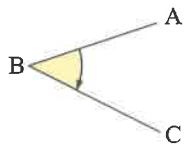
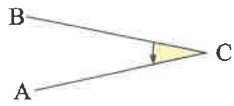
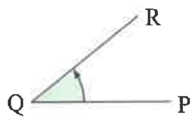


D

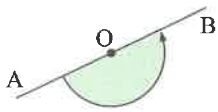


2 Name the shaded angle using three point notation, and classify the angle:

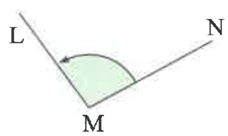
a



b



c



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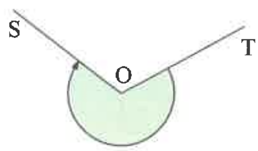
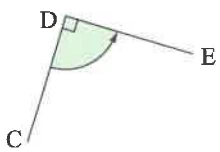
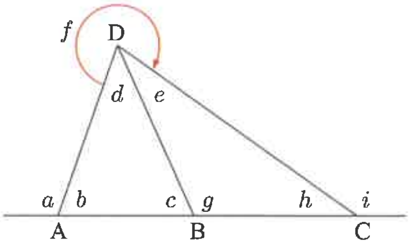


e



3 Look at the figure alongside.



a State the letter corresponding to:

i $B\widehat{A}D$ **ii** $D\widehat{B}C$ **iii** $A\widehat{D}B$

b Classify each angle as acute, obtuse, or reflex:

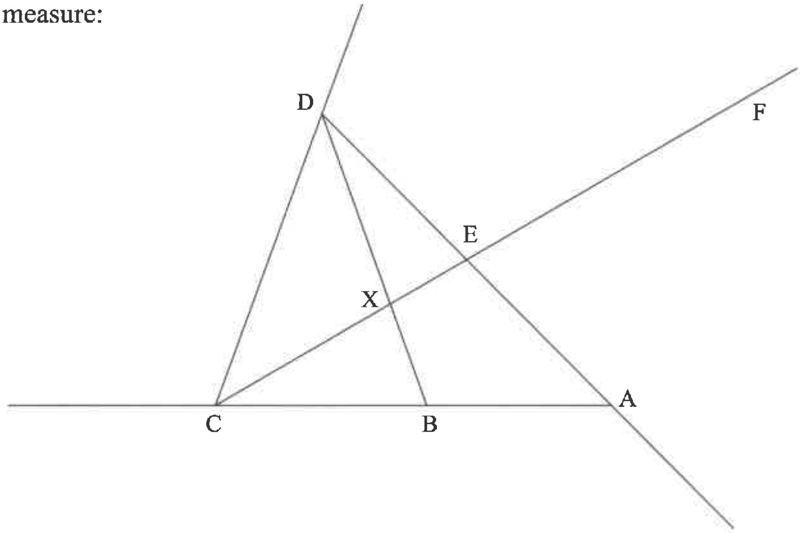
i f **ii** a **iii** h

4 Draw a free-hand sketch of:

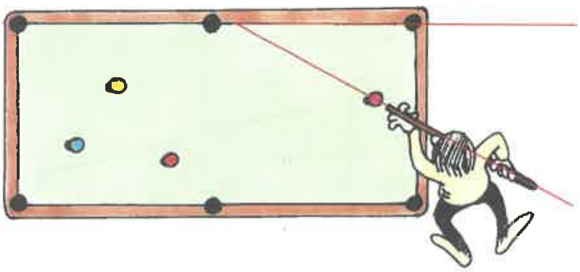
a obtuse $X\widehat{Y}Z$
b revolution B
c reflex $Y\widehat{S}P$
d right $C\widehat{Z}R$
e acute $J\widehat{K}L$
f straight $E\widehat{F}G$.

5 Use a protractor to measure:

a $C\widehat{B}D$
b $D\widehat{X}E$
c $A\widehat{D}C$
d $C\widehat{E}A$
e $F\widehat{E}D$
f $A\widehat{C}D$



6 Kit hits the billiard ball so that it follows the path shown. What *acute* angle will it make with the edge of the table?



7 Measure the marked angle between the top of the awning and the support post.

8 Use your ruler and protractor to draw the following angles:

a $L\widehat{M}N = 38°$ **b** $B\widehat{C}D = 84°$ **c** $A\widehat{B}C = 120°$

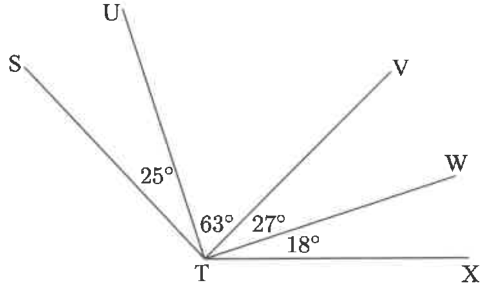
d $X\widehat{Y}Z = 159°$ **e** $J\widehat{K}L = 90°$ **f** $P\widehat{Q}R = 102°$

Ask a classmate to check your answers.

9 Find the size of each angle *without* using your protractor. Hence classify the angle as acute, a right angle, or obtuse.

a $S\widehat{T}U$ **b** $W\widehat{T}U$

c $X\widehat{T}V$ **d** $S\widehat{T}W$



ACTIVITY 1 MAKING A PROTRACTOR

Click on the icon to obtain instructions for this Activity.

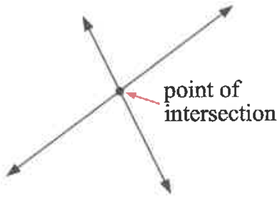
It explains how to make a protractor of your own, and provides activities for you to do in class.

INSTRUCTIONS

C PARALLEL AND PERPENDICULAR LINES

- Two lines are **parallel** if they never meet. We use the symbol $\|$ to mean "is parallel to".
- Two lines are **perpendicular** if they intersect at right angles. We use the symbol $\perp$ to mean "is perpendicular to".

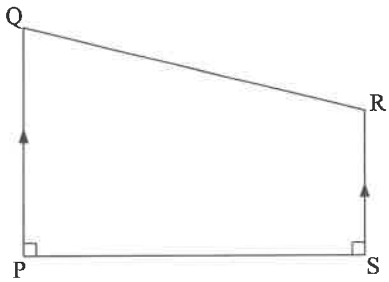
parallel lines



intersecting lines

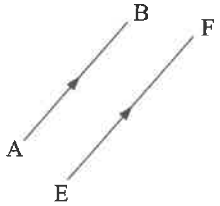
For example, in the figure alongside:

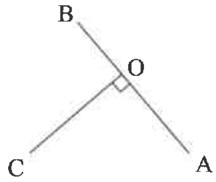
- [PQ] is parallel to [SR], so we write [PQ] $\|$ [SR].
- [PQ] is perpendicular to [PS], so we write [PQ] $\perp$ [PS].

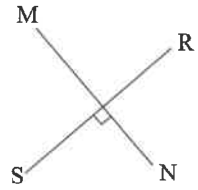


EXERCISE 3C

1 Write a statement using $\parallel$ or $\perp$:

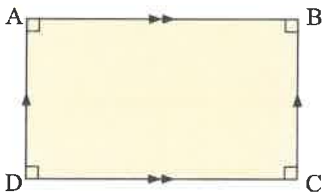
a


b


c


2 From the diagram, write:

a *two* statements using $\parallel$
b *four* statements using $\perp$.



3 Draw a diagram to show:

a [RS] $\perp$ [AB] **b** [CD] $\parallel$ [OP] **c** (AB) $\perp$ (BC)

4 Illustrate accurately:

a [AB] is 2 cm long, [AC] is 3 cm long, and [AB] $\perp$ [AC].
b [KL] is 4 cm long, [MN] is 5 cm long, [KL] $\parallel$ [MN], and [KL] is 2 cm from [MN].

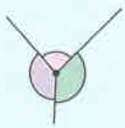
5 What can you say about A, B, and C if [AB] $\parallel$ [BC]? Illustrate your answer.

D ANGLE PROPERTIES

ANGLES AT A POINT

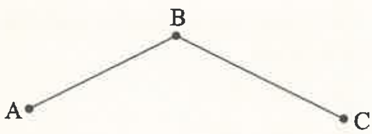
Since there are 360° in one complete turn:

Angles at a point add to 360°.



DISCUSSION

For any distinct points A, B, and C, what is the relationship between the size of $A\widehat{B}C$ and reflex $A\widehat{B}C$?

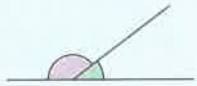


ANGLES ON A LINE

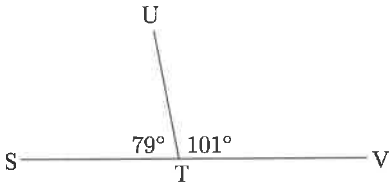
Since there are 180° in a straight angle:

Angles on a line add to 180°.

We say that angles which add to 180° are **supplementary angles.**



For example:



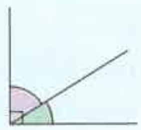
$S\widehat{T}U$ and $U\widehat{T}V$ are supplementary because $79° + 101° = 180°$.

ANGLES IN A RIGHT ANGLE

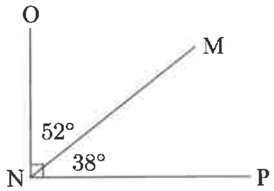
Since there are 90° in a right angle:

Angles in a right angle add to 90°.

We say that angles which add to 90° are **complementary angles.**



For example:



$M\widehat{N}O$ and $M\widehat{N}P$ are complementary because $52° + 38° = 90°$.

[ON] ⊥ [NP]

Example 1 — Self Tutor

a Are angles with sizes 37° and 53° complementary?
b What angle size is supplementary to 48°?

a $37° + 53° = 90°$, so the angles are complementary.
b The angle size supplementary to 48° is $180° - 48° = 132°$.

EXERCISE 3D

1 Add each pair of angles and hence state whether the angles are complementary, supplementary, or neither:

a 109°, 71° **b** 67°, 117° **c** 62°, 28°
d 155°, 31° **e** 25°, 55° **f** 64°, 116°

2 Find the size of the angle complementary to:

a 15° **b** 87° **c** 43°

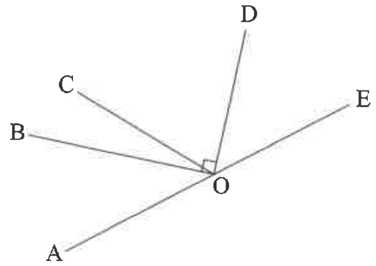
3 Find the size of the angle supplementary to:

a $129°$ **b** $57°$ **c** $90°$

4 Classify each pair of angles as complementary, supplementary, or neither:

a $C\widehat{O}A$ and $C\widehat{O}E$ **b** $A\widehat{O}D$ and $E\widehat{O}C$

c $B\widehat{O}C$ and $C\widehat{O}D$ **d** $C\widehat{O}E$ and $D\widehat{O}B$



5 Copy and complete:

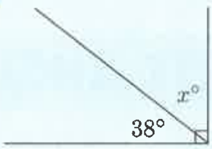
a the size of the angle complementary to $x°$ is ......

b the size of the angle supplementary to $y°$ is ......

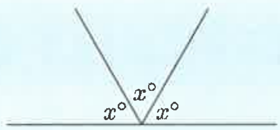
Example 2 **Self Tutor**

Find the unknown value without using a protractor:

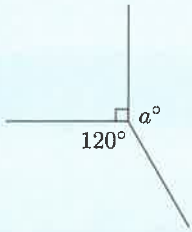
a



b



c



a The angles $38°$ and $x°$ are complementary.

$\therefore\ x = 90 - 38$

$\therefore\ x = 52$

b The three angles are supplementary.

$\therefore\ x = 180 \div 3$

$\therefore\ x = 60$

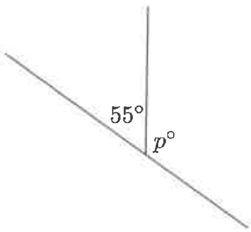
c Angles at a point sum to $360°$.

$\therefore\ a = 360 - 120 - 90$

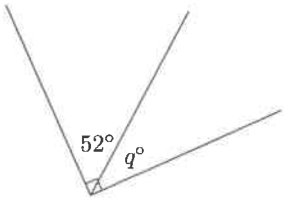
$\therefore\ a = 150$

6 Find the unknown value without using a protractor:

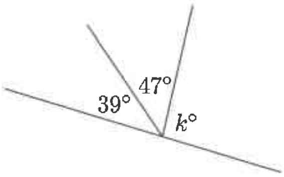
a



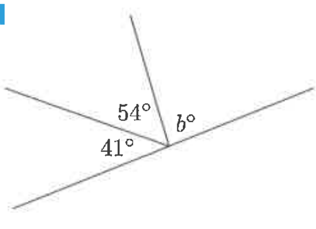
b



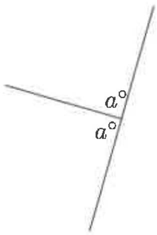
c



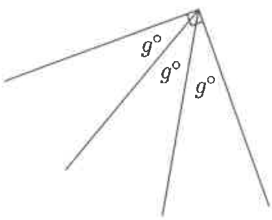
d



e

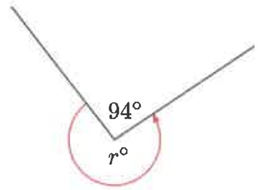


f

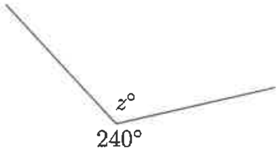


7 Find the unknown value without using a protractor:

a



b



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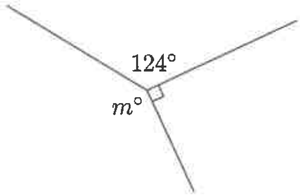
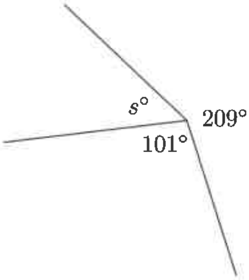
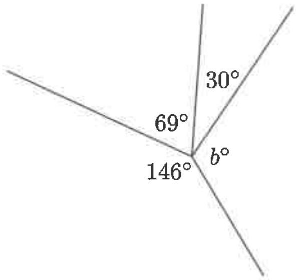


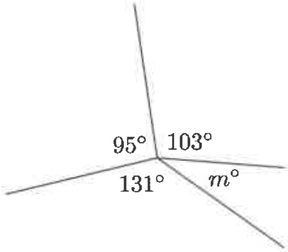
d



e



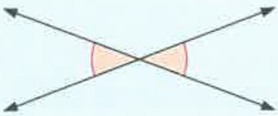
f



E VERTICALLY OPPOSITE ANGLES

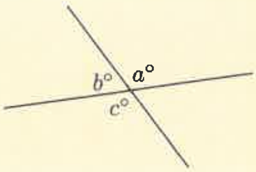
Vertically opposite angles are formed when two straight lines intersect. They are directly opposite each other through the vertex.

Vertically opposite angles are always *equal* in size.



vertically opposite angles

Proof:



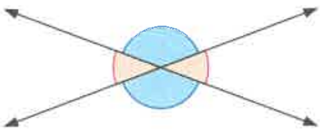
$a + b = 180$ {angles on a line}

and $c + b = 180$ {angles on a line}

$\therefore$ $a = c$

GEOMETRY PACKAGE

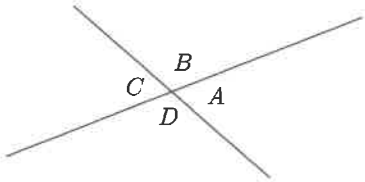
At any intersection of two straight lines, there are *two* pairs of equal vertically opposite angles.



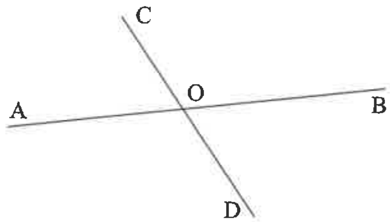
EXERCISE 3E

1 In each diagram, write down the pairs of equal vertically opposite angles:

a

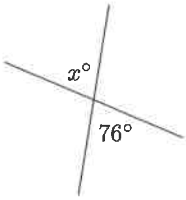


b

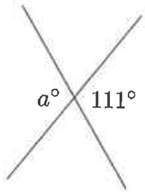


2 Find the unknown value:

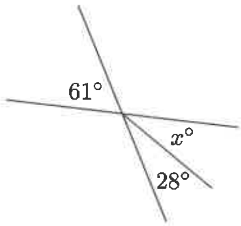
a



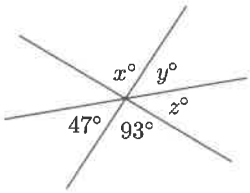
b



c



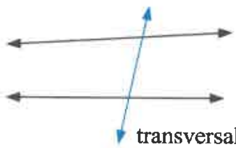
3 Find the unknown values.

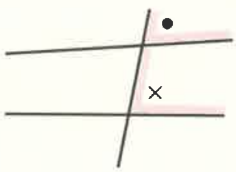
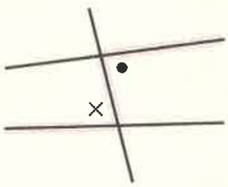
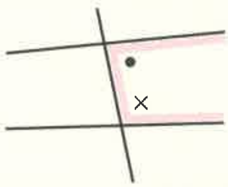


F ANGLE PAIRS

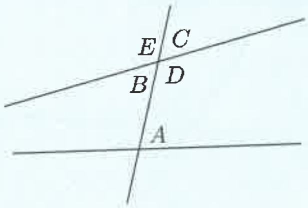
A third line that crosses two other straight lines is called a **transversal**.

When two straight lines are cut by a transversal, three different angle pairs are formed:



Corresponding angle pairs	Alternate angle pairs	Co-interior angle pairs
		
• and × are **corresponding angles**. They are on the *same side* of the transversal and the *same side* of the two straight lines.	• and × are **alternate angles**. They are on *opposite sides* of the transversal and *between* the two straight lines.	• and × are **co-interior angles**. They are on the *same side* of the transversal and *between* the two straight lines. Co-interior angles can also be called **allied angles**.

Example 3 — Self Tutor



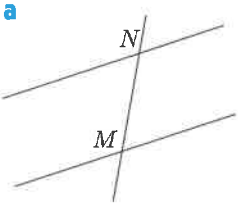
Classify each angle pair:

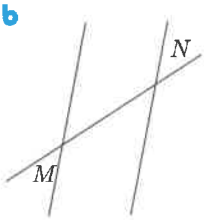
a A and D
b A and B
c D and E
d A and C

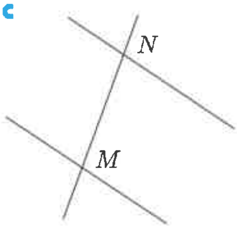
a A and D are co-interior angles.
b A and B are alternate angles.
c D and E are vertically opposite angles.
d A and C are corresponding angles.

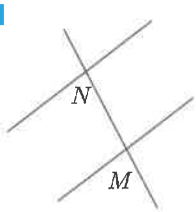
EXERCISE 3F

1 Decide whether M and N are corresponding angles:

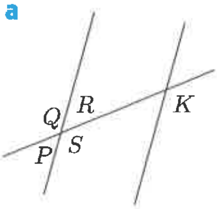
a


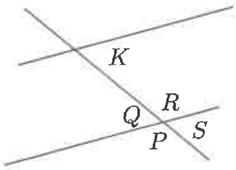
b


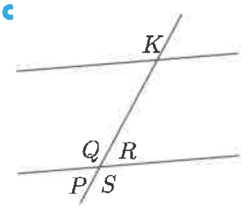
c


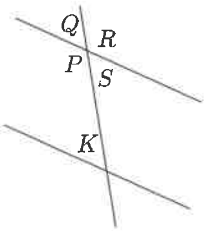
d


2 Which angle is corresponding to angle K?

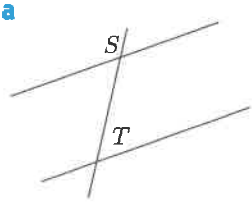
a


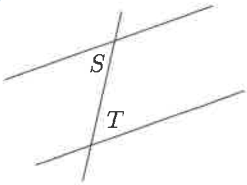
b


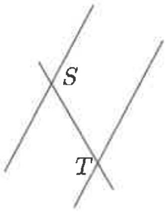
c


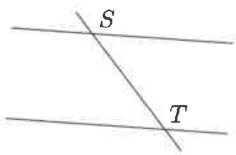
d


3 Decide whether S and T are alternate angles:

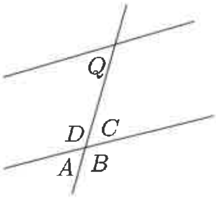
a


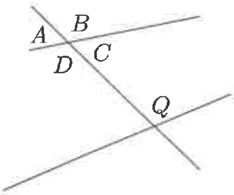
b


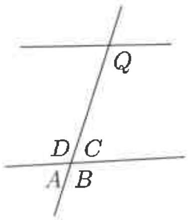
c


d


4 Which angle is alternate to angle Q?

a


b


c


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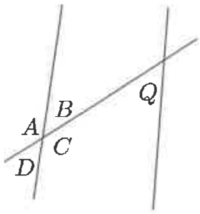

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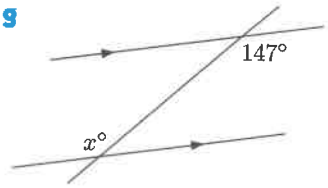

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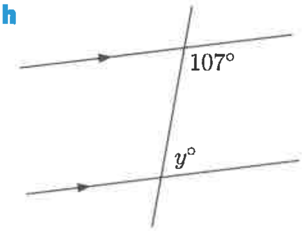
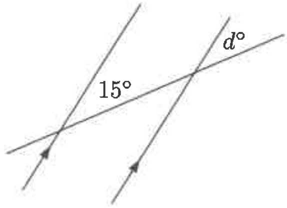
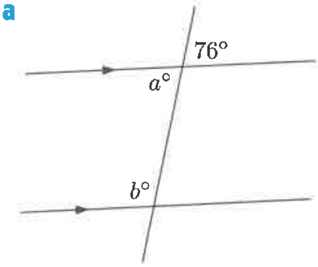


i

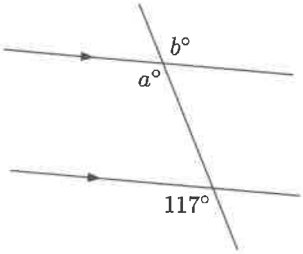


2 Find the unknown values in alphabetical order, giving brief reasons:

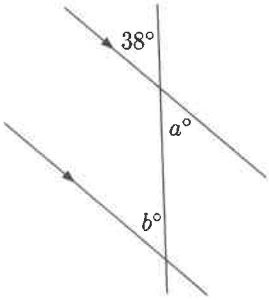
a



b



c



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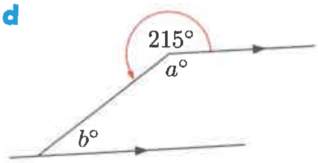
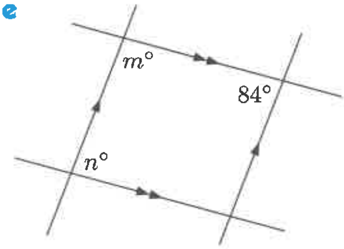
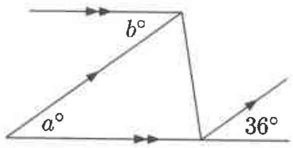


e

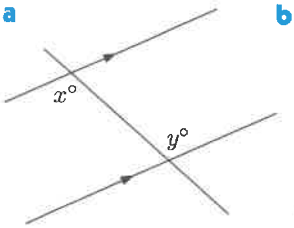


f

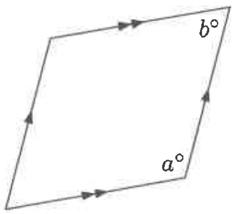


3 Write a statement connecting the unknown values, giving a brief reason:

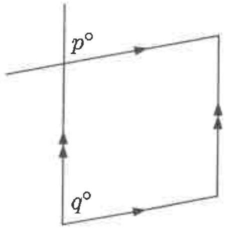
a



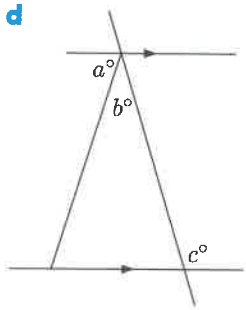
b



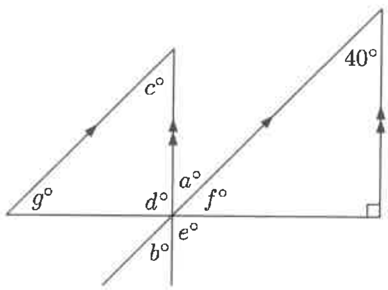
c



d



4 Working in alphabetical order, find the unknown values.
Give a reason for each answer.



H TESTS FOR PARALLELISM

Having seen the relationships between angle pairs when parallel lines are cut by a transversal, we can deduce these tests to determine whether two lines cut by a transversal are parallel:

Suppose two lines are cut by a transversal.

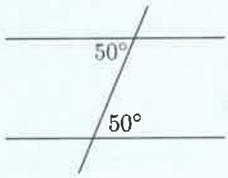
- If two corresponding angles are equal in size then the lines are parallel.
- If two alternate angles are equal in size then the lines are parallel.
- If two co-interior angles are supplementary then the lines are parallel.

GEOMETRY PACKAGE

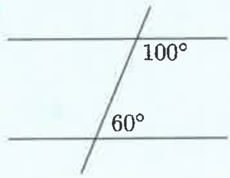
Example 5 Self Tutor

Decide whether the figure contains parallel lines. Explain your answers.

a



b



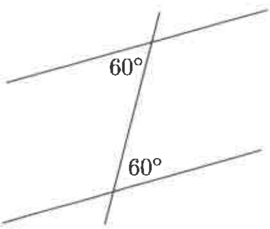
a These alternate angles are equal, so the lines are parallel.

b These co-interior angles add to 160°, so they are not supplementary.
$\therefore$ the lines are *not* parallel.

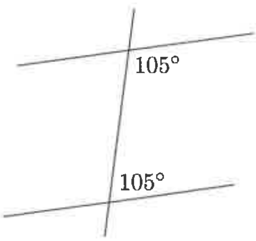
EXERCISE 3H

1 Use the tests for parallelism to decide whether each figure contains a pair of parallel lines. Explain your answers.

a



b



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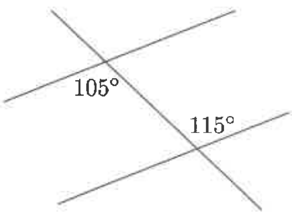
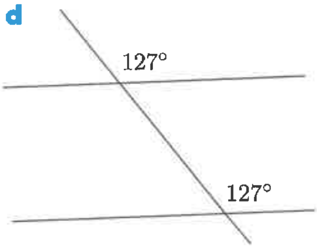
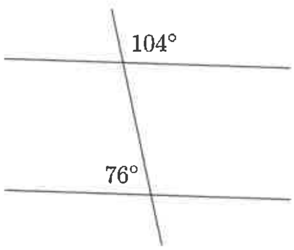


d



e



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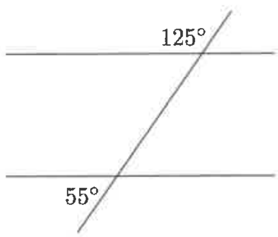
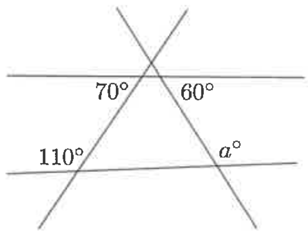
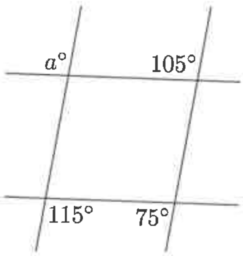


2 Find the value of a, giving reasons for your answer.

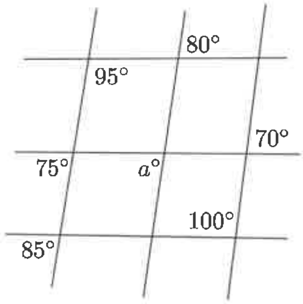
a



b



c



PUZZLE

Test your knowledge of geometry words in this printable crossword.

CROSSWORD

I GEOMETRIC CONSTRUCTION

In **geometric constructions**, we use a **ruler and compass only** to accurately draw diagrams.

When you perform geometric constructions:

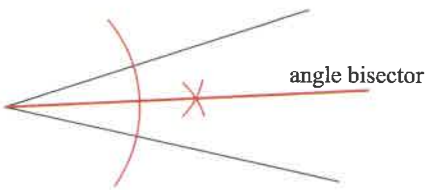
- use a sharp pencil
- do *not* erase your compass lines
- be careful with the sharp point of your compass.

Before you begin, you need to know that:

- The **radius** of your compass is the distance from the sharp point to the tip of your pencil.
- An **arc** is a curve which is *part* of a circle.

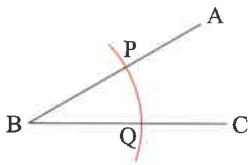
BISECTING ANGLES

When we **bisect** an angle, we divide it into two angles of equal size.

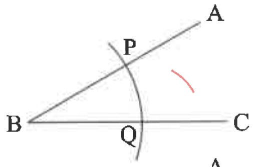


To bisect $A\widehat{B}C$, follow these steps:

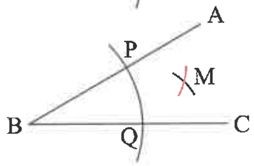
Step 1: With centre B, draw an arc which cuts [BA] and [BC] at P and Q respectively.



Step 2: With centre Q and radius [PQ], draw an arc within $A\hat{B}C$.

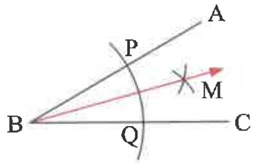


Step 3: With centre P and the same radius [PQ], draw another arc to intersect the previous one at M.

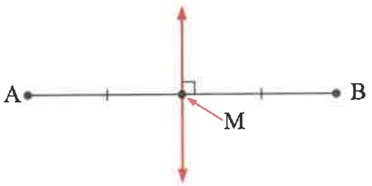


Step 4: Draw [BM].
[BM) bisects $A\hat{B}C$ with $A\hat{B}M = C\hat{B}M$.

DEMO



CONSTRUCTING A PERPENDICULAR BISECTOR



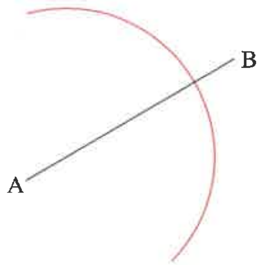
The red line in this figure is at right angles to [AB] so it is **perpendicular** to [AB].

It passes through M which is midway between A and B, so it also **bisects** [AB].

We say the red line is the **perpendicular bisector** of [AB].

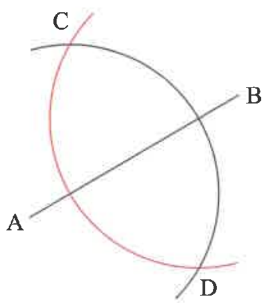
To construct the perpendicular bisector of [AB], follow these steps:

Step 1: Set your compass to a radius just less than AB.
With centre A, draw an arc of a circle as shown.

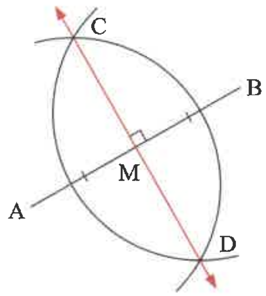


Step 2: Using centre B and the same radius, draw a second arc which intersects the first at C and D.

DEMO



Step 3: Use your ruler to draw the line (CD).
(CD) is the perpendicular bisector of [AB], intersecting [AB] at its midpoint M.

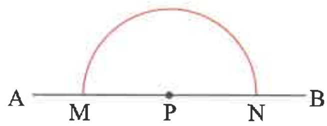


CONSTRUCTING A RIGHT ANGLE

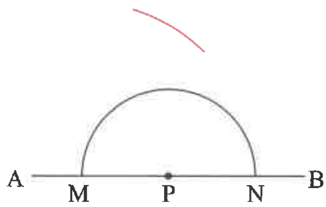
Constructing a right angle allows us to construct a line which is **perpendicular** to another line at any point we wish.

To construct a right angle at point P along [AB], follow these steps:

Step 1: Draw a semi-circle with centre P and convenient radius which cuts (AB) at M and N.

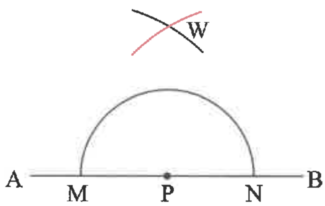


Step 2: With centre M and radius [MN], draw an arc above P.

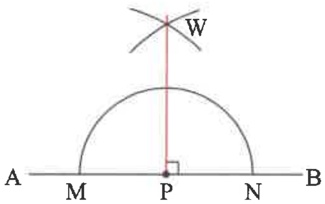


Step 3: With centre N and radius [MN], draw an arc to cut the first one at W.

DEMO

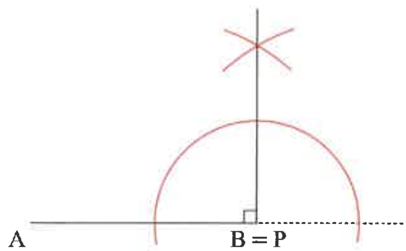


Step 4: Draw the line from P through W.
$W\widehat{P}A = W\widehat{P}B = 90°$ so [PW] $\perp$ [AB].



In some cases the point P may be close to one end of the line segment, or indeed be the end of the line segment. In these cases you will need to extend [AB].

For example, in the construction alongside, B and P are the same point. We say they **coincide**.



EXERCISE 3I

1 Use your protractor to accurately draw $A\hat{B}C$ of size $70°$.
Use a compass and ruler only to bisect $A\hat{B}C$.
Use a protractor to check the accuracy of your construction.

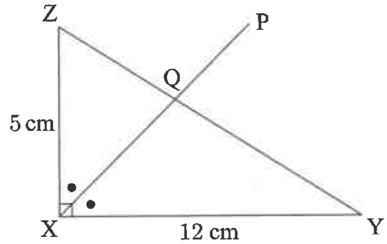
2 Draw a line segment [AB] of length 10 cm.
Mark C on [AB] so that $AC = 4$ cm.
Use a compass and ruler to construct a right angle at C *below* the line segment [AB].

3 **a** Draw a line segment [XY] of length 12 cm.
Use a compass and ruler to construct a $90°$ angle at X. Draw [XZ] of length 5 cm, and draw [ZY] as shown.

b Measure the length ZY.

c Bisect $Z\hat{X}Y$ using a compass and ruler only.

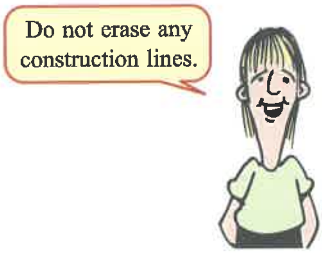
d Use a protractor to measure $X\hat{Q}Y$.



4 **a** Draw a line segment [AB] of length 4 cm.
Use a compass and ruler to construct a $90°$ angle at A.
Locate the point C so that $C\hat{A}B = 90°$ and $AC = 3$ cm. Draw [BC].

b Measure the length BC.

c Use a protractor to measure $C\hat{B}A$.



5 **a** Draw a line segment [PQ] of length 6 cm.
Use a compass and ruler to construct the perpendicular bisector of [PQ].

b Suppose your perpendicular bisector intersects [PQ] at M.
Check your perpendicular bisector by measuring: **i** MP **ii** MQ.

6 **a** Draw any triangle, and construct the perpendicular bisectors of its three sides.

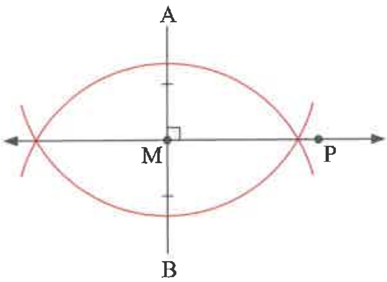
b Repeat **a** with a different triangle.

c Copy and complete:
"The three perpendicular bisectors of the sides of a triangle are ......".

7 **a** Draw a line segment [AB] of length 6 cm.
Construct the perpendicular bisector of [AB], meeting [AB] at M.
Locate P on the perpendicular bisector such that $MP = 4$ cm.

b Measure the lengths of [AP] and [BP]. What do you notice?

c Measure $M\hat{A}P$ and $M\hat{B}P$ with a protractor. What do you notice?

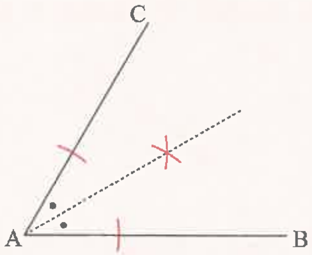


ACTIVITY 2 TRISECTING AN ANGLE

We have seen that we can **bisect** an angle into two equal angles using a compass and ruler.

It has been proven that an angle cannot be **trisected** using a compass and ruler only. In other words, we cannot perform a geometric construction to divide $C\widehat{A}B$ into *three* equal angles.

However, Japanese mathematician **Hisashi Abe** showed how to trisect an angle using **origami** or paper-folding.

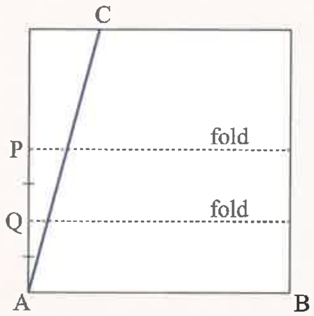


PRINTABLE PAPER

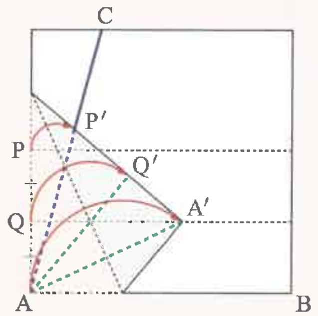
VIDEO DEMO

What to do:

1. Start with a square piece of paper 20 cm by 20 cm. Label the base [AB].
2. Choose any point C on the top and draw [AC]. We will trisect $C\widehat{A}B$.
3. Fold the page parallel to [AB] at some point P *about* half-way down the page.
4. By placing A onto P, fold the page parallel to [AB] at the point Q which is exactly half-way between A and P.



5. Fold the paper one more time so that P is placed onto [AC] *and* A is placed onto the fold through Q you made in **4**.
6. Mark on the page the positions of Q′ and A′.
7. Unfold the paper, and draw [AQ′] and [AA′]. $C\widehat{A}B$ is trisected by [AQ′] and [AA′].
8. Check your result using a protractor.



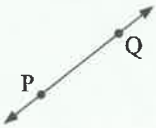
MULTIPLE CHOICE QUIZ

QUICK QUIZ

REVIEW SET 3A

1 Use bracket notation to describe:

a



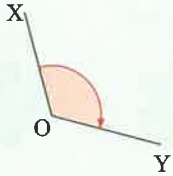
b

c

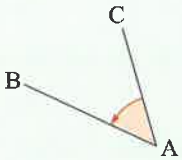
2 Draw a diagram to illustrate the statement: "[AB] and [CD] intersect at P."

3 Name the shaded angle using three point notation, and classify the angle:

a



b



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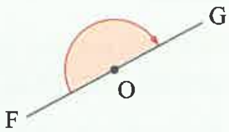
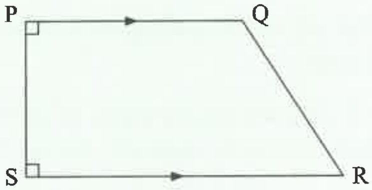


4 From the diagram, write:

a *one* statement using ‖

b *two* statements using ⊥ .



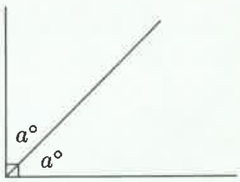
5 Find:

a the angle complementary to 53°

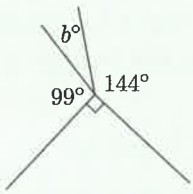
b the angle supplementary to 130°.

6 Find the unknown value without using a protractor:

a



b



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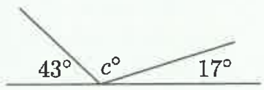
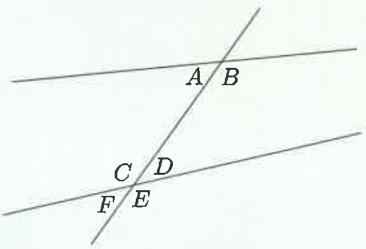


7



Name the angle:

a corresponding to B

b alternate to D

c co-interior to A

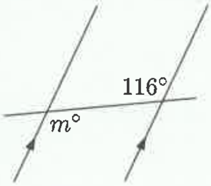
d vertically opposite E.

8 **a** Draw a line segment [PQ] of length 4 cm.
Construct the perpendicular bisector of [PQ], meeting [PQ] at X.

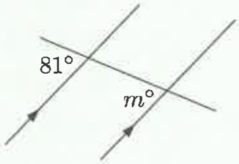
b Check your perpendicular bisector by measuring PX and QX.

9 Find the value of m, giving a brief reason.

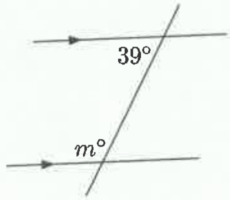
a



b

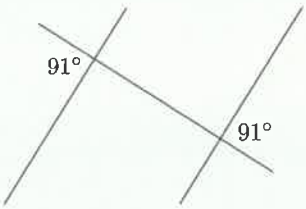


c

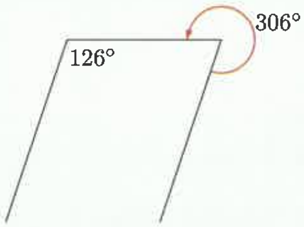


10 Decide whether each figure contains a pair of parallel lines. Explain your answers.

a



b

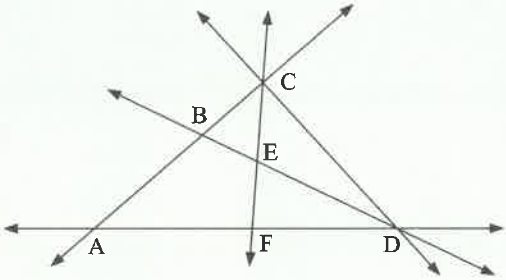


REVIEW SET 3B

1 What can be said about:

a points A, B, and C

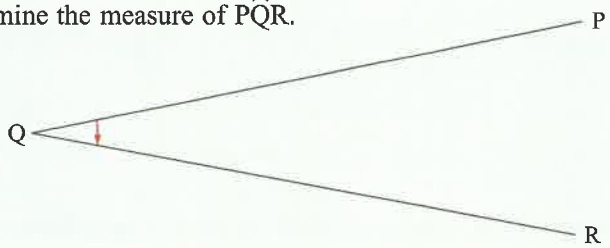
b lines (AF), (CD), and (BE)?



2 Draw and label the following angles:

a reflex $B\widehat{A}C$ **b** acute $P\widehat{Q}R$ **c** obtuse $T\widehat{R}S$

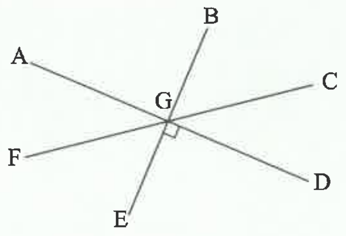
3 **a** Determine the measure of $P\widehat{Q}R$.



b Find, without using a protractor, the measure of reflex $P\widehat{Q}R$. Justify your answer.

4 Draw a diagram to show: **a** (PQ) $\parallel$ (RS) **b** [WX] $\perp$ [YZ]

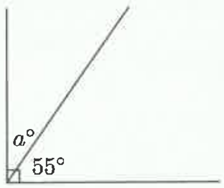
5 Answer the **Opening problem** on page **44**.

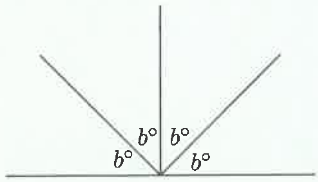
6 

Classify each pair of angles as complementary, supplementary, or neither:

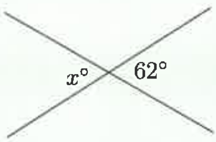
a $C\widehat{G}A$ and $C\widehat{G}D$
b $A\widehat{G}F$ and $A\widehat{G}B$
c $B\widehat{G}C$ and $C\widehat{G}D$
d $B\widehat{G}C$ and $E\widehat{G}F$

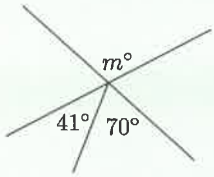
7 Find the unknown value without using a protractor:

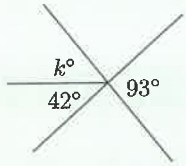
a 

b 

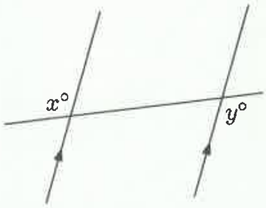
8 Find the unknown value:

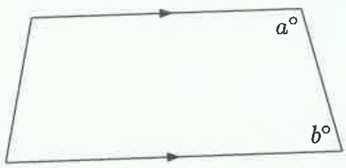
a 

b 

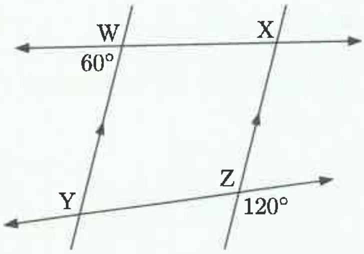
c 

9 Write a statement connecting the unknown values, giving a brief reason.

a 

b 

10 Decide whether (WX) is parallel to (YZ). Explain your answer.



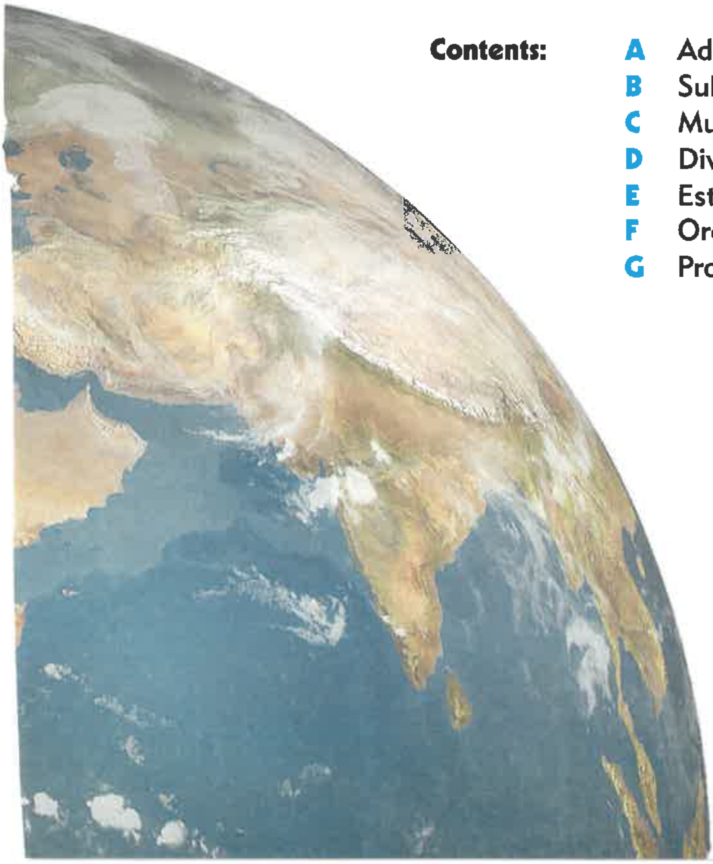
11 **a** Draw [AB] with length 4 cm. Construct a 90° angle at A, and locate point C on this perpendicular such that $AC = AB$. Draw [BC].

b Bisect $C\widehat{A}B$, and label the point where it intersects [BC] as X.

c Use a protractor to measure $A\widehat{X}B$. What do you notice?

Chapter 4

Number strategies and order of operations



Contents:

OPENING PROBLEM

A library has 59 bookcases labelled 1, 2, 3, ..., 59.

The even numbered bookcases each have 4 shelves, and the odd numbered bookcases each have 6 shelves.

Each shelf holds 21 books.

Things to think about:

- **a** What *operations* do we need to perform to calculate the total number of books in the library?
- **b** In which *order* do these operations need to be performed?
- **c** How can we *estimate* the total number of books in the library?

Calculators are useful tools which allow us to perform calculations very rapidly. However, it is important that we also have good number skills ourselves. We need to be able to:

- identify what operations we need to perform, and the order they need to be performed in
- write these operations in a language that is clear for other people to understand
- perform calculations by hand if we do not have a calculator with us
- estimate the size and form of the answer we are expecting, so we can identify if we have made a mistake using our calculator.

In this Chapter, we will therefore study:

- **number strategies** to help us perform operations both mentally and by hand without a calculator
- **estimation** to tell us whether a computed answer is *reasonable*
- an agreed **order of operations** which allows us to write our operations in a way others can understand
- contextual problems where we can practise these skills.

A ADDITION STRATEGIES

Our strategies for addition are based on:

- the property that the order in which numbers are added is not important
- the observation that it is easiest to add numbers with one significant figure.

Strategy	*Example*
1. Write one number in expanded form.	$23 + 38$ $= \underbrace{23 + \overbrace{30 + 8}}$ $= 53 + 8$ $= 61$
2. Write one number as the difference between two numbers with one significant figure each.	$98 + 137$ $= \overbrace{100 - 2} + 137$ $= \underbrace{100 + 137} - 2$ $= 237 - 2$ $= 235$
3. Change the order of addition to one which produces a number with one significant figure along the way.	$8 + 13 + 12$ $= \underbrace{8 + 12} + 13$ $= 20 + 13$ $= 33$

EXERCISE 4A

1 Find by writing one number in expanded form:

a $24 + 15$ **b** $33 + 18$ **c** $14 + 37$
d $35 + 26$ **e** $207 + 88$ **f** $1007 + 54$

2 Find by writing one number as the difference between two numbers with one significant figure each:

a $68 + 39$ **b** $57 + 47$ **c** $998 + 707$
d $234 + 490$ **e** $2996 + 315$ **f** $6990 + 5437$

3 Find by adding in a convenient order:

a $25 + 17 + 15$ **b** $18 + 24 + 12$ **c** $8 + 259 + 92$
d $39 + 16 + 14$ **e** $61 + 24 + 39$ **f** $342 + 137 + 63$

4 Find using an appropriate addition strategy:

a $89 + 106$ **b** $103 + 46$ **c** $82 + 15 + 18$
d $148 + 86$ **e** $316 + 198$ **f** $205 + 73 + 95$
g $63 + 241 + 37$ **h** $122 + 341 + 659$ **i** $79 + 321 + 418$
j $298 + 402 + 398$ **k** $604 + 95 + 296$ **l** $23 + 61 + 27 + 39$

5 A truck driver drove 296 km in the morning and 332 km in the afternoon.
Find the total distance travelled.

6 The stages of a cycling race are 138 km, 216 km, and 162 km.
Find the total length of the cycling race.

B SUBTRACTION STRATEGIES

Strategy	*Example*
1. Perform the subtraction one place value at a time.	$287-149$ $=\underbrace{287-100}-40-9$ $=\underbrace{187-40}-9$ $=147-9$ $=138$
2. Subtract a slightly larger number with one significant figure, then compensate by addition.	$77-29$ $=\underbrace{77-30}+1$ $=47+1$ $=48$

DISCUSSION

How do the strategies for subtraction compare with the strategies for addition?

EXERCISE 4B

1 Find by subtracting one place value at a time:

a $97-43$ **b** $85-38$ **c** $146-75$
d $128-103$ **e** $265-182$ **f** $304-127$

2 Find by subtracting a slightly larger number with one significant figure, then adding to compensate:

a $62-9$ **b** $43-28$ **c** $124-39$
d $318-97$ **e** $482-299$ **f** $3218-996$

3 Find, using an appropriate subtraction strategy:

a $73-39$ **b** $64-36$ **c** $94-47$
d $596-398$ **e** $214-126$ **f** $360-88$

4 Find the difference between:

a 28 and 59 **b** 241 and 158 **c** 137 and 912
d 284 and 169 **e** 398 and 617 **f** 1016 and 848.

5 What number must be increased by 288 to get 415?

6 The Burj Khalifa building in the United Arab Emirates is 828 m tall, whereas Taipei 101 in Taiwan is 509 m tall. How much taller is Burj Khalifa than Taipei 101?

C MULTIPLICATION STRATEGIES

Our strategies for multiplication are based on:

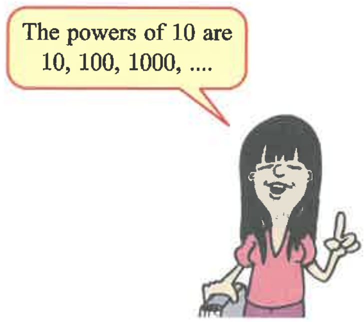
- the property that the order in which numbers are multiplied is not important
- the ease of multiplying by a power of 10.

Strategy	*Examples*	
1. Write one or more numbers as a factor pair involving a power of 10.	$8\times 70\,000$ $= \underbrace{8\times 7}\times 10\,000$ $= 56 \times 10\,000$ $= 560\,000$	400×60 $= 4\times 100\times 6\times 10$ $= \underbrace{4\times 6}\times \underbrace{100\times 10}$ $= 24 \times 1000$ $= 24\,000$
2. Change the order of multiplication to one which produces a number with one significant figure along the way.	$4\times 31\times 25$ $= 31\times \underbrace{4\times 25}$ $= 31\times 100$ $= 3100$	
3. Perform the multiplication one place value at a time.	43×12 $= 43\times 10+43\times 2$ $= 430+86$ $= 430+80+6$ $= 516$	*or* $\begin{array}{rrrr} & & 4 & 3 \\ \times & & 1 & 2 \\ \hline & & 8 & 6 \\ + & {}_{1}4 & 3 & 0 \\ \hline & 5 & 1 & 6 \end{array}$
4. Multiply by a slightly larger number with one significant figure then compensate by subtraction.	65×98 $= 65\times 100-65\times 2$ $= 6500-130$ $= 6370$	{98 lots of 65} {100 lots of 65 minus 2 lots of 65}

EXERCISE 4C

1 Find by writing one or both numbers as a factor pair involving a power of 10:

- **a** 7×40
- **b** 60×8
- **c** 14×20
- **d** 9×7000
- **e** 600×12
- **f** 15×2000
- **g** 400×50
- **h** 90×700
- **i** 600^2



2 Find by multiplying in a convenient order:

a $2 \times 13 \times 5$ **b** $4 \times 9 \times 25$ **c** $20 \times 5 \times 31$ **d** $3 \times 40 \times 5$

e $6 \times 7 \times 50$ **f** $8 \times 3 \times 25$ **g** $4 \times 7 \times 50$ **h** $8 \times 9 \times 125$

3 Find by performing the multiplication one place value at a time:

a 86×4 **b** 136×5 **c** 63×18 **d** 35×22

e 18×140 **f** 82×21 **g** 36×44 **h** 215×16

4 Find by multiplying by a slightly larger number with one significant figure, then subtracting to compensate:

a 16×9 **b** 28×38 **c** 19×106 **d** 7×99

e 39×24 **f** 12×998 **g** 196×43 **h** 25×2997

5 Find, using an appropriate multiplication strategy:

a 37×200 **b** 34×72 **c** 16×2003

d 70×800 **e** $125 \times 6 \times 4$ **f** 6×79

g 124×23 **h** 21×198 **i** 209×13

6 Find:

a $200 \times 37 \times 50$ **b** $40 \times 8 \times 125 \times 25$ **c** $5 \times 80 \times 20 \times 125$

7 A concert hall has 39 rows of seats. There are 35 seats in each row. How many people can be seated in the concert hall?

8 Adrian sold 14 crates of oranges, each weighing 22 kg. If Adrian received \$3 per kg of oranges, how much did he receive in total?

9 A squad of 18 rugby players attend a training camp. Each player takes 4 pairs of socks with them. In total, how many socks are taken to the camp?

D DIVISION STRATEGIES

In any division we can identify a **divisor**, a **dividend**, and a **quotient**.

divisor ÷ dividend = quotient

Our strategies for division are based on recognising factors and multiples.

Strategy	*Examples*	
1. Divide both the divisor and dividend by a common factor.	$4800 \div 80$ $= 480 \div 8$ $= 60$	$98 \div 14$ $= 49 \div 7$ $= 7$
2. Look for an easily recognisable multiple of the dividend which is close to the divisor, then add or subtract to compensate.	$153 \div 3$ $= 150 \div 3 + 3 \div 3$ $= 50 + 1$ $= 51$	$147 \div 3$ $= 150 \div 3 - 3 \div 3$ $= 50 - 1$ $= 49$

EXERCISE 4D

1 Find by first dividing both the divisor and dividend by a common factor which is a power of 10:

a $90 \div 30$ **b** $480 \div 60$ **c** $4900 \div 70$

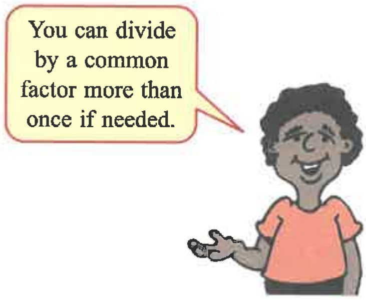
d $7200 \div 800$ **e** $1080 \div 120$ **f** $32\,000 \div 80$

2 Find by first dividing both the divisor and dividend by a common factor:

a $60 \div 15$ **b** $56 \div 28$ **c** $70 \div 14$

d $208 \div 8$ **e** $216 \div 12$ **f** $408 \div 24$

g $210 \div 14$ **h** $450 \div 18$ **i** $720 \div 15$



3 Find by using a multiple of the dividend which is close to the divisor, then adding or subtracting to compensate:

a $205 \div 5$ **b** $186 \div 3$ **c** $294 \div 6$

d $217 \div 7$ **e** $154 \div 7$ **f** $324 \div 6$

4 Dana sells coal in 15 kg bags. If she has 480 kg of coal, how many bags can she sell?

5 A truck is transporting 35 desks to a school. The total mass of the desks is 770 kg. Find the mass of each desk.

6 Lincoln wants to buy a new mobile phone which costs \$189. It will take him exactly 21 weeks to save this amount. How much money is Lincoln able to save each week?

ACTIVITY 1 LONG DIVISION

If the division strategies we have seen do not help perform a division, we can use **long division**.

To find $456 \div 19$ by long division, we follow these steps:

• Starting from the left, consider as many digits of the divisor as necessary to form a number greater than the dividend.	$19 \overline{)\,456}$	45 is greater than 19.
• Find the largest multiple of the dividend which is less than or equal to this number.	$\begin{array}{r} 2 \\ 19 \overline{)\,456} \\ 38 \end{array}$	The largest multiple of 19 less than or equal to 45 is $19 \times 2 = 38$. We write the 2 above the line, and 38 below the 45.
• Subtract this multiple from the number.	$\begin{array}{r} 2 \\ 19 \overline{)\,456} \\ -38 \\ \hline 7 \end{array}$	$45 - 38 = 7$

- Bring the next digit of the divisor down. Repeat the process until all digits of the divisor have been considered.

$$\begin{array}{r|l} & \;\;\;\;\;\;2\;4 \\ \hline 19 & 4\;5\;6 \\ & -3\;8\downarrow \\ \cline{2-2} & \;\;\;\;7\;6 \\ & \;\;-7\;6 \\ \cline{2-2} & \;\;\;\;\;\;\;\;0 \end{array}$$

The largest multiple of 19 less than or equal to 76 is $19 \times 4 = 76$.

So, $456 \div 19 = 24$.

What to do:

1 Find using long division:

a $448 \div 14$ **b** $644 \div 28$ **c** $364 \div 13$

d $306 \div 17$ **e** $665 \div 35$ **f** $1292 \div 34$

2 How many 38 seat buses are needed to transport 646 students to the athletics stadium?

E ESTIMATION

We previously used **rounding** to estimate individual numbers.

Estimation is also a useful skill when we perform operations.

- An estimate can give us a good idea of a number of items when we do not have the time to count them all.
  For example, in a football stadium, we might look around and see there are *about* 20 sections of seating with *about* 20 rows in each section and *about* 50 seats in each row.
  So, the stadium seats *about* $20 \times 20 \times 50$
  $= 400 \times 50$
  $= 20\,000$ people.
- When we perform calculations with bigger numbers, it is easy to make mistakes. It is also easy to make mistakes using a calculator by simply pressing the wrong button.
  To help identify when we have made an error, it is useful to **estimate** the answer. Our estimate should tell us whether the computed answer is *reasonable*.

When estimating, we usually **round** each number to **one significant figure** and evaluate the result. We call this a **one figure approximation**.

Example 1 Self Tutor

Estimate using a one figure approximation:

a 7235×591 **b** $3946 \div 79$

a 7235×591
$\approx 7000 \times 600$
$\approx 4\,200\,000$

b $3946 \div 79$
$\approx 4000 \div 80$
≈ 50

EXERCISE 4E

1 Estimate using a one figure approximation:

a $4326 + 7709$ b $8862 + 4019$ c $41\,965 + 57\,840$
d $786 - 512$ e $9283 - 6147$ f $38\,616 - 24\,762$

2 Estimate using a one figure approximation:

a $683 + 417 + 216 + 891$ b $2311 + 6884 + 3908 + 9780$

3 Estimate using a one figure approximation:

a 22×394 b 218×74 c 58×3102
d 2360×21 e 3904×541 f 376×2308
g $77\,203 \times 409$ h $193 \times 21\,067$ i $38 \times 451\,807$

4 Estimate using a one figure approximation:

a $612 \div 29$ b $230 \div 54$ c $3864 \div 792$
d $8586 \div 299$ e $5890 \div 28$ f $7136 \div 19$
g $64\,183 \div 595$ h $38\,076 \div 822$ i $27\,830 \div 426$

5 A mathematics teacher gives a test to his class of 31 students every week. Estimate the number of tests the teacher will mark in 39 weeks.

6 There are 11 biscuits in a pack. Estimate how many biscuits are in a crate containing 138 packs.

7 Arma went shopping for clothes with her mother. The items they bought cost \$22, \$29, \$59, \$34, and \$49. Estimate the total amount they spent.

8 559 eggs are divided equally between 43 baskets. Estimate the number of eggs in each basket.

9 Mark has just picked 291 avocados from a tree in his orchard. The orchard has 9 rows of avocado trees, with 21 trees in each row.
Estimate the number of avocados in Mark's orchard.

10 a Use the table to estimate the population density of:
i Tokyo ii Taipei.
b Which city is more densely populated?

	Population	*Land area* (km^2)
Tokyo	37 393 000	2194
Taipei	2 646 000	272

11 Use a one figure approximation to estimate the answer to each calculation. Hence decide whether the computed answer is reasonable, or whether a mistake has been made.

	Calculation	*Computed answer*
a	833×6842	5 699 386
b	775×902	69 950
c	$12\,390 \div 21$	590
d	$9\,252\,250 \div 425$	2177

F ORDER OF OPERATIONS

When two or more different operations are carried out, the answer could vary depending on the **order** in which the operations are performed.

For example, consider the expression $16 - 10 \div 2$.

Ilsa subtracted first then divided:

$$\begin{aligned} &16 - 10 \div 2 \\ &= 6 \div 2 \\ &= 3 \end{aligned}$$

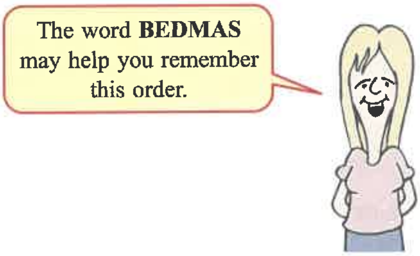
Lily divided first then subtracted:

$$\begin{aligned} &16 - 10 \div 2 \\ &= 16 - 5 \\ &= 11 \end{aligned}$$

Which answer is correct, 3 or 11?

To avoid confusion, there is a set of rules for the **order of operations**:

- Perform the operations within **Brackets** first.
- Calculate any part involving **Exponents**.
- Starting from the left, perform all **Divisions** and **Multiplications** as you come to them.
- Restart from the left, performing all **Additions** and **Subtractions** as you come to them.



The rule of BEDMAS does *not* mean that division should be performed before multiplication, or that addition should be performed before subtraction.

- If an expression contains only $\times$ and $\div$ operations, we work from left to right.
- If an expression contains only $+$ and $-$ operations, we work from left to right.

Using the rule of BEDMAS, Lily's method is correct, and

$$\begin{aligned} &16 - 10 \div 2 \quad \{\div \text{ before } -\} \\ &= 16 - 5 \\ &= 11. \end{aligned}$$

Example 2 **Self Tutor**

Evaluate:

a $3 + 7 - 5$ **b** $6 \times 3 \div 2$

a
$$\begin{aligned} &3 + 7 - 5 \quad \{+ \text{ and } - \text{ from the left}\} \\ &= 10 - 5 \\ &= 5 \end{aligned}$$

b
$$\begin{aligned} &6 \times 3 \div 2 \quad \{\times \text{ and } \div \text{ from the left}\} \\ &= 18 \div 2 \\ &= 9 \end{aligned}$$

EXERCISE 4F

1 Evaluate:

a $9 + 4 - 5$ **b** $9 - 4 + 5$ **c** $9 - 4 - 5$

d $3 \times 12 \div 6$ **e** $12 \div 6 \times 3$ **f** $6 \times 12 \div 3$

g $9 - 3 + 5$ **h** $9 \div 3 \times 5$ **i** $3 + 9 - 5$

Example 3 **Self Tutor**

Evaluate:

a $23 - 10 \div 2$ **b** $7 \times 8 - 6 \times 5$

a $23 - 10 \div 2$ {$\div$ before $-$}
$= 23 - 5$
$= 18$

b $7 \times 8 - 6 \times 5$ {$\times$ before $-$}
$= 56 - 30$
$= 26$

2 Evaluate:

a $8 + 9 \times 3$ **b** $6 \times 3 + 7$ **c** $14 - 2 \times 4$
d $21 - 6 \times 1$ **e** $3 \times 2 - 2$ **f** $30 - 2 \times 2 \times 3$
g $30 \div 2 + 6$ **h** $4 + 3 + 2 \times 6$ **i** $12 - 3 \times 3 + 2$
j $26 - 9 \div 3$ **k** $5 + 12 \div 2$ **l** $15 \div 3 + 16 \div 2$

Example 4 **Self Tutor**

Evaluate: $3 + (11 - 7) \times 2$

$3 + (11 - 7) \times 2$ {brackets first}
$= 3 + 4 \times 2$ {$\times$ before $+$}
$= 3 + 8$
$= 11$

3 Evaluate:

a $15 + (9 - 3)$ **b** $(15 + 9) - 3$ **c** $(12 \div 6) - 2$
d $12 \div (6 - 2)$ **e** $(11 - 8) - 3$ **f** $11 - (8 - 3)$
g $36 - (9 \div 3)$ **h** $(36 - 9) \div 3$ **i** $24 - (6 + 10) - 3$
j $(24 - 6) + (10 - 3)$ **k** $(20 \div 10) \div 2$ **l** $20 \div (10 \div 2)$
m $16 - (4 \times 3) - 2$ **n** $30 \times 6 \div (5 - 2)$ **o** $28 - (3 \times 8) \div 6$

Example 5 **Self Tutor**

Evaluate: $3 \times (6 - 2)^2$

$3 \times (6 - 2)^2$ {brackets first}
$= 3 \times 4^2$ {exponent next}
$= 3 \times 16$
$= 48$

4 Evaluate:

a 2×5^2 **b** $4^3 + 2^2$ **c** $18 - (9 \div 3)^2$
d $18 - 9 \div 3^2$ **e** $(18 - 9) \div 3^2$ **f** $(18 - 9 \div 3)^2$
g $(7 - 2)^2 - 4^2$ **h** $3 \times (4^2 - 9) - 2$ **i** $16 - 2^3 + 3^2$

5 Evaluate using your calculator:

a $16+25\times 9$ **b** $(16+25)\times 9$ **c** $112\div 7+7$

d $112\div(7+7)$ **e** $14+3^2\times 7$ **f** $(14+3^2)\times 7$

6 Replace each $\square$ with either $+$, $-$, $\times$, or $\div$ to make each statement true:

a $5\square 9\div 3=8$ **b** $7\square 11-21=56$ **c** $18-16\square 2=10$

d $17\square 3^2=8$ **e** $13\square 4\times 2=5$ **f** $4\square 13-6\square 7=10$

7 Insert brackets to make each statement true:

a $3\times 4+2\times 5=90$ **b** $3\times 4-5\times 4=28$ **c** $4\times 16-1-6=54$

d $6+7\times 2\div 5=4$ **e** $4+4\div 2+2=5$ **f** $3+11-5\div 3=3$

ACTIVITY 2

Click on the icon to run the BEDMAS Challenge.

How fast can you go?

BEDMAS CHALLENGE

G PROBLEM SOLVING

In this Section we consider some real-world problems where more than one operation is involved.

We need to decide:
- what operations we need to perform
- which order we need to perform them in.

This allows us to write a **mathematical expression** involving numbers and operations, for what we want to calculate.

If necessary, we can use brackets to make sure the operations are performed in the correct order.

Example 6 Self Tutor

At Jody's party there were 5 bags of lollies. Three bags contained 24 lollies each, and the other bags contained 30 lollies each. There were six girls and five boys at the party, and the lollies were shared equally between them.

a Write an *expression* for the number of lollies each child received.

b Calculate the number of lollies each child received.

a The total number of lollies was $3\times 24+2\times 30$.
The total number of children was $6+5$.
So, the number of lollies each child received was $(3\times 24+2\times 30)\div(6+5)$.

b
$$\begin{aligned}&(3\times 24+2\times 30)\div(6+5)\\ &=(72+60)\div 11\\ &=132\div 11\\ &=12\end{aligned}$$
Each child received 12 lollies.

EXERCISE 4G

1 My bank account balance was \$1080. I withdrew amounts of \$427 and \$173. I was then paid \$769.

- **a** Write an *expression* for my current account balance.
- **b** Calculate my current account balance.

2 Alysia stands on some scales with a 15 kg dumbbell in each hand. The scales read 92 kg.

- **a** Write an *expression* for Alysia's mass.
- **b** Calculate Alysia's mass.

3 At an equestrian competition, Marcus and McLain finished equal first. The first prize of \$23 000 and the second prize of \$12 500 were shared equally between them.

- **a** Write an *expression* for the amount each person received.
- **b** Calculate the amount each person received.

4 June buys three shirts costing \$20 each, and a \$30 scarf. She pays for the items with a \$100 note.

- **a** Write an *expression* for the amount of change June will receive.
- **b** Calculate the amount of change June will receive.

5 A tank contains 70 litres of water. Ashley adds 4 buckets of water to the tank, and Tiffany adds 7 buckets of water to the tank. Each bucket holds 8 litres of water.

- **a** Write an *expression* for the total amount of water now in the tank.
- **b** Calculate the amount of water now in the tank.

6 Max has won the lottery. The money is paid to him as a lump sum of £60 000, plus 8 weekly payments of £15 000. He decides to give the money away, sharing it equally between his 2 sons, 4 daughters, and 3 grandchildren.

- **a** Write an *expression* for the amount each person will receive.
- **b** Calculate the amount each person will receive.

7 In the **Opening Problem**, a library has 59 bookcases labelled 1, 2, 3, ..., 59.
The even numbered bookcases each have 4 shelves, and the odd numbered bookcases each have 6 shelves. Each shelf holds 21 books.

- **a** Write an expression involving numbers and symbols, for the total number of books in the library.
- **b** Use a one figure approximation to *estimate* the total number of books in the library.
- **c** Find the *exact* number of books in the library.

GLOBAL CONTEXT — THE ABACUS

GLOBAL CONTEXT

Global context: Scientific and technical innovation

Statement of inquiry: The abacus is a tool that can be used to perform arithmetic calculations.

Criterion: Communicating

QUICK QUIZ

MULTIPLE CHOICE QUIZ

REVIEW SET 4A

1 Find using an appropriate addition strategy:

a $57+25$ **b** $85+96$ **c** $304+68$
d $23+46+27$ **e** $516+290$ **f** $147+316+84$

2 Find the difference between:

a 84 and 39 **b** 141 and 267.

3 Find using an appropriate multiplication strategy:

a 5×90 **b** $25\times 13\times 4$ **c** 23×13
d $50\times 8\times 70$ **e** 19×9 **f** 38×25

4 A pie cart vendor sells 11 dozen pies at €5 each. How much money does he make?

5 Find:

a $1500\div 30$ **b** $270\div 45$ **c** $424\div 8$

6 Placemats are shipped from a warehouse to a kitchenware store in boxes of 12. How many boxes are required for a shipment of 420 placemats?

7 Estimate using a one figure approximation:

a $2746+5149$ **b** 18×52 **c** $781\div 22$

8 In one week, a parking inspector issued 87 fines of \$48 each. Use a one figure approximation to estimate the total value of the fines issued.

9 In the last month, Jamal has put purchases worth \$81, \$53, \$119, \$263, and \$47 on his credit card.

a Use a one figure approximation to estimate the total credit card bill.
b Use an appropriate strategy to find the exact credit card bill.
c Find the difference between your estimate and the exact value.

10 Evaluate:

a $3 + 16 \div 2^2$ **b** $5 \times 4 + 18 \div 3$ **c** $3 \times 25 + 3 \times (7 - 2)$

11 Insert brackets to make each statement true:

a $2 + 12 \div 4 - 2 = 8$ **b** $30 \div 5 + 1 + 4 = 9$

12 The students at a dance school are split into 6 equal classes of 15 students. When one of the teachers retires, the students in her class are distributed equally among the remaining classes.

a Write an expression for the number of students there will now be in each class.

b Evaluate this expression to find the new class size.

13 A new hotel has 11 floors. The ground floor has no guest rooms, 6 floors each have 20 rooms, and the rest each have 25 rooms.

a Write an expression for the total number of guest rooms in the hotel.

b Calculate the total number of guest rooms in the hotel.

c In the opening promotional sale, each room is available for \$99 per night. The hotel is fully booked for the first three nights. Use an appropriate strategy to find the total revenue for the hotel during this time.

REVIEW SET 4B

1 Use an appropriate strategy to find:

a $452 + 198$ **b** $125 \times 7 \times 8$ **c** $596 \div 4$

2 A busker earned \$78 on Friday, \$145 on Saturday, and \$122 on Sunday. Find the total amount earned.

3 Find using an appropriate subtraction strategy:

a $295 - 136$ **b** $94 - 69$ **c** $752 - 396$

4 By how much is 838 greater than 562?

5 At a zoo, the elephant weighs 2750 kg and the giraffe weighs 795 kg. How much heavier is the elephant than the giraffe?

6 Daniel delivers 160 newspapers on his paper route. He performs this task 4 times each week, for 50 weeks each year. How many newspapers does he deliver each year?

7 Find using an appropriate division strategy:

a $6300 \div 70$ **b** $144 \div 18$ **c** $490 \div 5$

8 Estimate using a one figure approximation:

a $7591 - 4620$ **b** $5781 \div 381$ **c** 437×68

9 Scott calculated the value of 622×76 to be 4712. Use a one figure approximation to decide whether Scott's calculated answer is reasonable.

10 A teacher spends 252 minutes marking 12 assignments.

a *Estimate*, using a one figure approximation, the average time spent marking each assignment.

b Use an appropriate strategy to find the *exact* average time spent marking each assignment.

11 Evaluate:

a $2^2 \times (7-3) + 5 \times 8$

b $(13 - 2 \times 5)^2$

12 Replace each $\square$ with either $+$, $-$, $\times$, or $\div$ to make each statement true:

a $6 \square 1 + 7 = 12$

b $20 - 8 \square 2 \square 4 = 12$

13 A small farming group has 16 members. They plan to split a potato harvest equally, so that each member receives 3 kg of potatoes. However, four of the members do not want any potatoes, so the harvest is redistributed among the remaining members.

a Write an expression for the weight (in kg) of potatoes that each remaining member will receive.

b Calculate the weight of potatoes that each remaining member receives.

c One member uses all of his potatoes to make gnocchi, which he sells in 1 kg packs for $14. If 1 kg of potatoes makes 1 kg of gnocchi, how much money will he make from the sale of all his packs?

Chapter 5

Positive and negative numbers

Contents:

OPENING PROBLEM

This is the Super League Greece football premiership table at the end of the 2019/2020 regular season.

Team	W	D	L	GF	GA	GD	Pts
Olympiacos	20	6	0	53	9	44	66
PAOK	18	5	3	50	23	27	59
AEK Athens	15	6	5	42	22	20	51
Panathinaikos	12	8	6	35	23	12	44
OFI	10	4	12	35	35	0	34
Aris	8	10	8	38	32	6	34
Atromitos	9	5	12	31	36	−5	32
AEL	7	9	10	28	33		30
Xanthi	8	6	12		32	−11	30
Asteras Tripolis	8	6	12	33	37	−4	30
Lamia	5	12	9	19	33	−14	27
Volos	7	6	13	23		−19	27
Panetolikos	3	8	15	20	42	−22	17
Panionios	4	5	17	16	45	−29	11

A team's *goal difference* (GD) is found by subtracting their *goals against* (GA) from their *goals for* (GF).

Things to think about:

a What does it mean if a team's goal difference is:
  i greater than zero **ii** zero **iii** less than zero?

b What is the goal difference of AEL?

c How many goals were scored:
  i *for* Xanthi **ii** *against* Volos?

d Find the sum of the goal differences for all of the teams. Can you explain this result?

There are many situations when it is sensible to talk about numbers less than zero.

For example:

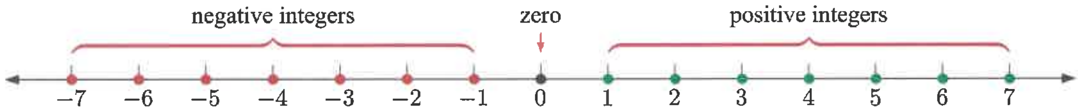
- A credit card balance less than zero means that you *owe* the bank money.
- A temperature less than zero means it is colder than the freezing point of water.
- An elevator floor number less than zero indicates a floor below ground.

Values less than zero are described using **negative numbers**.

A THE NUMBER LINE

The **integers** are the set of all whole numbers.

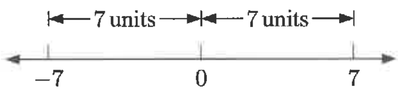
We can represent the set of integers on a **number line** which extends in both directions from zero.



- The **counting numbers** or **positive integers** 1, 2, 3, 4, 5, ... are the whole numbers to the right of zero.
  They can be written with a **positive sign** (+) before the numbers. However, we normally see them with no sign and *assume* they are positive.
- The **negative integers** −1, −2, −3, −4, −5, ... are the whole numbers to the left of zero.
  They are written with a **negative sign** (−) before numerals.
  −1 is read as "minus one" or "negative one".
- Zero is neither positive nor negative.

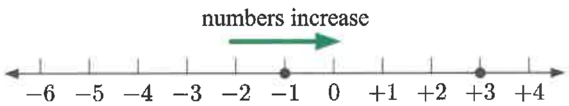
Pairs of numbers like −7 and 7 are the same distance from zero, but on opposite sides of zero.

They are called **opposites**.



ORDERING NUMBERS

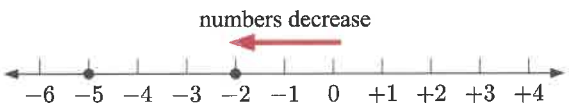
As you move along the number line from *left* to *right*, the numbers *increase*. In a set of numbers, the number furthest to the right is the *greatest* number.



+3 is *greater than* −1 because it is further to the right on the number line.

We write $3 > -1$.

As you move along the number line from *right* to *left*, the numbers *decrease*. In a set of numbers, the number furthest to the left is the *least* number.



−5 is *less than* −2 because it is further to the left on the number line.

We write $-5 < -2$.

Example 1 **Self Tutor**

Arrange the numbers $5, -3, 0, -6$, and 4 in descending order.

In descending order, the numbers are $5, 4, 0, -3$, and -6.

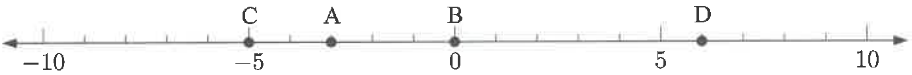
EXERCISE 5A

1 Write down the opposite of:

a -3 **b** $+8$ **c** -10 **d** 7 **e** 19 **f** -14

2 Write down the values of A, B, C, and D:

a



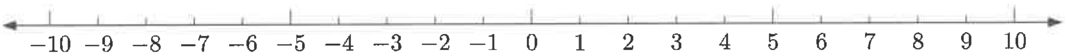
b

3 Place each set of numbers on a number line:

a $-9, 0, 2, 5$ **b** $2, 6, 8, -3, -4$

c $9, -4, -9, 1, 6, -6$ **d** $-3, 2, 5, -5, 0, -1$

4



Use the number line to help you complete each statement with $<$ or $>$:

a 3 ...... -4 **b** -7 ...... -5 **c** -6 1

d -2 ...... 0 **e** -3 ...... -10 **f** 0 ...... -9

5 Decide whether each statement is true or false:

a $6 > -2$ **b** $-4 < -5$ **c** $7 > -8$

d $-6 < 3$ **e** $-13 > 5$ **f** $-20 < -12$

6 Place each set of numbers on a number line, and hence write them in ascending order:

a $4, -2, 1, -1$ **b** $0, -10, 8, -7, -2$

7 Place each set of numbers on a number line, and hence write them in descending order:

a $5, -3, 0, -4$ **b** $-5, 8, -9, -2, 6$

8 The maximum temperatures of six cities on a particular day were:
Ulaanbaatar 3°C, Singapore 33°C, Melbourne 19°C, Oslo -4°C, Moscow -6°C, Tokyo 1°C.
Write these temperatures in order from coldest to hottest.

B WORDS INDICATING POSITIVE AND NEGATIVE

There are many words we commonly use which are *opposites*. We can often associate them with positive and negative.

- Some words refer to our **position** on the number line. They tell us the **sign** of the quantity we are talking about.
  For example:
  - A temperature *above* zero is positive.
  - A temperature *below* zero is negative.
- Some words refer to the **direction** we are moving on the number line. They tell us the **operation** we are performing.
  For example:
  - If a quantity is *increasing*, we are moving to the right, which is the positive direction.
  - If a quantity is *decreasing*, we are moving to the left, which is the negative direction.

ACTIVITY 1

What to do:

1 The words in this table refer to a position.
Copy the table, then use the opposites of the given words to complete it.

Positive (+)	*Negative* (−)
above	
	south of
in front of	
	early
east of	
	left of

2 The words in this table refer to a direction.
Copy the table, then use the opposites of the given words to complete it.

Positive (+)	*Negative* (−)
increasing	
	downwards
profit	
	cooling by
rise	
	backwards

EXERCISE 5B

1 Write each temperature as a positive or negative number:

a 21°C above zero b 17°C below zero c 30°C below zero

2 Assuming north is the positive direction, write each position as a positive or negative number:

a 3 km south **b** 15 km north **c** 250 m south

3 Write as a positive or negative number:

a 9 points in front **b** 6 points behind **c** 15 minutes late
d 5 minutes early **e** 20 m to the left **f** 40 m to the right

4 Write the opposite of:

a 7 km to the west **b** 12 m behind **c** 1 floor above ground level

5 Copy and complete:

	Statement	*Number*	*Opposite of statement*	*Opposite number*
a	winning by 5 goals	$+5$		
b	25 km to the east			
c	The clock is 3 minutes slow.			
d	200 m below sea level			
e	60 g underweight			

6 Write the opposite of:

a moving 7 steps south **b** cooling by 6°C **c** gaining \$50

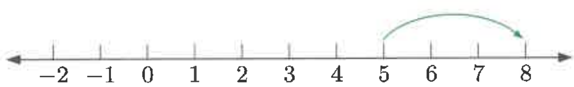
7 Copy and complete:

	Statement	*Operation*	*Opposite of statement*	*Opposite operation*
a	losing 4 kg	subtract 4 kg		
b	3 floors upwards			
c	warming by 6°C			
d	walking 8 m left			
e	falling 20 cm			

C ADDITION AND SUBTRACTION ON THE NUMBER LINE

When we **add** a positive number, we move to the right on the number line. The quantity is increasing.

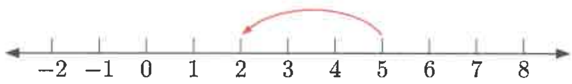
For example, in $5 + 3$, we start at 5 and move 3 units to the right.



$5 + 3 = 8$

When we **subtract** a positive number, we move to the left on the number line. The quantity is decreasing.

For example, in $5-3$, we start at 5 and move 3 units to the left.



$5-3=2$

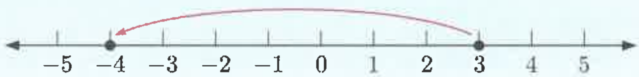
We can continue to apply these rules when our starting number is negative.

Example 2 — Self Tutor

Use a number line to find:

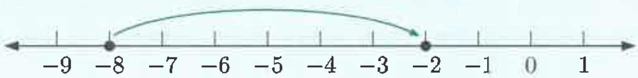
a $3-7$ **b** $-8+6$ **c** $2-5+8$

a We start at 3 and move 7 units to the left.



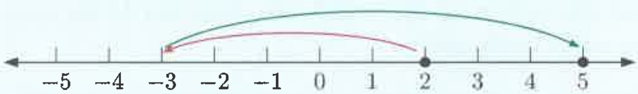
So, $3-7=-4$

b We start at -8 and move 6 units to the right.



So, $-8+6=-2$

c We start at 2, move 5 units to the left, and then 8 units to the right.



So, $2-5+8=5$

EXERCISE 5C

1 Use a number line to find:

a $7-4$ **b** $4-7$ **c** $-7+4$ **d** $-4+7$

2 Use a number line to find:

a $6-6$ **b** $-6+6$ **c** $6+6$ **d** $-6-6$

3 Use a number line to find:

a $5-7$ **b** $-4+3$ **c** $-3+6$ **d** $0-5$
e $-2-7$ **f** $3-9$ **g** $-6+4$ **h** $-5-6$

4 Find:

a $-9+5$ **b** $4-10$ **c** $-2-8$ **d** $-4+11$

5 Use a number line to find:

a $3+2-8$ **b** $-1+3-4$ **c** $-6+2+7$ **d** $4-9+5$

e $-4-7+3$ **f** $7-8-5$ **g** $-9+3-6$ **h** $-5-4-8$

Example 3 Self Tutor

At 8 pm, the temperature in Oslo was -4°C. Between 8 pm and midnight, the temperature rose by 2°C, then fell by 5°C.

What was the temperature at midnight?

The temperature started at -4.

Rising 2°C means we add 2.

Falling 5°C means we subtract 5.

So, the result is $-4+2-5=-7$.

The temperature at midnight was -7°C.

6 At midnight, the temperature in Beijing was 2°C. Between midnight and 9 am, the temperature fell by 7°C, then rose by 3°C. What was the temperature at 9 am?

7 A maintenance worker entered an elevator on the 3rd floor below ground level. The elevator went up 5 floors, then down 3 floors. How many floors above or below ground level is the worker now?

8 Alanna is 4 km south of her campsite. She hikes 3 km south then 7 km north. Where is Alanna now?

9 A diver is 6 m below the surface of the water. He descends 10 m, ascends 3 m, and then descends 7 m. Where is the diver now?

Example 4 Self Tutor

Find the combined effect of spending \$10 and then earning \$5.

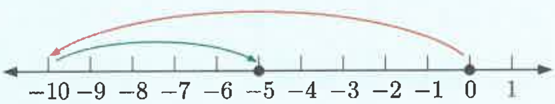
Suppose we start at 0.

Spending \$10 means we subtract 10.

Earning \$5 means we add 5.

So, the result is $0-10+5=-5$.

The combined effect is spending \$5.



10 Find the combined effect of travelling:

a 9 km south and then 4 km north **b** 7 km west and then 13 km east.

11 The staff at a zoo are monitoring the weight of a newborn hippopotamus. Find the combined effect of:

a losing 3 kg then gaining 8 kg **b** gaining 2 kg, losing 4 kg, then gaining 7 kg.

12 Find the combined effect of:

- **a** a rise of 12°C, followed by a fall of 15°C
- **b** going up 8 floors in a lift, then going down 9 floors
- **c** earning \$20, and then spending \$50
- **d** a loss of £50, followed by a profit of £90
- **e** winning by 13 points in the first half, then losing by 27 points in the second half.

13 Find the combined effect of walking 4 km north, then 5 km west, then 4 km south, then 7 km east.

D ADDING AND SUBTRACTING NEGATIVE NUMBERS

INVESTIGATION ADDING AND SUBTRACTING NEGATIVE NUMBERS

What to do:

1 **a** Copy and complete these additions:

$4+3=\ldots\ldots$
$4+2=\ldots\ldots$
$4+1=\ldots\ldots$
$4+0=\ldots\ldots$

b As the number being added to 4 decreases by 1, what happens to the final answer?

c Continue the pattern for these additions:

$4+-1=\ldots\ldots$
$4+-2=\ldots\ldots$
$4+-3=\ldots\ldots$
$4+-4=\ldots\ldots$

d Suggest a rule for adding a negative number.

2 **a** Copy and complete these subtractions:

$4-3=\ldots\ldots$
$4-2=\ldots\ldots$
$4-1=\ldots\ldots$
$4-0=\ldots\ldots$

b As the number being subtracted from 4 decreases by 1, what happens to the final answer?

c Continue the pattern for these subtractions:

$4--1=\ldots\ldots$
$4--2=\ldots\ldots$
$4--3=\ldots\ldots$
$4--4=\ldots\ldots$

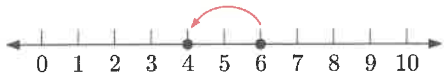
d Suggest a rule for subtracting a negative number.

We have previously seen that:

- to add a positive number, we move to the right on the number line
- to subtract a positive number, we move to the left on the number line.

To add or subtract a *negative* number, we move in the opposite direction.

- To add a negative number, we move to the *left*.
  To find $6 + -2$, we start at 6 and move 2 units to the left.

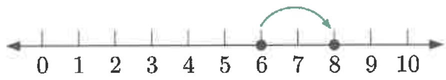


$$6 + -2 = 4$$

Since we know $6 - 2 = 4$, we conclude that:

Adding a **negative** number is the same as **subtracting** its **opposite**.

- To subtract a negative number, we move to the *right*.
  To find $6 - -2$, we start at 6, and move 2 units to the right.



$$6 - -2 = 8$$

Since we know $6 + 2 = 8$, we conclude that:

Subtracting a **negative** number is the same as **adding** its **opposite**.

Example 5 — Self Tutor

Find:

a $2 + -5$ **b** $2 - -5$ **c** $-2 + -5$ **d** $-2 - -5$

a $2 + -5$
$= 2 - 5$
$= -3$

b $2 - -5$
$= 2 + 5$
$= 7$

c $-2 + -5$
$= -2 - 5$
$= -7$

d $-2 - -5$
$= -2 + 5$
$= 3$

EXERCISE 5D

1 Find:

a $4 + -3$ **b** $4 - -3$ **c** $-4 + -3$ **d** $-4 - -3$
e $2 + -6$ **f** $2 - -6$ **g** $-2 + -6$ **h** $-2 - -6$
i $10 + -5$ **j** $10 - -5$ **k** $-10 + -5$ **l** $-10 - -5$

2 Find:

a $1 + -2$ **b** $-2 + -6$ **c** $6 + -8$ **d** $-3 - -7$
e $3 + -13$ **f** $4 - -5$ **g** $-7 - -10$ **h** $-9 + -8$
i $-6 + -3$ **j** $-7 + -5$ **k** $-6 - -9$ **l** $-11 - -17$

3 Find:

a $6 - 8$ **b** $-6 + 8$ **c** $-6 + -8$ **d** $6 - -8$
e $-6 - 8$ **f** $6 + -8$ **g** $6 + 8$ **h** $-6 - -8$

4 Find:

a $4+7+-10$ **b** $8--2+-4$ **c** $-4--6--1$
d $-12--9+5$ **e** $-4--5+-8$ **f** $-3--10+10$
g $-3-7+-7$ **h** $-10+-8+-9$ **i** $-1+10--7$

Example 6 Self Tutor

Find the difference between:

a -3 and 7 **b** -6 and -14.

a Since $-3<7$, the difference between -3 and 7 is
$$\begin{aligned} &7--3\\ &=7+3\\ &=10\end{aligned}$$

b Since $-6>-14$, the difference between -6 and -14 is
$$\begin{aligned} &-6--14\\ &=-6+14\\ &=8\end{aligned}$$

5 Find the difference between:

a -5 and 4 **b** -2 and -5
c 8 and -1 **d** -4 and -2
e 8 and -8 **f** -7 and 8
g -3 and -12 **h** -15 and 6.

The **difference** between two numbers is the *distance* between them on the number line.

6 Deidre and Ian's apartment is 7 m above ground level.
The apartment lift is currently at ground level.
The car park is 5 m below ground level.
The rubbish skip is 10 m below ground level.

a Write an integer to describe the position of each object compared with ground level.

b Find the distance between:
  i the lift and the car park
  ii the apartment and the rubbish skip
  iii the car park and the apartment
  iv the car park and the rubbish skip.

7 Salt Lake City recorded these maximum temperatures over a week:

Mon	Tue	Wed	Thu	Fri	Sat	Sun
5°C	−3°C	−7°C	2°C	1°C	−4°C	−1°C

a Which day had the:
  i warmest maximum temperature **ii** coolest maximum temperature?

b Find the difference between the maximum temperatures of:
  i Monday and Saturday **ii** Wednesday and Sunday **iii** Tuesday and Thursday.

8 Answer the **Opening Problem** on page **86**.

ACTIVITY 2 MAGIC SQUARES

4	3	8
9	5	1
2	7	6

A **magic square** is a square grid filled with **consecutive whole numbers** so that each row, column, and diagonal has the same sum.

For example, this magic square contains the numbers 1 to 9, and has the **magic sum** of 15 along every row, column, and diagonal.

What to do:

1 Copy and complete these magic squares:

a

4		8
	7	
		10

b

		7	12
15		9	6
	5		
8	11	2	

2 Magic squares may also contain negative numbers.

2	-5	0
-3	-1	1
-2	3	-4

a Check that the square alongside is a magic square. What is the magic sum?

b Make a new magic square by adding 2 to each number in the magic square given. State the new magic sum.

c Make a new magic square by subtracting 3 from each number in the magic square given. State the new magic sum.

3 Copy and complete these magic squares:

a

-4		0
	-1	
		2

b

3			-9
-8			
-7		-4	5
6		-1	-6

c

4	11		-5	2
		-6	1	
-9	-7	0	7	
-3			8	
-2			-11	-4

E MULTIPLYING NEGATIVE NUMBERS

We can think of 3×2 as "3 lots of 2", which is $2+2+2=6$

$\therefore\ 3 \times 2 = 6$

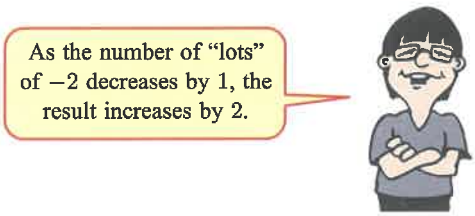
Likewise, we can think of 3×-2 as "3 lots of -2", which is $-2 + -2 + -2 = -6$

$\therefore\ 3 \times -2 = -6$

We can change the order in which two numbers are multiplied without changing the result. We can therefore conclude that $-2 \times 3 = -6$.

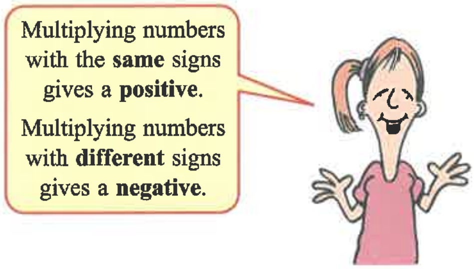
We can also produce this pattern of products:

$$\begin{array}{rl}
3 \times -2 = -6 & \\
 & +2 \\
2 \times -2 = -4 & \\
 & +2 \\
1 \times -2 = -2 & \\
 & +2 \\
0 \times -2 = 0 & \\
 & +2 \\
-1 \times -2 = 2 & \\
 & +2 \\
-2 \times -2 = 4 & \\
 & +2 \\
-3 \times -2 = 6 &
\end{array}$$



Our observations lead us to the following **rules for multiplication**:

- A **positive** times a **positive** gives a **positive**.
- A **positive** times a **negative** gives a **negative**.
- A **negative** times a **positive** gives a **negative**.
- A **negative** times a **negative** gives a **positive**.



Example 7 **Self Tutor**

Find:

a 7×5 **b** 7×-5 **c** -7×5 **d** -7×-5

a $7 \times 5 = 35$ **b** $7 \times -5 = -35$ **c** $-7 \times 5 = -35$ **d** $-7 \times -5 = 35$

EXERCISE 5E

1 Find:

a 6×4 **b** 6×-4 **c** -6×4 **d** -6×-4
e 4×-6 **f** -4×6 **g** 4×6 **h** -4×-6

2 Find:

a 5×-2 **b** -10×3 **c** -1×-6 **d** 5×-10
e -6×8 **f** 5×-9 **g** -8×11 **h** 3×-11
i 9×-9 **j** -12×-2 **k** 11×-5 **l** -6×-7

3 Determine the missing number in each product:

a $-2 \times \square = -2$ **b** $\square \times 5 = -10$ **c** $\square \times 1 = -11$
d $8 \times \square = -32$ **e** $-3 \times \square = 18$ **f** $8 \times \square = -16$
g $-2 \times \square = -8$ **h** $9 \times \square = -9$ **i** $\square \times -7 = -42$
j $\square \times -10 = 30$ **k** $4 \times \square = -12$ **l** $\square \times 12 = -120$

4 In a video game, players are awarded 3 points for capturing another player's flag, but lose 2 points if their own flag is captured.

- **a** In the first round of a competition, Tim captured 1 flag but had his own flag captured 3 times. Find Tim's score after Round 1.
- **b** Tim achieves the same score in each of the first 4 rounds. Find Tim's score after Round 4.

5 Find:

a $(-2)^2$	**b** $2 \times 5 \times -6$	**c** $3 \times -3 \times 4$	**d** $-5 \times 4 \times -2$
e $4 \times -3 \times -6$	**f** $-1 \times -9 \times -7$	**g** $5 \times (-4)^2$	**h** $(-3)^3$

6 Evaluate:

a $(-1)^2$ **b** $(-1)^3$ **c** $(-1)^4$ **d** $(-1)^5$ **e** $(-1)^6$ **f** $(-1)^7$

What do you notice?

7 Write down *all* the factor pairs of each integer. You should include negatives.
For example: -3 and -4 is a factor pair of 12, as $-3 \times -4 = 12$.

a 6	**b** -9	**c** 15	**d** -14
e -12	**f** 13	**g** -24	**h** 36

F DIVIDING NEGATIVE NUMBERS

We have seen previously how multiplication and division are related.

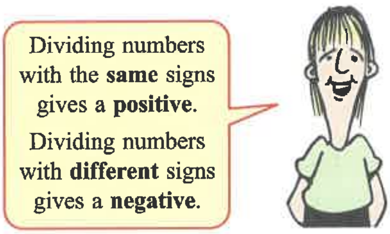
For example, since $9 \times 4 = 36$, we know $36 \div 9 = 4$.

We can also use this principle to perform divisions with negative numbers:

- Since $-9 \times -4 = 36$, we conclude that $36 \div -9 = -4$.
- Since $9 \times -4 = -36$, we conclude that $-36 \div 9 = -4$.
- Since $-9 \times 4 = -36$, we conclude that $-36 \div -9 = 4$.

Our observations lead us to the following **rules for division**:

- A **positive** divided by a **positive** gives a **positive**.
- A **positive** divided by a **negative** gives a **negative**.
- A **negative** divided by a **positive** gives a **negative**.
- A **negative** divided by a **negative** gives a **positive**.



Example 8 Self Tutor

Find:

a $18 \div 6$	**b** $18 \div -6$	**c** $-18 \div 6$	**d** $-18 \div -6$
a $18 \div 6 = 3$	**b** $18 \div -6 = -3$	**c** $-18 \div 6 = -3$	**d** $-18 \div -6 = 3$

EXERCISE 5F

1 Find:

- **a** $15 \div 3$ **b** $15 \div -3$ **c** $-15 \div 3$ **d** $-15 \div -3$
- **e** $45 \div 9$ **f** $45 \div -9$ **g** $-45 \div 9$ **h** $-45 \div -9$
- **i** $44 \div 4$ **j** $44 \div -4$ **k** $-44 \div 4$ **l** $-44 \div -4$

2 Find:

- **a** $-16 \div 8$ **b** $20 \div -4$ **c** $-21 \div -3$ **d** $30 \div -6$
- **e** $-35 \div 5$ **f** $-72 \div -8$ **g** $9 \div -9$ **h** $-49 \div 7$
- **i** $-100 \div -10$ **j** $84 \div -12$ **k** $-66 \div 6$ **l** $-132 \div -11$

3 Determine the missing number in each division:

- **a** $12 \div \square = -3$ **b** $\square \div -2 = 3$ **c** $-4 \div \square = 1$
- **d** $\square \div 5 = -5$ **e** $-18 \div \square = 2$ **f** $\square \div 4 = -3$
- **g** $\square \div -2 = 4$ **h** $30 \div \square = -6$ **i** $36 \div \square = -4$
- **j** $-15 \div \square = -5$ **k** $\square \div -4 = 7$ **l** $72 \div \square = -9$
- **m** $\square \div 10 = -12$ **n** $\square \div -12 = -12$ **o** $-96 \div \square = -8$

4 One night in the Gobi Desert, the temperature drops from 33°C to −12°C in five hours. What is the average temperature change per hour?

5 An aircraft takes off from a runway 1350 m below the nearby mountain top. After 4 minutes it is 1962 m above the mountain. Find the average gain in altitude per minute.

The *average* temperature change per hour is the total temperature change divided by the number of hours.

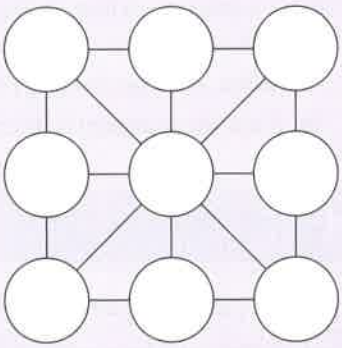
PUZZLE

What to do:

1 Find the average of the integers from −4 up to 4.

2 Arrange the integers from −4 up to 4 in this grid so that every row, every column, and both of the main diagonals add up to the same number.

3 Explain how the possible solutions to this puzzle are related.



G ORDER OF OPERATIONS

To evaluate more complicated expressions involving negative numbers, we must follow the **order of operations** rules.

Example 9 Self Tutor

Evaluate:

a $5 + -8 \times 3$ **b** $-5 - 15 \div (2 - 7)$

a $5 + -8 \times 3$ {$\times$ before $+$}
$= 5 + -24$
$= -19$

b $-5 - 15 \div (2 - 7)$ {brackets first}
$= -5 - 15 \div -5$ {$\div$ before $-$}
$= -5 - -3$
$= -2$

EXERCISE 5G

1 Evaluate:

a $3 + 4 \div -2$ **b** $-1 + -3 \times 2$ **c** $8 \div -2 + 5$
d $4 - 3 \times 2$ **e** $6 \times 5 \div -3$ **f** $16 \div -8 \times -7$
g $2 - 6 \div -3$ **h** $-2 \times 4 + -7$ **i** $-40 \div 4 \div -2$
j $7 - 3 \times -3$ **k** $-4 \times -5 - 12$ **l** $3 - 6 \div -6$

2 Evaluate:

a $(-3 + 4) \times -7$ **b** $15 \div (4 - 7)$ **c** $-3 \times (-2 + 5)$
d $-7 - (-1 - 3)$ **e** $-28 \div (2 \times -2)$ **f** $-36 \div (18 \div -3)$
g $-5 \times (1 - 4)^2$ **h** $(10 - 8 \div -4)^2$ **i** $(-8)^2 \div (-5 + 3 \times 1)$

3 Do -3^2 and $(-3)^2$ have the same value? Explain your answer.

4 Min's company makes an \$18 000 profit each month for four months, and then a \$45 000 loss for each of the next three months.

a Write an *expression* for the company's *average* monthly profit or loss during this period.

b Find the company's average monthly profit or loss during this period.

H CALCULATOR USE

We can use our calculator to perform operations with negative numbers. The negative numbers are entered on the calculator using a key such as $\boxed{(-)}$ or $\boxed{+/-}$.

For example, to find $22 - -45$, we press 22 $\boxed{-}$ $\boxed{(-)}$ 45 $\boxed{=}$.

EXERCISE 5H

1. Use your calculator to evaluate:
   a $-35+61-47$ b $-26--41+38$ c $-92-16+57$
   d $-12--87-129$ e 38×-25 f $-1280\div 320$
   g $-48\div -12\times -6$ h $-630\times 8\div -36$ i $-23\times 17--51$
2. In windy conditions, a helicopter falls 30 m, rises 45 m, falls 20 m, rises 10 m, falls 15 m, then rises 2 m. How far is it now above or below its original position?
3. Regina's credit card balance was $-\$645$. This means she *owed* the bank \$645. She bought a television for \$490 and a heater for \$185, then made a credit card repayment of \$820. What is her credit card balance now?

MULTIPLE CHOICE QUIZ

QUICK QUIZ

REVIEW SET 5A

1. Write down the opposite of:
   a 4 b -7 c 12 d -15
2. Place each set of numbers on a number line, and hence write them in descending order:
   a $-2, 3, -4, 1$ b $0, -5, 7, -3, 2$
3. Write as a positive or negative number:
   a 4 points behind b 5 floors above ground c 7°C below zero
4. Use a number line to find:
   a $5-10$ b $-3+9$ c $-8+6$ d $-7-10$
5. Find the combined effect of:
   a travelling 12 km south and then 4 km north
   b a profit of \$30 followed by a loss of \$50.
6. Find: a $-6--11$ b $5+-7--8$ c $-3-12--8$
7. Beck, Cathy, Emily, and Ying agreed to meet at a coffee shop. Beck was 9 minutes late, Cathy was 4 minutes early, Emily was 17 minutes late, and Ying was 10 minutes early.

   a Who arrived first?
   b Who arrived closest to the agreed time?
   c Find the difference between the arrival times of:
      i Beck and Ying ii Cathy and Emily
      iii Beck and Emily.

8 Find:

a 3×-2 **b** -9×5 **c** -11×-7 **d** $(-8)^2$

9 Find:

a $-24 \div 3$ **b** $18 \div -9$ **c** $-72 \div -8$ **d** $-108 \div 12$

10 Determine the missing number in each statement:

a $\square \times -4 = 28$ **b** $\square \div 5 = -8$ **c** $81 \div \square = -9$

11 Evaluate:

a $36 \div -2 \div -3$ **b** $20 - 8 \div -2$ **c** $9 \times (2 + -5)$

12 Use your calculator to evaluate:

a $-471 + 295 - 356$ **b** -17×27 **c** $94 \times -30 \div -4$

13 In golf, a player's score is expressed relative to the *par* score for the course. For a par 72 course, a player who completes the course in 75 strokes receives a score of 3 over par, or $+3$. A player who takes 68 strokes receives a score of 4 under par, or -4.

a For a par 72 course, find the score for a player who completes the round in:

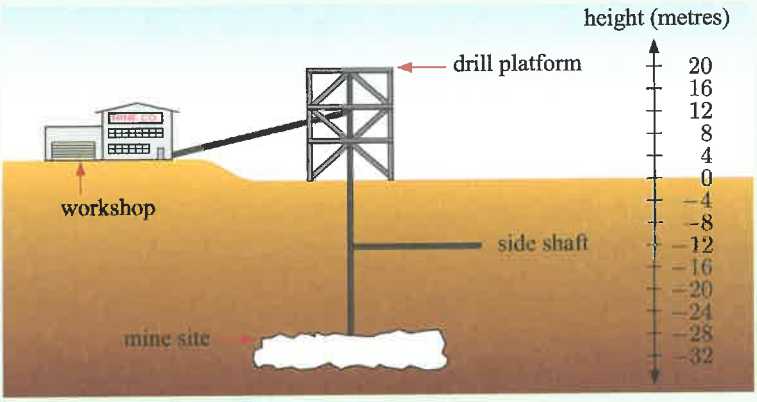
i 70 strokes **ii** 78 strokes **iii** 67 strokes.

b A tournament is played over 4 rounds.

i Trevor shoots a score of -3 for each of the rounds. What is his score at the end of the tournament?

ii Wayne shoots scores of -5, $+2$, and -4 for the first 3 rounds. What score does he need in the final round to beat Trevor?

14



The illustration shows some important parts of a mine.

a Write an integer to describe the position of these objects compared with ground level:

i the top of the drill platform **ii** the bottom of the workshop

iii the side shaft **iv** the top of the mine site.

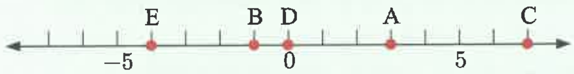
b How much higher is the drill platform than the side shaft?

c How much lower is the mine site than the workshop?

d What is the difference in height between the mine site and the drill platform?

REVIEW SET 5B

1 Write down the values of A, B, C, D, and E:



2 Complete each statement with $<$ or $>$:

a $0 \ldots\ldots -6$ **b** $-5 \ldots\ldots 3$ **c** $-7 \ldots\ldots -10$

3 Write the opposite of warming by 12°C.

4 Find:

a $-6+10$ **b** $2-9+1$ **c** $-4+10-8$

5 A boat is 8 km west of the harbour. The boat sails 7 km west, then 11 km east. Where is the boat now?

6 Find:

a $8--3$ **b** $-5+-9$ **c** $-7+4--12$

7 Which is the greater distance:

A rising from 77 m below sea level to 12 m above sea level, or

B falling from 409 m above sea level to 321 m above sea level?

8 Find:

a 8×-9 **b** $-4\times -12\times 5$ **c** $(-4)^3$

d $32\div -4$ **e** $-63\div 7$ **f** $-80\div -8$

9 In a mathematics competition, students are awarded 3 points for a right answer, and penalised 4 points for a wrong answer. Amy gave 6 wrong answers and 24 right answers. Sean gave 14 wrong answers and 16 right answers.

a How many points did each student score?

b By how many points did Amy beat Sean?

10 The minimum temperatures for a week in Beijing were:
−5°C, −4°C, −5°C, 2°C, 3°C, 3°C, −1°C.
Find the average minimum temperature for that week.

11 Evaluate:

a $7-3\times -7$ **b** $(3-9)^2\div(2\times -3)$ **c** $-48\div(2+7\times -2)$

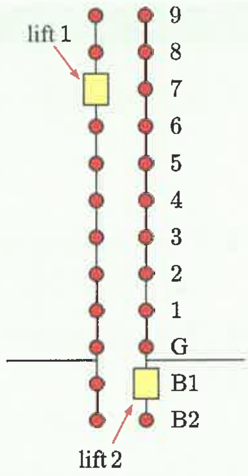
12

Credit Card Statement

Opening Balance	\$ 21 CR
Shoe Store	− \$136
Zhao Restaurant	− \$ 67
Wages	\$228 CR
Electricity	− \$268

Consider the credit card statement opposite. The letters CR indicate a *credit* or positive number. Calculate the closing balance in the account at the end of this period.

13 A new office building has two lifts. There are two floors of underground carparking, and 9 floors above ground. The lifts can move up and down by 1 floor every 5 seconds.



- **a** Lift 1 is on floor 7 and lift 2 is at B1. How far apart are the lifts?
- **b** Lift 1 moves down for 15 seconds. Where is it now?
- **c** Lift 2 moves up for 25 seconds. Where is it now?
- **d** Suppose the lifts are both at floor 3. Lift 1 starts moving up at the same time as lift 2 starts moving down.
  - **i** Write an expression for the distance between the lifts after 10 seconds.
  - **ii** Find the distance between the lifts after 10 seconds.
  - **iii** Will lift 1 reach the top floor before lift 2 reaches the bottom floor?

14 Previous experience has taught Rapunzel that using one's hair to escape is a bad idea. She instead knots bedsheets together, keeping the knots at equal intervals. From her window to the ground is a distance of 25 knots.

- **a** If her window is 0, what integer represents the ground?
- **b** A climbing plant on the side of the tower reaches 11 knots up from the ground. What integer represents the top of the plant?
- **c** Rapunzel starts the climb from her window. She climbs down 6 knots each minute.
  - **i** What is her position after 2 minutes?
  - **ii** How far above or below the top of the plant is she?

Chapter 6

Fractions

Contents:

OPENING PROBLEM

Billy is picking apples at a farm. He picks $\frac{1}{6}$ of the apples in the first hour, $\frac{1}{4}$ of the apples in the second hour, and $\frac{1}{2}$ of the *remaining* apples in the third hour.

Things to think about:

a What fraction of the apples did Billy pick in the first two hours?

b What fraction of apples remained to be picked after two hours?

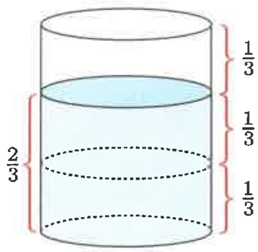
c What fraction of *all* the apples were picked in the third hour?

d There were 480 apples in total. How many apples did Billy pick in the first three hours?

A FRACTIONS

Fractions are used to represent the division of wholes into equal parts.

A container is divided into three equal parts. Two of the parts are filled with water.



We say the container is *two thirds* full, and write this as the fraction $\frac{2}{3}$.

We say the container is *one third* empty, and write this as the fraction $\frac{1}{3}$.

Notice that:

$$\underbrace{\frac{2}{3}}_{\text{filled part of container}} + \underbrace{\frac{1}{3}}_{\text{empty part of container}} = \underbrace{\frac{3}{3}}_{\text{whole container}} = 1$$

A written fraction includes a **numerator**, a **bar**, and a **denominator**.

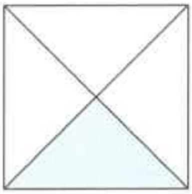
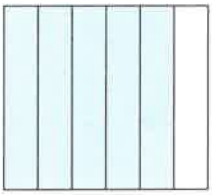
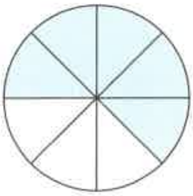
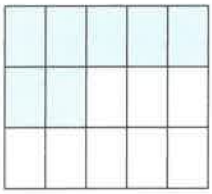
The **denominator** is the number of **equal** parts in a whole.

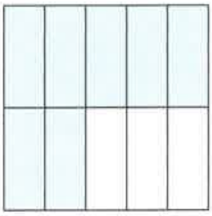
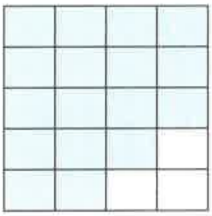
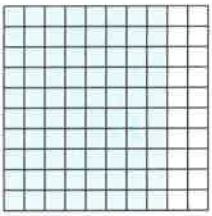
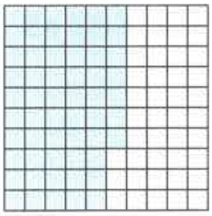
The **numerator** is the number of parts we are looking at.

$$\frac{2}{3} \quad \begin{array}{l} \leftarrow \text{numerator} \\ \leftarrow \text{bar} \\ \leftarrow \text{denominator} \end{array}$$

EXERCISE 6A

1 What fraction of the diagram is shaded?

a 
b 
c 
d 

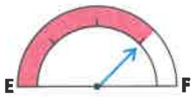
e 
f 
g 
h 

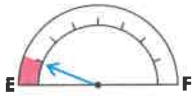
2 Draw a diagram to represent:

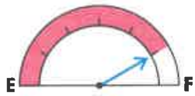
a $\frac{3}{5}$ **b** $\frac{6}{8}$ **c** $\frac{2}{7}$ **d** $\frac{7}{10}$

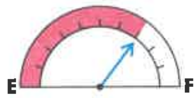
e $\frac{0}{3}$ **f** $\frac{13}{20}$ **g** $\frac{11}{12}$ **h** $\frac{6}{9}$

3 Write down the fraction represented by each fuel gauge:

a 

b 

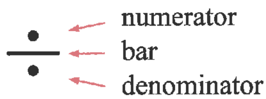
c 

d 

4 There are 23 goldfish in my outside pond. Of these, nine are gold, seven are silver, five are black, and two are white. State what fraction of my goldfish are:

a gold **b** black **c** silver
d gold or silver **e** not black **f** neither black nor white.

B FRACTIONS AS DIVISION



Notice how the division symbol looks like a fraction.

In fact, a fraction is actually a different way to write a division.

$$\frac{2}{3} = 2 \div 3$$

Thinking of a fraction as a division gives meaning to fractions in which the numerator or denominator is negative.

Example 1 **Self Tutor**

Write as a division:

a $\frac{5}{6}$ **b** $\frac{-4}{7}$

a $\frac{5}{6} = 5 \div 6$ **b** $\frac{-4}{7} = -4 \div 7$

EXERCISE 6B

1 Write as a fraction:

a $1 \div 2$ **b** $1 \div 5$ **c** $4 \div 7$ **d** $8 \div 9$
e $2 \div 3$ **f** $9 \div 10$ **g** $9 \div 6$ **h** $20 \div 4$

2 Write as a division:

a $\frac{3}{5}$ **b** $\frac{2}{7}$ **c** $\frac{6}{10}$ **d** $\frac{5}{8}$ **e** $\frac{13}{3}$

3 Write as a fraction:

a $-2 \div 3$ **b** $4 \div -5$ **c** $-6 \div -7$ **d** $10 \div -12$
e $-11 \div 22$ **f** $-23 \div -5$ **g** $16 \div -8$ **h** $-18 \div 2$

4 Write as a division:

a $\frac{-1}{8}$ **b** $\frac{-4}{-6}$ **c** $\frac{3}{-9}$ **d** $\frac{-10}{-2}$ **e** $\frac{-24}{6}$

Example 2 **Self Tutor**

Write as a division, and hence evaluate:

a $\frac{18}{6}$ **b** $\frac{18}{-6}$ **c** $\frac{-18}{6}$ **d** $\frac{-18}{-6}$

a $\frac{18}{6} = 18 \div 6$
$= 3$

b $\frac{18}{-6} = 18 \div -6$
$= -3$

c $\frac{-18}{6} = -18 \div 6$
$= -3$

d $\frac{-18}{-6} = -18 \div -6$
$= 3$

Dividing numbers with the **same** signs gives a **positive**.

Dividing numbers with **different** signs gives a **negative**.

5 Write as a division, and hence evaluate:

a $\frac{8}{2}$ **b** $\frac{15}{5}$ **c** $\frac{24}{8}$ **d** $\frac{10}{10}$ **e** $\frac{16}{4}$
f $\frac{42}{6}$ **g** $\frac{60}{20}$ **h** $\frac{77}{11}$ **i** $\frac{72}{9}$ **j** $\frac{132}{11}$

6 Write as a division, and hence evaluate:

a $\frac{25}{5}$ **b** $\frac{25}{-5}$ **c** $\frac{-25}{5}$ **d** $\frac{-25}{-5}$

e $\frac{27}{9}$ **f** $\frac{-27}{9}$ **g** $\frac{27}{-9}$ **h** $\frac{-27}{-9}$

7 By performing each division, explain why $-\frac{4}{2} = \frac{-4}{2} = \frac{4}{-2}$.

8 Write as a division, and hence evaluate:

a $\frac{-36}{9}$ **b** $\frac{72}{-8}$ **c** $\frac{-63}{7}$ **d** $\frac{-60}{-12}$ **e** $\frac{108}{-9}$

Example 3 Self Tutor

Evaluate:

a $\frac{28-4}{3\times 4}$ **b** $\frac{5-7\times 3}{11-9}$

a $\frac{(28-4)}{(3\times 4)}$

$= \frac{24}{12}$

$= 24 \div 12$

$= 2$

b $\frac{(5-7\times 3)}{(11-9)}$

$= \frac{(5-21)}{2}$

$= \frac{-16}{2}$

$= -16 \div 2$

$= -8$

9 Evaluate:

a $\frac{7+3}{5}$ **b** $\frac{12}{8-2}$ **c** $\frac{2\times -6}{4}$ **d** $\frac{3+11}{10-3}$

e $\frac{3+9}{2\times 3}$ **f** $\frac{20-2}{1+5}$ **g** $\frac{5\times 6}{4-7}$ **h** $\frac{6\times 8}{15-3}$

i $\frac{17-3}{2\times 3+1}$ **j** $\frac{6-5\times 10}{3+8}$ **k** $\frac{25-3\times 5}{10-8}$ **l** $\frac{3-31}{2\times 3-10}$

DISCUSSION

Look at the fraction in **Example 3**, part **a**.

Mark evaluates this fraction by writing:

$$\frac{28-4}{3\times 4}$$
$$= (28-4) \div (3\times 4)$$
$$= 24 \div 12$$
$$= 2$$

- Is Mark's reasoning correct?
- Would Mark obtain the correct answer if he forgot the brackets?

ACTIVITY — USING A CALCULATOR

When we enter operations into a calculator, it automatically uses the rule of BEDMAS. However, we need to be careful with more complicated fractions because we need to divide *all of the numerator* by *all of the denominator*. To make sure the calculator knows what we mean, we insert brackets around the numerator and the denominator.

For example, consider the fraction $\dfrac{5+7}{6-3}$.

If we type in $5\ \boxed{+}\ 7\ \boxed{\div}\ 6\ \boxed{-}\ 3$, the calculator will think we want $5+\frac{7}{6}-3$ and it will therefore give us an incorrect answer.

We need to insert brackets around both the numerator and denominator, giving $\dfrac{(5+7)}{(6-3)}$.

We therefore type in $\boxed{(}\ 5\ \boxed{+}\ 7\ \boxed{)}\ \boxed{\div}\ \boxed{(}\ 6\ \boxed{-}\ 3\ \boxed{)}$, and the calculator will give us the correct answer 4.

What to do:

1 Evaluate each expression, then check your answer using a calculator:

a $8+\dfrac{16}{8}$ **b** $\dfrac{8+16}{8}$ **c** $9-\dfrac{12}{6}$

d $\dfrac{12}{9-6}$ **e** $\dfrac{4+10}{15-8}$ **f** $15-\dfrac{8}{4}+10$

2 Evaluate using a calculator:

a $\dfrac{45-2\times 3}{15-2}$ **b** $\dfrac{30-4}{3\times 4+1}$ **c** $\dfrac{18\times 6+17}{5\times(8-3)}$

C PROPER AND IMPROPER FRACTIONS

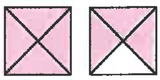
A fraction which has numerator **less** than its denominator is called a **proper fraction**.

A fraction which has numerator **greater** than its denominator is called an **improper fraction**.

For example:

- $\frac{1}{4}$ is a proper fraction

- $\frac{7}{4}$ is an improper fraction.



$\frac{7}{4}=\frac{4}{4}+\frac{3}{4}=1+\frac{3}{4}$

When an improper fraction is written as a whole number and a proper fraction, it is called a **mixed number**.

For example, we can write $\frac{7}{4}$ as the mixed number $1\frac{3}{4}$.

Example 4 — Self Tutor

Write:

a $2\frac{3}{5}$ as an improper fraction **b** $-\frac{27}{4}$ as a mixed number.

a $2\frac{3}{5} = 2 + \frac{3}{5}$

$= \frac{10}{5} + \frac{3}{5}$

$= \frac{13}{5}$

b $\frac{27}{4} = \frac{24}{4} + \frac{3}{4}$

$= 6 + \frac{3}{4}$

$= 6\frac{3}{4}$

$\therefore\ -\frac{27}{4} = -6\frac{3}{4}$

EXERCISE 6C

1 Write as an improper fraction:

a $2\frac{1}{2}$ **b** $1\frac{1}{4}$ **c** $2\frac{4}{5}$ **d** $4\frac{3}{8}$ **e** $5\frac{2}{7}$

f $3\frac{3}{10}$ **g** $1\frac{15}{16}$ **h** $5\frac{7}{12}$ **i** $4\frac{3}{20}$ **j** $6\frac{7}{100}$

2 Write as a mixed number:

a $\frac{9}{5}$ **b** $\frac{5}{4}$ **c** $\frac{21}{10}$ **d** $\frac{31}{3}$ **e** $\frac{23}{6}$

f $\frac{25}{7}$ **g** $\frac{64}{9}$ **h** $\frac{103}{10}$ **i** $\frac{73}{8}$ **j** $\frac{91}{12}$

3 Write as a mixed number:

a $-\frac{6}{5}$ **b** $\frac{-8}{3}$ **c** $\frac{7}{-4}$ **d** $\frac{-11}{5}$ **e** $\frac{13}{-3}$

4 71 eggs are to be packed into cartons of 6.

a How many complete cartons can be filled?

b What fraction of a carton can be filled by the eggs left over?

c Write $\frac{71}{6}$ as a mixed number.

5 89 books are to be packed into cartons of 12.

a How many complete cartons can be filled?

b What fraction of a carton can be filled by the books left over?

c Write $\frac{89}{12}$ as a mixed number.

6 Ella had 15 m of ribbon which she cut into four equal lengths. Express the length of each ribbon as a mixed number of metres.

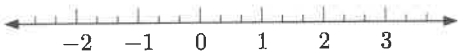
7 Trevor is building a fence between two existing posts 29 m apart. He places 5 equally spaced posts between them then connects the posts with wire.
Find the space between the posts as a mixed number.

D FRACTIONS ON A NUMBER LINE

On a number line, we can divide the gaps between the whole numbers into any number of equal parts we like.

This allows us to include fractions with any denominator we like.

For example, each gap on this number line has been divided into 3, so the number line shows thirds.

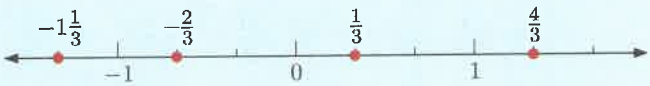


Example 5 Self Tutor

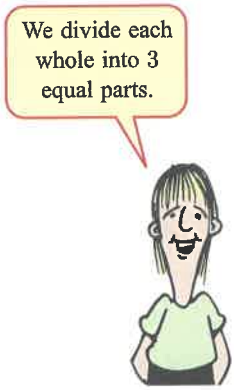
a Place $\frac{1}{3}$, $-1\frac{1}{3}$, $\frac{4}{3}$, and $-\frac{2}{3}$ on a number line.

b Hence write the numbers in ascending order.

a



b In ascending order, the numbers are: $-1\frac{1}{3}$, $-\frac{2}{3}$, $\frac{1}{3}$, $\frac{4}{3}$.



EXERCISE 6D

1 Place on a number line:

a $\frac{2}{8}$, $\frac{3}{8}$, $\frac{5}{8}$

b $\frac{3}{4}$, $\frac{5}{4}$, $1\frac{3}{4}$

c $\frac{2}{5}$, $\frac{3}{5}$, $\frac{8}{5}$

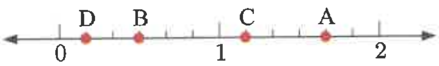
d $\frac{3}{10}$, $\frac{9}{10}$, $\frac{12}{10}$, $1\frac{9}{10}$

e $\frac{5}{6}$, $1\frac{2}{6}$, $\frac{11}{6}$, $2\frac{3}{6}$

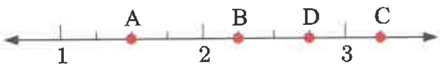
f $\frac{6}{7}$, $1\frac{5}{7}$, $\frac{16}{7}$, $2\frac{3}{7}$

2 Find the fractions represented by the points A, B, C, and D on each number line:

a

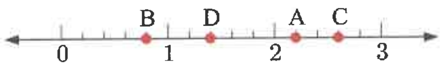


b



c

d



3 **a** Place $1\frac{4}{9}$, $\frac{10}{9}$, $2\frac{2}{9}$, and $\frac{15}{9}$ on a number line.

b Hence write the numbers in ascending order.

4 **a** Place $\frac{4}{7}$, $\frac{12}{7}$, $1\frac{1}{7}$, and $\frac{11}{7}$ on a number line.

b Hence write the numbers in descending order.

5 Place on a number line:

a $-\frac{4}{5}, -\frac{2}{5}, \frac{1}{5}, \frac{3}{5}$ **b** $-\frac{3}{4}, -\frac{1}{4}, \frac{3}{4}, \frac{5}{4}$ **c** $-\frac{5}{3}, -\frac{1}{3}, \frac{2}{3}, \frac{4}{3}$

6 **a** Place $-1\frac{1}{2}$, $2\frac{1}{2}$, $-\frac{6}{2}$, and $\frac{1}{2}$ on a number line.

b Hence write the numbers in ascending order.

7 **a** Place $\frac{4}{3}$, $1\frac{2}{3}$, $-\frac{2}{3}$, $-\frac{1}{3}$, $\frac{2}{3}$ on a number line.

b Hence write the numbers in descending order.

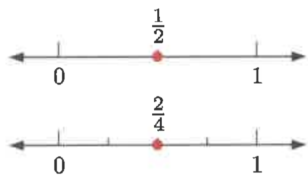
E EQUAL FRACTIONS

Two fractions are **equal** if they have the same value.

Equal fractions lie at the same place on the number line.

For example, the fractions $\frac{1}{2}$ and $\frac{2}{4}$ each lie halfway between 0 and 1 on the number line.

So, $\frac{1}{2} = \frac{2}{4}$.



Notice that:

- If we multiply the numerator and denominator of $\frac{1}{2}$ by 2, we obtain $\frac{2}{4}$.

$$\frac{1}{2} = \frac{2}{4} \quad (\times 2)$$

- If we divide the numerator and denominator of $\frac{2}{4}$ by 2, we obtain $\frac{1}{2}$.

$$\frac{2}{4} = \frac{1}{2} \quad (\div 2)$$

Multiplying or dividing both the numerator and the denominator by the same non-zero number produces an equal fraction.

Example 6 **Self Tutor**

Write $\frac{6}{10}$ with denominator:

a 30 **b** 5

a $\frac{6}{10} = \frac{6 \times 3}{10 \times 3}$ $\{10 \times 3 = 30\}$

$= \frac{18}{30}$

b $\frac{6}{10} = \frac{6 \div 2}{10 \div 2}$ $\{10 \div 2 = 5\}$

$= \frac{3}{5}$

EXERCISE 6E

1 Decide whether each statement is true or false:

a $\frac{4}{5} = \frac{8}{10}$ **b** $\frac{1}{3} = \frac{3}{8}$ **c** $\frac{6}{15} = \frac{2}{5}$ **d** $\frac{16}{24} = \frac{2}{4}$ **e** $\frac{12}{18} = \frac{10}{16}$

2 Write $\frac{2}{5}$ with denominator:

a 10 **b** 20 **c** 50 **d** 100

3 Write $\frac{10}{30}$ with denominator:

a 3 **b** 60 **c** 15 **d** 300

4 Write with denominator 12:

a $\frac{1}{4}$ **b** $\frac{2}{3}$ **c** $\frac{5}{6}$ **d** $\frac{9}{2}$ **e** $\frac{20}{24}$

5 Write with denominator 20:

a $\frac{1}{5}$ **b** $\frac{3}{4}$ **c** $\frac{13}{10}$ **d** $\frac{26}{40}$ **e** $\frac{36}{80}$

6 Write with denominator 100:

a $\frac{3}{10}$ **b** $\frac{4}{5}$ **c** $\frac{11}{25}$ **d** $\frac{43}{50}$ **e** $\frac{140}{200}$

F LOWEST TERMS

A fraction is written in **lowest terms** or **simplest form** if it is written with the smallest possible integer numerator and denominator.

To write a fraction in lowest terms, we divide both the numerator and denominator by their **highest common factor** (HCF).

Example 7 Self Tutor

Write in lowest terms:

a $\frac{7}{21}$ **b** $\frac{100}{90}$

a $\frac{7}{21} = \frac{7 \div 7}{21 \div 7}$ {HCF of 7 and 21 is 7}

$= \frac{1}{3}$

b $\frac{100}{90} = \frac{100 \div 10}{90 \div 10}$ {HCF of 100 and 90 is 10}

$= \frac{10}{9}$

EXERCISE 6F

1 Write in lowest terms:

a $\frac{3}{9}$ **b** $\frac{2}{8}$ **c** $\frac{8}{10}$ **d** $\frac{4}{6}$ **e** $\frac{6}{12}$

f $\frac{18}{24}$ **g** $\frac{16}{20}$ **h** $\frac{18}{21}$ **i** $\frac{20}{25}$ **j** $\frac{11}{33}$

k $\frac{21}{30}$ **l** $\frac{14}{35}$ **m** $\frac{15}{27}$ **n** $\frac{32}{50}$ **o** $\frac{36}{96}$

2 Write as an improper fraction in lowest terms:

a $\frac{6}{4}$ **b** $\frac{8}{6}$ **c** $\frac{16}{10}$ **d** $\frac{14}{8}$ **e** $\frac{20}{12}$

f $\frac{35}{15}$ **g** $\frac{40}{24}$ **h** $\frac{56}{16}$ **i** $\frac{60}{48}$ **j** $\frac{84}{49}$

3 Write in lowest terms:

a $-\frac{2}{6}$ **b** $-\frac{8}{10}$ **c** $\frac{-4}{14}$ **d** $\frac{10}{-25}$ **e** $-\frac{30}{20}$

f $\frac{-9}{15}$ **g** $\frac{10}{-20}$ **h** $-\frac{18}{28}$ **i** $\frac{-28}{12}$ **j** $\frac{40}{-24}$

4 Evaluate, writing your answer in lowest terms:

a $\frac{3\times 2}{13-4}$ **b** $\frac{4-7}{6+2}$ **c** $\frac{3\times 5}{4-9}$ **d** $\frac{5+3}{8+2\times 2}$

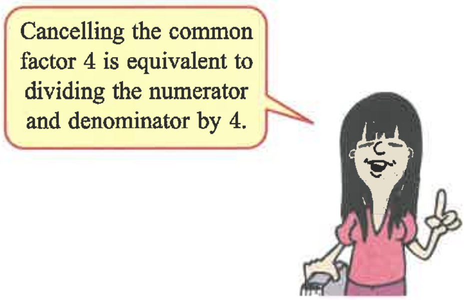
e $\frac{2-3\times 2}{-8\div 2}$ **f** $\frac{6+4}{5+3\times 5}$ **g** $\frac{18+3\times 4}{7+11}$ **h** $\frac{3+5\times 7}{1-40\div 2}$

G CANCELLING COMMON FACTORS

We have seen that whenever we divide both the numerator and denominator of a fraction by a non-zero number, we obtain an equal fraction.

An alternative way to represent this process is to write the numerator and denominator as a product of factors, so that we see the factors they have in common. We then *cancel* the common factors, which is equivalent to dividing by them.

For example: $\frac{8}{12} = \frac{2\times\cancel{4}}{3\times\cancel{4}} = \frac{2}{3}$



With practice, we can cancel the common factor immediately without writing the products of factors first.

$$\frac{\cancel{8}^{\,2}}{\cancel{12}_{\,3}} = \frac{2}{3}$$

Example 8 **Self Tutor**

Write $\frac{18}{42}$ in lowest terms by cancelling common factors.

The HCF of 18 and 42 is 6.

So, $\frac{18}{42} = \frac{3 \times \cancel{6}}{7 \times \cancel{6}} = \frac{3}{7}$ *or* $\frac{{}^{3}\cancel{18}}{{}_{7}\cancel{42}} = \frac{3}{7}$

EXERCISE 6G

1 Cancel the common factors and hence write the resulting fraction:

a $\frac{2 \times 3}{5 \times 3}$ **b** $\frac{9 \times 5}{2 \times 5}$ **c** $\frac{3 \times 2}{2 \times 7}$ **d** $\frac{-4 \times 3}{5 \times 3}$ **e** $\frac{3 \times 4}{4 \times 3}$ **f** $\frac{-1 \times 5}{-1 \times 6}$

2 Write as a whole number by cancelling common factors:

a $\frac{21}{3}$ **b** $\frac{30}{6}$ **c** $\frac{-12}{4}$ **d** $\frac{-6}{-3}$ **e** $\frac{84}{7}$ **f** $\frac{132}{11}$

3 Write in lowest terms by cancelling common factors:

a $\frac{3}{6}$ **b** $\frac{4}{16}$ **c** $\frac{10}{15}$ **d** $\frac{8}{-12}$ **e** $\frac{21}{28}$ **f** $\frac{-30}{35}$

g $\frac{20}{60}$ **h** $\frac{24}{-32}$ **i** $\frac{18}{27}$ **j** $\frac{-36}{45}$ **k** $\frac{42}{63}$ **l** $\frac{60}{108}$

4 Josephine has written a fraction in which the numerator and denominator are different prime numbers. Is Josephine's fraction in lowest terms? Explain your answer.

H ONE QUANTITY AS A FRACTION OF ANOTHER

When we compare quantities in the real world, we can write them as fractions with the quantities in the **same units**. We then write the fraction in lowest terms.

Example 9 **Self Tutor**

Write 80 cents as a fraction of \$3.

The fraction is $\frac{80 \text{ cents}}{\$3}$

$= \frac{80 \text{ cents}}{300 \text{ cents}}$ {\$3 is 300 cents}

$= \frac{4 \times \cancel{20}}{15 \times \cancel{20}}$ {HCF of 80 and 300 is 20}

$= \frac{4}{15}$

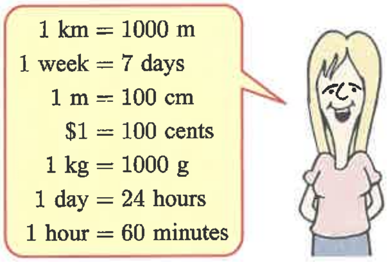
EXERCISE 6H

1 Write:

- **a** 7 kg as a fraction of 35 kg
- **b** 3 hours as a fraction of 15 hours
- **c** \$27 as a fraction of \$45
- **d** 45° as a fraction of 360°
- **e** 75 m as a fraction of 400 m
- **f** 13 cards as a fraction of 52 cards.

2 By first writing the quantities with the same units, write:

- **a** 200 m as a fraction of 1 km
- **b** 8 days as a fraction of 2 weeks
- **c** 75 cm as a fraction of 2 m
- **d** 60 cents as a fraction of \$1
- **e** 400 g as a fraction of 2 kg
- **f** 8 hours as a fraction of 3 days.



3 What fraction of one hour is:

- **a** 20 minutes
- **b** 15 minutes
- **c** 24 minutes?

4 What fraction of one day is:

- **a** 3 hours
- **b** 4 hours
- **c** 16 hours?

5 Nicky scored 21 marks out of 30 for her mathematics test. Write her result as a fraction in lowest terms.

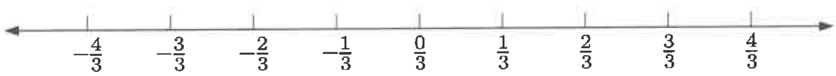
6 Enrique spent €3 on a drink and €5 on a sandwich. What fraction of €20 did he spend?

7 Jo used 150 g from her 1 kg bag of rice to make dinner. What fraction of her rice did she use?

I COMPARING FRACTIONS

To *compare* fractions, we can compare their positions on the number line in the same way we did whole numbers.

Notice that for fractions with the same denominator, the numerators tell us the order of the fractions.



For example:

- $-\frac{4}{3} < -\frac{1}{3}$ because $-\frac{4}{3}$ is to the left of $-\frac{1}{3}$ on the number line
- $\frac{2}{3} > -\frac{1}{3}$ because $\frac{2}{3}$ is to the right of $-\frac{1}{3}$ on the number line.

If we want to compare fractions with *different* denominators, we first write them with the same denominator.

> The **lowest common denominator** (LCD) of a set of fractions is the lowest common multiple (LCM) of their denominators.

The lowest common denominator is usually the most convenient choice.

Example 10 — Self Tutor

Use $<$ or $>$ to complete:

a $\frac{4}{5} \;......\; \frac{7}{9}$

b $\frac{13}{6} \;......\; 2\frac{2}{9}$

a The LCM of 5 and 9 is 45.

$\therefore$ the LCD is 45.

$$\frac{4}{5} = \frac{4 \times 9}{5 \times 9} = \frac{36}{45}$$

and $\frac{7}{9} = \frac{7 \times 5}{9 \times 5} = \frac{35}{45}$

$\frac{36}{45} > \frac{35}{45}$, so $\frac{4}{5} > \frac{7}{9}$.

b $2\frac{2}{9}$ as an improper fraction is $\frac{20}{9}$.

The LCM of 6 and 9 is 18.

$\therefore$ the LCD is 18.

$$\frac{13}{6} = \frac{13 \times 3}{6 \times 3} = \frac{39}{18}$$

and $\frac{20}{9} = \frac{20 \times 2}{9 \times 2} = \frac{40}{18}$

$\frac{39}{18} < \frac{40}{18}$, so $\frac{13}{6} < 2\frac{2}{9}$.

EXERCISE 6I

1. Use $<$ or $>$ to complete:

   a $\frac{5}{8} \;......\; \frac{3}{8}$
   b $\frac{12}{5} \;......\; \frac{9}{5}$
   c $-\frac{2}{7} \;......\; \frac{1}{7}$
   d $-\frac{3}{8} \;......\; -\frac{5}{8}$
   e $-\frac{2}{5} \;......\; -\frac{4}{5}$
   f $\frac{13}{6} \;......\; \frac{11}{6}$

2. Use $<$ or $>$ to complete:

   a $\frac{3}{5} \;......\; \frac{11}{15}$
   b $\frac{2}{9} \;......\; \frac{1}{3}$
   c $-\frac{1}{6} \;......\; -\frac{2}{11}$
   d $\frac{7}{10} \;......\; \frac{19}{25}$
   e $-\frac{8}{11} \;......\; -\frac{3}{4}$
   f $\frac{7}{12} \;......\; \frac{9}{16}$
   g $-\frac{11}{8} \;......\; -\frac{8}{6}$
   h $\frac{16}{5} \;......\; \frac{10}{3}$

3. Use $<$ or $>$ to complete:

   a $2\frac{2}{3} \;......\; \frac{7}{3}$
   b $\frac{11}{4} \;......\; 2\frac{1}{2}$
   c $-1\frac{2}{3} \;......\; -\frac{7}{4}$
   d $\frac{32}{9} \;......\; 3\frac{1}{3}$
   e $-1\frac{7}{8} \;......\; -\frac{11}{6}$
   f $\frac{27}{10} \;......\; 2\frac{3}{4}$

4. Last night, Lex read $\frac{3}{8}$ of a book, and his classmate Tan read $\frac{1}{3}$ of the same book. Who read a greater fraction of the book?

5. Rewrite each set of fractions in ascending order:

   a $\frac{1}{2}, \frac{3}{4}, \frac{1}{3}, \frac{3}{5}$
   b $-\frac{2}{5}, \frac{1}{3}, -\frac{2}{3}, \frac{1}{4}$
   c $\frac{2}{3}, \frac{5}{8}, \frac{7}{9}, \frac{11}{15}$
   d $1\frac{5}{6}, \frac{11}{5}, 2\frac{1}{8}, \frac{-12}{-6}$

CHALLENGE

Write each set of fractions in ascending order:

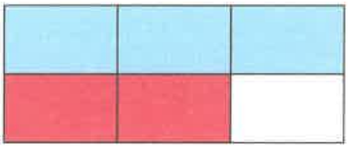
a $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}$

b $\frac{9}{10}, \frac{8}{9}, \frac{7}{8}, \frac{6}{7}, \frac{5}{6}, \frac{4}{5}, \frac{3}{4}, \frac{2}{3}, \frac{1}{2}$

c $\frac{1}{3}, \frac{4}{9}, \frac{3}{6}, \frac{2}{5}, \frac{4}{7}, \frac{3}{10}$

J ADDING AND SUBTRACTING FRACTIONS

In the diagram alongside, we see that:



- $\frac{1}{2}$ or $\frac{3}{6}$ is shaded blue
- $\frac{1}{3}$ or $\frac{2}{6}$ is shaded red.

In total, $\frac{5}{6}$ of the diagram is shaded.

$$\underbrace{\frac{1}{2}}_{\text{blue}} + \underbrace{\frac{1}{3}}_{\text{red}} = \frac{3}{6} + \frac{2}{6} = \underbrace{\frac{5}{6}}_{\text{total shaded}}$$

$\frac{1}{6}$ of the diagram is unshaded.

$$\underbrace{1}_{\text{one whole}} - \underbrace{\frac{5}{6}}_{\text{total shaded}} = \frac{6}{6} - \frac{5}{6} = \underbrace{\frac{1}{6}}_{\text{total unshaded}}$$

To **add** or **subtract** fractions:

- If necessary, write the fractions with the lowest common denominator.
- Add or subtract the new numerators. The denominator stays the same.

Example 11

Self Tutor

Find: **a** $\frac{2}{5} + \frac{1}{4}$ **b** $\frac{5}{6} - \frac{1}{2}$

a
$$\begin{aligned} & \frac{2}{5} + \frac{1}{4} \\ &= \frac{2 \times 4}{5 \times 4} + \frac{1 \times 5}{4 \times 5} \quad \{\text{LCD} = 20\} \\ &= \frac{8}{20} + \frac{5}{20} \\ &= \frac{8+5}{20} \\ &= \frac{13}{20} \end{aligned}$$

b
$$\begin{aligned} & \frac{5}{6} - \frac{1}{2} \\ &= \frac{5}{6} - \frac{1 \times 3}{2 \times 3} \quad \{\text{LCD} = 6\} \\ &= \frac{5}{6} - \frac{3}{6} \\ &= \frac{5-3}{6} \\ &= \frac{{}^{1}\cancel{2}}{{}_{3}\cancel{6}} \quad \{\text{HCF of 2 and 6 is 2}\} \\ &= \frac{1}{3} \end{aligned}$$

EXERCISE 6J

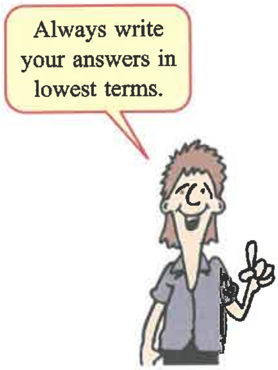
1 Find:

a $\frac{2}{9}+\frac{5}{9}$ **b** $\frac{3}{5}-\frac{1}{5}$ **c** $\frac{4}{7}+\frac{5}{7}$

d $\frac{7}{3}-\frac{4}{3}$ **e** $\frac{4}{15}+\frac{8}{15}$ **f** $\frac{19}{4}-\frac{3}{4}$

g $-\frac{1}{4}+\frac{3}{4}$ **h** $-\frac{2}{3}-\frac{1}{3}$ **i** $1+\frac{3}{5}+\frac{1}{5}$

j $1-\frac{3}{8}+\frac{5}{8}$ **k** $\frac{2}{4}-\frac{3}{4}+\frac{1}{4}$ **l** $-\frac{2}{5}+\frac{4}{5}-\frac{1}{5}$



2 Find:

a $\frac{1}{4}+\frac{1}{8}$ **b** $\frac{9}{10}-\frac{3}{5}$ **c** $\frac{13}{14}-\frac{6}{7}$ **d** $\frac{7}{6}-\frac{2}{3}$

e $\frac{1}{2}-\frac{1}{3}$ **f** $\frac{3}{4}+\frac{3}{5}$ **g** $-\frac{1}{3}+\frac{3}{4}$ **h** $\frac{5}{7}-\frac{2}{3}$

i $\frac{3}{4}-\frac{1}{6}$ **j** $\frac{1}{8}+\frac{5}{12}$ **k** $-\frac{3}{5}-\frac{1}{2}$ **l** $\frac{3}{15}+\frac{7}{20}$

m $\frac{16}{9}-\frac{1}{6}$ **n** $\frac{3}{10}-\frac{1}{8}$ **o** $\frac{3}{2}+\frac{1}{4}-1$ **p** $2-\frac{4}{3}+\frac{1}{4}$

Example 12

Self Tutor

Find:

a $2\frac{1}{3}+1\frac{1}{4}$ **b** $2\frac{2}{3}-1\frac{1}{6}$ **c** $5-2\frac{2}{7}$

a
$$\begin{aligned}&2\tfrac{1}{3}+1\tfrac{1}{4}\\&=\frac{7}{3}+\frac{5}{4}\\&=\frac{7\times 4}{3\times 4}+\frac{5\times 3}{4\times 3}\quad\{\text{LCD}=12\}\\&=\frac{28}{12}+\frac{15}{12}\\&=\frac{28+15}{12}\\&=\frac{43}{12}\\&=3\tfrac{7}{12}\end{aligned}$$

b
$$\begin{aligned}&2\tfrac{2}{3}-1\tfrac{1}{6}\\&=\frac{8}{3}-\frac{7}{6}\\&=\frac{8\times 2}{3\times 2}-\frac{7}{6}\quad\{\text{LCD}=6\}\\&=\frac{16}{6}-\frac{7}{6}\\&=\frac{16-7}{6}\\&=\frac{\cancel{9}^{\,3}}{\cancel{6}_{\,2}}\\&=\frac{3}{2}\\&=1\tfrac{1}{2}\end{aligned}$$

c
$$\begin{aligned}&5-2\tfrac{2}{7}\\&=\frac{35}{7}-\frac{16}{7}\\&=\frac{35-16}{7}\\&=\frac{19}{7}\\&=2\tfrac{5}{7}\end{aligned}$$

3 Find:

a $2\frac{3}{5}+1\frac{4}{5}$ **b** $3\frac{1}{3}-1\frac{2}{3}$ **c** $3\frac{2}{3}-1\frac{1}{2}$ **d** $1\frac{5}{7}+1\frac{3}{14}$

e $3\frac{4}{5}-2\frac{9}{10}$ **f** $1\frac{1}{9}+2\frac{5}{6}$ **g** $4\frac{2}{3}-3\frac{5}{6}$ **h** $-2\frac{3}{4}+3\frac{1}{6}$

4 Find:

a $2-1\frac{1}{4}$ **b** $4-2\frac{2}{3}$ **c** $-6+4\frac{3}{7}$ **d** $9-5\frac{7}{10}$

5 Find:

a the sum of $\frac{4}{5}$ and $\frac{5}{7}$

b the number $1\frac{1}{2}$ more than $\frac{3}{8}$

c the number $\frac{5}{6}$ less than $\frac{19}{18}$

d the difference between $4\frac{9}{10}$ and $2\frac{1}{2}$.

6 Over 2 successive days Toby paves $\frac{3}{10}$ and $\frac{5}{12}$ of his driveway.

a On which day did Toby pave a greater fraction of his driveway?

b What fraction of his driveway has been paved so far?

7 Workers at an office eat $\frac{3}{5}$ of a cake at morning tea and $\frac{3}{8}$ of it at afternoon tea. What fraction of cake remains?

8 In a basketball game, Jacob gets $\frac{4}{9}$ of the rebounds and Adam gets $\frac{3}{10}$ of the rebounds for their team. What fraction of the rebounds did the rest of the team get?

9 Kelly practised guitar for $1\frac{3}{4}$ hours in the morning, and $2\frac{1}{2}$ hours in the afternoon. For how long did she practise in total?

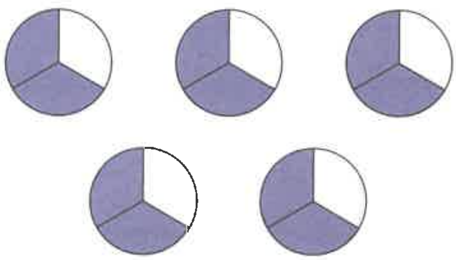
K MULTIPLYING A FRACTION BY A WHOLE NUMBER

The multiplication $\frac{2}{3}\times 5$ means "5 lots of $\frac{2}{3}$".

We start with 5 wholes and shade $\frac{2}{3}$ of each.

In total, we have shaded $\frac{10}{3}$.

We see that $\frac{2}{3}\times 5=\frac{2\times 5}{3}=\frac{10}{3}$.



To multiply a fraction by a whole number, the numerator is multiplied by the whole number. The denominator remains the same.

Example 13 **Self Tutor**

Find:

a $\frac{3}{5} \times 2$ **b** $\frac{1}{6} \times 24$ **c** $\frac{3}{4}$ of 30

a $\frac{3}{5} \times 2$
$= \frac{3 \times 2}{5}$
$= \frac{6}{5}$

b $\frac{1}{6} \times 24$
$= \frac{1 \times \cancel{24}^{4}}{\cancel{6}_{1}}$
$= 4$

c $\frac{3}{4}$ of 30
$= \frac{3}{4} \times 30$
$= \frac{3 \times \cancel{30}^{15}}{\cancel{4}_{2}}$
$= \frac{45}{2}$ or $22\frac{1}{2}$

The word "of" suggests we multiply.

EXERCISE 6K

1 Find:

a $\frac{1}{4} \times 3$ **b** $\frac{2}{7} \times 2$ **c** $\frac{4}{5}$ of 3 **d** $\frac{3}{8} \times 5$

2 Find:

a $\frac{2}{3} \times 6$ **b** $\frac{3}{4}$ of 16 **c** $\frac{2}{9} \times 27$ **d** $\frac{3}{5}$ of 60

3 Find:

a $\frac{3}{10} \times 5$ **b** $\frac{4}{9} \times 3$ **c** $\frac{3}{14}$ of 7 **d** $\frac{5}{6} \times 9$

e $\frac{7}{8}$ of 12 **f** $\frac{7}{12} \times 16$ **g** $\frac{4}{9}$ of 15 **h** $\frac{7}{9} \times 33$

4 Find:

a $\frac{2}{3}$ of \$63 **b** $\frac{3}{7}$ of 35 kg **c** $\frac{7}{10}$ of 20 km

5 Julie owes Grace $\frac{3}{5}$ of \$135. How much does she owe Grace?

6 In a test consisting of 60 questions, Tim answered $\frac{3}{4}$ of the questions correctly. How many questions did Tim answer correctly?

7 The price of a shirt is $\frac{2}{15}$ of the cost of a suit. If the suit costs \$375, find the price of the shirt.

You may use your calculator to evaluate fractions *if you need to.*

8 **a** Find $\frac{3}{5} \times -1$.

b Write $\frac{3}{-5}$ with denominator 5 by multiplying its numerator and denominator by -1.

c What is the relationship between $-\frac{3}{5}$, $\frac{-3}{5}$, and $\frac{3}{-5}$?

L MULTIPLYING FRACTIONS

Ella took $\frac{2}{5}$ of a lasagne to school for her lunch. She only ate $\frac{2}{3}$ of what she took.

Ella ate $\frac{2}{3}$ of $\frac{2}{5}$ of the whole lasagne, which we find by the multiplication $\frac{2}{3} \times \frac{2}{5}$.

Suppose we divide the whole lasagne into 5 fifths, then divide each fifth into 3 thirds.

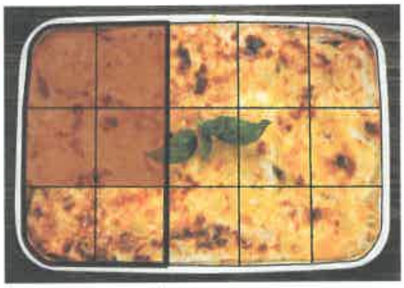
In total there are $3 \times 5 = 15$ pieces.

Ella ate 2 pieces from each of the 2 fifths, so she ate $2 \times 2 = 4$ pieces.

Ella ate $\frac{4}{15}$ of the whole lasagne.

$$\frac{2}{3} \times \frac{2}{5} = \frac{2 \times 2}{3 \times 5} = \frac{4}{15}$$

$\frac{2}{3}$

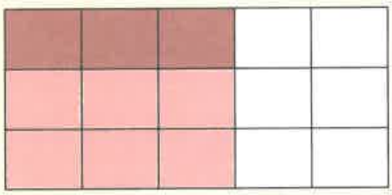


$\frac{2}{5}$ taken to school

INVESTIGATION 1 MULTIPLYING FRACTIONS

What to do:

1 Use the shaded rectangle alongside to show that $\frac{1}{3} \times \frac{3}{5} = \frac{3}{15}$.



2 Using the method in **1**, copy and complete this fraction multiplication table.

	$\frac{1}{4}$	$\frac{2}{3}$	$\frac{3}{5}$
$\frac{1}{2}$			
$\frac{1}{3}$			$\frac{1}{3} \times \frac{3}{5} = \frac{3}{15}$
$\frac{3}{4}$			

3 Use your results from **2** to write a rule for multiplying fractions.

From the **Investigation**, you should have found that:

To **multiply** two fractions, we multiply the two numerators to get the new numerator, and multiply the two denominators to get the new denominator.

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

To make multiplication easier, we can **cancel** any **common factors** in the numerator and denominator *before* we multiply.

Example 14 Self Tutor

Find:

a $\frac{2}{11} \times \frac{3}{5}$ **b** $\frac{4}{9} \times \frac{3}{5}$ **c** $\frac{4}{9} \times 1\frac{7}{8}$

a $\frac{2}{11} \times \frac{3}{5}$
$= \frac{2 \times 3}{11 \times 5}$
$= \frac{6}{55}$

b $\frac{4}{9} \times \frac{3}{5}$
$= \frac{4 \times \cancel{3}^{1}}{{}_{3}\cancel{9} \times 5}$
$= \frac{4}{15}$

c $\frac{4}{9} \times 1\frac{7}{8}$
$= \frac{4}{9} \times \frac{15}{8}$
$= \frac{{}^{1}\cancel{4} \times \cancel{15}^{5}}{{}_{3}\cancel{9} \times \cancel{8}_{2}}$
$= \frac{5}{6}$

EXERCISE 6L

1 Find:

a $\frac{1}{2} \times \frac{1}{3}$ **b** $\frac{1}{2} \times \frac{3}{5}$ **c** $\frac{2}{3} \times \frac{2}{3}$ **d** $\frac{4}{3} \times \frac{1}{5}$

e $\frac{2}{9} \times \frac{2}{3}$ **f** $\frac{1}{6} \times \frac{5}{2}$ **g** $\frac{5}{3} \times \frac{2}{3}$ **h** $\frac{7}{10} \times \frac{9}{10}$

i $\frac{3}{4} \times 1\frac{1}{4}$ **j** $1\frac{1}{3} \times \frac{2}{5}$ **k** $2\frac{1}{3} \times 1\frac{3}{4}$ **l** $3\frac{1}{2} \times 2\frac{3}{4}$

2 Evaluate, giving your answer in lowest terms:

a $\frac{3}{4} \times \frac{1}{3}$ **b** $\frac{1}{2} \times \frac{2}{3}$ **c** $\frac{4}{5} \times \frac{5}{4}$ **d** $\frac{5}{6} \times \frac{2}{3}$

e $\frac{3}{4} \times \frac{6}{7}$ **f** $\frac{2}{3} \times \frac{9}{4}$ **g** $\frac{5}{6} \times \frac{3}{10}$ **h** $\frac{3}{8} \times \frac{2}{9}$

i $\frac{3}{11} \times \frac{44}{9}$ **j** $\frac{3}{5} \times 1\frac{2}{3}$ **k** $\frac{4}{7} \times \frac{21}{16}$ **l** $\frac{9}{7} \times 1\frac{1}{6}$

3 Find the product of:

a $\frac{3}{8}$ and $\frac{1}{3}$ **b** $5\frac{1}{2}$ and $\frac{3}{11}$ **c** $\frac{5}{7}$ and 28 **d** $\frac{3}{10}$ and $\frac{11}{100}$.

4 Find:

a $\frac{3}{4}$ of $\frac{2}{5}$ **b** $\frac{1}{6}$ of $\frac{3}{8}$

c $\frac{4}{5}$ of $\frac{5}{12}$ **d** $\frac{2}{3}$ of $\frac{9}{8}$

e $\frac{3}{5}$ of $3\frac{3}{4}$ **f** $\frac{5}{8}$ of $3\frac{1}{5}$

5 Find:

a $\frac{1}{2} \times \frac{2}{3} \times \frac{1}{2}$ **b** $\frac{1}{3} \times \frac{2}{3} \times \frac{3}{4}$ **c** $\frac{2}{3} \times \frac{1}{4} \times \frac{3}{7}$

d $\frac{2}{3} \times \frac{1}{4} \times \frac{3}{5}$ **e** $\frac{3}{8} \times \frac{4}{3} \times \frac{2}{5}$ **f** $\frac{1}{2} \times \frac{4}{5} \times 3$

6 Find:

a $-\frac{1}{2} \times \frac{5}{6}$ **b** $\frac{3}{4} \times \left(-\frac{2}{9}\right)$ **c** $-\frac{4}{7} \times 1\frac{2}{3}$ **d** $-\frac{6}{5} \times \left(-\frac{3}{8}\right)$

7 Josie worked for $7\frac{1}{2}$ hours yesterday. She spent $\frac{2}{3}$ of the time serving customers. How many hours did she spend serving customers?

8 Penelope drank $\frac{1}{4}$ of a bottle of juice on Monday, and $\frac{2}{5}$ of the *remaining* juice on Tuesday. What fraction of the bottle did Penelope drink on Tuesday?

9 Johnny has $3\frac{1}{2}$ bottles of cordial. Each full bottle contains $2\frac{3}{5}$ litres of lemon cordial. $\frac{1}{20}$ of the lemon cordial is lemon juice. Find the total amount of lemon juice in Johnny's bottles.

M RECIPROCALS

Two numbers are **reciprocals** of each other if their product is one.

INVESTIGATION 2 RECIPROCALS

What to do:

1 Find:

a $\frac{1}{4} \times \frac{4}{1}$ **b** $\frac{1}{3} \times 3$ **c** $\frac{2}{5} \times \frac{5}{2}$

d $\frac{5}{3} \times \frac{3}{5}$ **e** $\frac{3}{7} \times \frac{7}{3}$ **f** $\frac{-4}{3} \times \frac{3}{-4}$

2 Use your observations from **1** to write a rule for finding the reciprocal of a fraction.

From the **Investigation**, you should have found that:

The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

Example 15 | **Self Tutor**

Find the reciprocal of $1\frac{5}{8}$.

$1\frac{5}{8} = \frac{13}{8}$

$\therefore$ the reciprocal of $1\frac{5}{8}$ is $\frac{8}{13}$.

EXERCISE 6M

1. Find the reciprocal of:
   a $\frac{3}{4}$ **b** $\frac{2}{3}$ **c** $\frac{5}{6}$ **d** $\frac{4}{7}$ **e** $\frac{8}{3}$ **f** $\frac{18}{5}$
2. Find the reciprocal of:
   a $1\frac{1}{2}$ **b** $2\frac{2}{3}$ **c** $2\frac{1}{5}$ **d** $4\frac{3}{4}$ **e** $1\frac{7}{8}$ **f** $5\frac{1}{6}$
3. Find the reciprocal of:
   a $-\frac{3}{4}$ **b** $-\frac{1}{3}$ **c** $-\frac{5}{6}$ **d** $-\frac{12}{5}$ **e** $-1\frac{1}{8}$ **f** $-2\frac{4}{5}$

N DIVIDING FRACTIONS

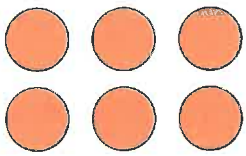
Compare these division stories:

- To find $6 \div 2$, we ask the question "How many twos are there in 6?"

  There are 3 lots of 2 in 6,

  so $6 \div 2 = 3$

  Notice that $6 \div 2 = 6 \times \frac{1}{2}$.

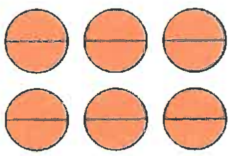


- To find $6 \div \frac{1}{2}$, we ask the question "How many halves are there in 6?"

  There are 12 lots of $\frac{1}{2}$ in 6,

  so $6 \div \frac{1}{2} = 12$

  Notice that $6 \div \frac{1}{2} = 6 \times 2$.

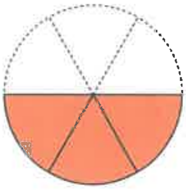


- To find $\frac{1}{2} \div \frac{1}{6}$, we ask the question "How many sixths are there in $\frac{1}{2}$?"

  There are 3 lots of $\frac{1}{6}$ in $\frac{1}{2}$,

  so $\frac{1}{2} \div \frac{1}{6} = 3$

  Notice that $\frac{1}{2} \div \frac{1}{6} = \frac{1}{2} \times 6$.



We conclude that:

To **divide** by a number, we multiply by its reciprocal.

Example 16 Self Tutor

Find: **a** $\frac{5}{4} \div \frac{2}{3}$ **b** $1\frac{1}{3} \div 1\frac{3}{5}$

a
$$\begin{aligned} &\frac{5}{4} \div \frac{2}{3} \\ &= \frac{5}{4} \times \frac{3}{2} \\ &= \frac{5 \times 3}{4 \times 2} \\ &= \frac{15}{8} \end{aligned}$$

b
$$\begin{aligned} &1\frac{1}{3} \div 1\frac{3}{5} \\ &= \frac{4}{3} \div \frac{8}{5} \\ &= \frac{4}{3} \times \frac{5}{8} \\ &= \frac{^1\cancel{4} \times 5}{3 \times \cancel{8}_2} \\ &= \frac{5}{6} \end{aligned}$$

EXERCISE 6N

1 Find:

a $\frac{3}{4} \div \frac{1}{4}$ **b** $\frac{2}{3} \div \frac{1}{3}$ **c** $\frac{5}{2} \div \frac{3}{2}$ **d** $\frac{2}{5} \div \frac{8}{5}$

2 Find:

a $\frac{2}{3} \div \frac{1}{2}$ **b** $\frac{2}{5} \div \frac{1}{6}$ **c** $\frac{3}{5} \div \frac{2}{3}$ **d** $\frac{2}{5} \div \frac{3}{7}$

3 Find:

a $\frac{5}{8} \div 2$ **b** $\frac{1}{2} \div 1\frac{1}{3}$ **c** $\frac{3}{10} \div 1\frac{1}{2}$ **d** $\frac{5}{6} \div 3$

e $\frac{4}{5} \div 8$ **f** $2\frac{1}{2} \div 1\frac{3}{4}$ **g** $3\frac{3}{10} \div 1\frac{5}{6}$ **h** $2\frac{3}{4} \div \frac{2}{3}$

4 Find:

a the average of $\frac{3}{4}$ and $\frac{3}{8}$

b the quotient of $1\frac{1}{2}$ and $\frac{3}{7}$.

5 Find:

a $-\frac{2}{3} \div \frac{3}{4}$

b $\frac{1}{6} \div \left(-\frac{3}{8}\right)$

c $-\frac{2}{5} \div 1\frac{1}{2}$

d $-\frac{4}{5} \div \left(-\frac{8}{3}\right)$

6 To perform the division $\frac{3}{5} \div \frac{2}{5}$, Neil used the method:

$$\begin{aligned}\frac{3}{5} \div \frac{2}{5} &= \frac{\frac{3}{5}}{\frac{2}{5}} \\ &= \frac{3 \times \cancel{5}^{1}}{2 \times \cancel{5}^{1}} \\ &= \frac{3}{2}\end{aligned}$$

a Check that Neil's method gives the correct answer by using the usual method of division of fractions.

b Use Neil's method to find $\frac{9}{7} \div \frac{5}{7}$.

Example 17 **Self Tutor**

Carla the cat eats $\frac{2}{3}$ of a tin of cat food for each meal.

How many meals are in 12 tins of cat food?

$$\begin{aligned}\text{Number of meals} &= 12 \div \frac{2}{3} \\ &= \frac{12}{1} \times \frac{3}{2} \\ &= \frac{{}^{6}\cancel{12} \times 3}{1 \times \cancel{2}_{1}} = 18\end{aligned}$$

7 60 kg of pine nuts are poured into packets so that each packet contains $\frac{3}{4}$ kg of pine nuts. How many packets will be filled?

8 3600 L of water are poured into bottles which hold $1\frac{1}{4}$ L each. How many bottles will be filled?

9 John says that his income is now $3\frac{1}{2}$ times what it was 20 years ago. If his current annual income is \$63 000, what was his income 20 years ago?

10 Tony's orange tree produces a large number of oranges. He keeps one third of them for himself, and shares the rest between his four children. What fraction of the total number of oranges does each child receive?

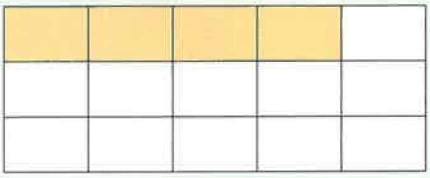
QUICK QUIZ

MULTIPLE CHOICE QUIZ

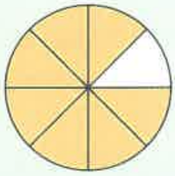
REVIEW SET 6A

1 State the fraction represented by:

a



b



2 Write as a fraction:

a $5 \div 6$ **b** $8 \div 2$ **c** $30 \div 7$

3 Write as a mixed number:

a $\frac{29}{3}$ **b** $\frac{38}{5}$ **c** $\frac{13}{-4}$

4 Place the fractions $-\frac{1}{5}$, $\frac{2}{5}$, $1\frac{1}{5}$, and $\frac{8}{5}$ on a number line.

5 Write:

a $\frac{3}{7}$ with denominator 28 **b** $\frac{15}{40}$ with denominator 8.

6 Write in lowest terms:

a $\frac{3}{18}$ **b** $-\frac{24}{44}$ **c** $\frac{45}{25}$

7 Use $<$ or $>$ to complete:

a $\frac{2}{5}$ ...... $\frac{7}{20}$ **b** $\frac{5}{9}$ ...... $\frac{7}{11}$ **c** $-\frac{3}{8}$ ...... $-\frac{5}{12}$

8 Write:

a 10 minutes as a fraction of 30 minutes **b** 17 cm as a fraction of 1 m

c 45 cents as a fraction of $5.

9 Find:

a $\frac{2}{3} + \frac{2}{5}$ **b** $\frac{7}{10} - \frac{2}{3}$ **c** $2\frac{1}{2} + 3\frac{5}{6}$ **d** $7 - 4\frac{3}{5}$

10 Find:

a $\frac{2}{7} \times \frac{5}{6}$ **b** $\frac{1}{3} \div \frac{4}{9}$ **c** $1\frac{3}{4} \times 2$ **d** $\frac{3}{5} \div 2\frac{3}{4}$

11 Find:

a the product of $\frac{5}{11}$ and $1\frac{1}{2}$

b the average of $\frac{5}{8}$ and $\frac{5}{6}$.

12 Only $\frac{3}{8}$ of a class brought lunch to school yesterday. If there are 24 students in the class, how many students brought lunch to school?

13 An abalone diver has a daily catch limit. He catches $\frac{1}{5}$ of his limit in the first hour, and $\frac{1}{3}$ of his limit in the second hour.

a What fraction of his limit has he caught so far?

b What fraction of his limit is he yet to catch?

14 Laura and Nora buy a 10 kg sack of flour.

a Laura uses 500 g of flour each day. What fraction of the sack is this?

b Nora uses 200 g of flour each day. What fraction of the sack is this?

c What total fraction of the sack is used each day?

d Laura and Nora will buy their next sack of flour in a fortnight. Will their current sack last until then?

15 Ray's apricot tree produced 684 apricots this year.

a Last year the tree produced $\frac{11}{12}$ of this year's harvest. How many apricots did it produce last year?

b $\frac{1}{3}$ of this year's harvest were pecked by birds and spoiled. How many apricots were not spoiled?

c From this year's harvest of good apricots, Ray gave 72 apricots to his mum, and sold $\frac{3}{4}$ of the remainder. How many apricots did Ray have left?

16 Answer the **Opening Problem** on page **106**.

REVIEW SET 6B

1 Aaron is cooking sausages on a barbecue. Four are chicken, seven are beef, six are pork, and three are lamb. State what fraction of Aaron's sausages are:

a beef **b** chicken **c** beef or lamb.

2 Evaluate:

a $\dfrac{20+16}{4\times 3}$ **b** $\dfrac{5\times 10}{13-11}$ **c** $\dfrac{5-2\times 13}{3+4}$

3 Write as an improper fraction: **a** $1\frac{9}{10}$ **b** $7\frac{1}{5}$

4 Find the fractions represented by the points A, B, and C:

5 Write with denominator 24:

a $\frac{1}{3}$ **b** $\frac{5}{8}$ **c** $\frac{7}{12}$

6 Write $\frac{28}{80}$ in lowest terms by cancelling common factors.

7 What fraction of 105 kg is 30 kg?

8 Write the fractions $\frac{4}{15}$, $\frac{2}{5}$, and $\frac{1}{3}$ in ascending order.

9 What number is $\frac{3}{7}$ less than $2\frac{1}{2}$?

10 Find:

a $\frac{5}{8} \times 7$ **b** $\frac{3}{4} \times 10$ **c** $\frac{2}{5}$ of 30

11 Find:

a $-\frac{3}{4} + \frac{5}{6}$ **b** $5\frac{2}{5} - 1\frac{1}{3}$ **c** $-\frac{2}{7} \times \frac{5}{8}$ **d** $3\frac{1}{2} \div 1\frac{1}{4}$

12 350 kg of plastic is moulded to make garden pots weighing $1\frac{2}{5}$ kg each. How many pots are made?

13 Cheryl ate $\frac{1}{5}$ of her block of chocolate yesterday and $\frac{1}{3}$ of the *remaining* chocolate today. What fraction of the block did she eat today?

14 At the start of a picnic, three full bottles of soft drink were placed in a cooler. At the end of the picnic, one bottle was still full, one was $\frac{3}{4}$ full, and the other was $\frac{1}{3}$ full.

a How many bottles of soft drink were consumed?

b What *fraction* of the soft drink was consumed?

c Each bottle contained $1\frac{1}{2}$ litres of soft drink when full. How many litres of soft drink remains?

15 William is copying music and video files to his 120 GB storage drive.

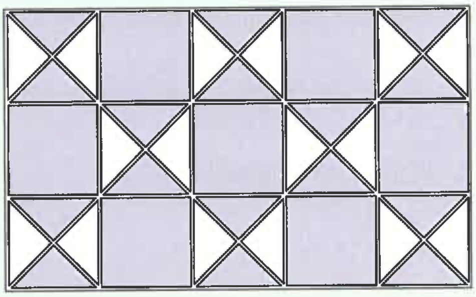
a If William's video files are each $\frac{2}{5}$ GB in size, how many video files would fit on the drive?

b A music file is $\frac{3}{80}$ the size of a video file.

i Find the size of a music file.

ii How many music files would fit on the empty drive?

16 Leo is tiling his bathroom wall with square and triangular tiles. He has completed the section shown.



a What fraction is covered by square tiles?

b What fraction is covered by white tiles?

c A triangular tile is $\frac{2}{3}$ of the cost of a square tile. If square tiles cost \$3 each, what is the total cost of the tiles needed for this section?

Chapter 7

Decimals

Contents:

OPENING PROBLEM

Jevonte noticed this nutritional information label on his half-loaf of bread.

Nutrition		
Typical values	100 g contains	Each slice (typically 44 g) contains
Energy	985 kJ 235 kcal	435 kJ 105 kcal
Unsaturated fat	1.2 g	0.6 g
Saturated fat	0.3 g	0.1 g
Carbohydrate	45.5 g	20.0 g
Fibre	2.8 g	1.2 g
Protein	7.7 g	3.4 g
Salt	1.0 g	0.4 g

Things to think about:

a In total, how much fat is in each 100 g of bread?

b In each 100 g of bread, how much more protein is there than fibre?

c If the half-loaf weighs 400 g, how much carbohydrate does it contain?

d How many slices of bread would Jevonte need to eat to consume 6 g of fibre?

Decimal numbers are widely used in everyday life. We see them frequently when using money, and in measurements of length, time, and weight.

A DECIMAL NUMBERS

The number system we use today is a **place value system**.

For example, the whole number 731 represents 7 hundreds, 3 tens, and 1 unit.

hundreds	tens	units
7	3	1

We say the place value system is in "base 10" because each place value is 10 times the place value to its right, and $\frac{1}{10}$ of the place value to its left.

Using a **decimal point** allows us to represent values between the whole numbers. The decimal point separates the whole number part from the fraction part.

For example, the **place value table** for 731.245 and 24.059 is shown below:

	whole number part				fraction part		
	hundreds	tens	units		tenths	hundredths	thousandths
731.245	7	3	1	.	2	4	5
24.059		2	4	.	0	5	9

We can write each number in **expanded form** by using its place values:

$$731.245 \text{ represents } 700 + 30 + 1 + \frac{2}{10} + \frac{4}{100} + \frac{5}{1000}$$

$$24.059 \text{ represents } 20 + 4 + \frac{0}{10} + \frac{5}{100} + \frac{9}{1000}$$

We normally leave out the $+\frac{0}{10}$ as it has no value. However, it is essential to include the 0 in the decimal number so that the digits 5 and 9 have the correct place values.

When the whole number part of a decimal number is zero, we write a zero in the units place. This gives more emphasis to the decimal point.

For example, we write 0.75 instead of .75.

Example 1 **Self Tutor**

Write in expanded form: 7.802

$$7.802 = 7 + \frac{8}{10} + \frac{2}{1000}$$

EXERCISE 7A

1 Between which two whole numbers does each decimal number lie?

a 7.4 **b** 15.1 **c** 9.46 **d** 22.803

2 In the decimal number 0.5416, state the value of the digit:

a 5 **b** 4 **c** 1 **d** 6

3 State the value of the digit 7 in:

a 1723 **b** 3.7128 **c** 23.07 **d** 88.0672
e 0.8713 **f** 73 066 **g** 81.794 **h** 19.3017

4 Write in expanded form:

a 4.2 **b** 7.53 **c** 9.18 **d** 3.03
e 0.234 **f** 1.059 **g** 5.0061 **h** 0.000 71
i 2.501 **j** 0.0771 **k** 11.912 **l** 0.0101

Example 2 **Self Tutor**

Write as a decimal number:

a $\frac{3}{10} + \frac{7}{100}$ **b** $4 + \frac{9}{10} + \frac{6}{1000}$

a $\frac{3}{10} + \frac{7}{100} = 0.37$ **b** $4 + \frac{9}{10} + \frac{6}{1000} = 4.906$

5 Write as a decimal number:

a $\frac{7}{10}$ **b** $\frac{1}{10}+\frac{5}{100}$ **c** $\frac{5}{10}+\frac{4}{100}+\frac{9}{1000}$

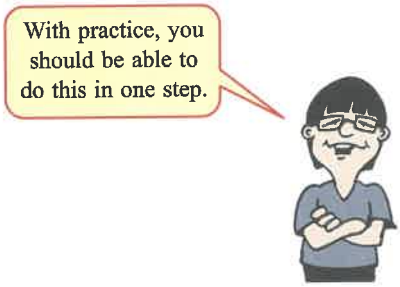
d $\frac{3}{100}$ **e** $\frac{1}{10}+\frac{5}{1000}$ **f** $\frac{6}{100}+\frac{7}{1000}$

g $\frac{8}{100}+\frac{4}{1000}$ **h** $\frac{3}{1000}+\frac{9}{10\,000}$ **i** $\frac{6}{10}+\frac{1}{100}+\frac{5}{1000}+\frac{5}{10\,000}$

Example 3 **Self Tutor**

Write $4\frac{137}{1000}$ as a decimal number.

$$4\frac{137}{1000} = 4+\frac{100}{1000}+\frac{30}{1000}+\frac{7}{1000}$$
$$= 4+\frac{1}{10}+\frac{3}{100}+\frac{7}{1000}$$
$$= 4.137$$



6 Write as a decimal number:

a $\frac{71}{100}$ **b** $\frac{13}{100}$ **c** $\frac{54}{100}$ **d** $\frac{267}{1000}$

e $\frac{506}{1000}$ **f** $\frac{97}{1000}$ **g** $\frac{803}{1000}$ **h** $\frac{22}{1000}$

7 Write as a fraction with denominator 100:

a 0.03 **b** 0.17 **c** 0.39 **d** 0.72 **e** 0.93

8 Write as a fraction with denominator 1000:

a 0.006 **b** 0.080 **c** 0.092 **d** 0.205 **e** 0.713

9 Write as a decimal number:

a $7\frac{6}{10}$ **b** $3\frac{67}{100}$ **c** $12\frac{17}{100}$ **d** $2\frac{5}{100}$

e $1\frac{461}{1000}$ **f** $6\frac{39}{1000}$ **g** $12\frac{1}{1000}$ **h** $3\frac{7}{10\,000}$

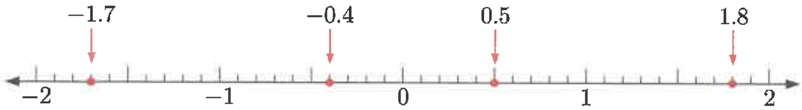
i $5\frac{390}{1000}$ **j** $7\frac{203}{10\,000}$ **k** $\frac{721}{100}$ **l** $\frac{3723}{1000}$

B DECIMAL NUMBERS ON A NUMBER LINE

We can place decimal numbers on a number line in the same way as with whole numbers and fractions. We divide each interval according to the place value of the smallest decimal place.

Extending the number line in both directions allows us to include both positive and negative decimal numbers.

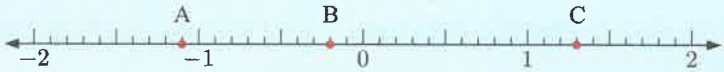
For example, when representing -1.7, -0.4, 0.5, and 1.8, the place value of the smallest decimal place is tenths. We therefore divide each whole into tenths.



Example 4 — Self Tutor

What decimal numbers are at A, B, and C?

a



b

a Each division on the number line represents 0.1.
$A = -1.1$, $B = -0.2$, $C = 1.3$

b Each division on the number line represents 0.01.
$A = 3.07$, $B = 3.25$, $C = 3.34$

EXERCISE 7B

1 Write down the measurement indicated on each device:

a

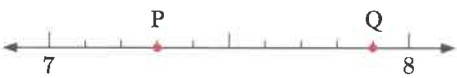
b

c

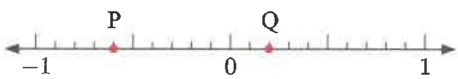
kilograms

2 What decimal numbers are at P and Q?

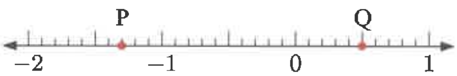
a



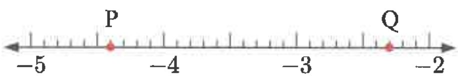
b



c



d

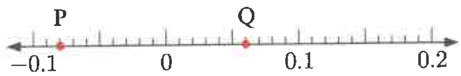


3 What decimal numbers are at P and Q?

a

b

c



d

4 Place on a number line:

a 1.4, 1.06, 1.66, 1.89

b 0.7, 0.35, 0.82, 0.11

5 Draw a number line to illustrate these times: 53.4 seconds, 61.9 seconds, 57.1 seconds, and 63.2 seconds.

Example 5 — Self Tutor

a Place the decimal numbers 0.6, −0.3, 1.2, −0.8, and 0.1 on a number line.

b Hence write the numbers in ascending order.

a

b In ascending order, the numbers are: −0.8, −0.3, 0.1, 0.6, 1.2.

6 **a** Place the decimal numbers 2, 1.7, 2.6, 1.4, and 2.3 on a number line.

b Hence write the numbers in ascending order.

7 **a** Place the decimal numbers −0.1, 0.04, 0.14, −0.03, and 0.1 on a number line.

b Hence write the numbers in descending order.

C ORDERING DECIMAL NUMBERS

To compare decimal numbers without having to construct a number line, we compare digits starting with the largest place values.

Example 6 — Self Tutor

Write in ascending order: 8.66, 8.6, 8.606

Each number has 8 units and 6 tenths.

8.66 has more hundredths than 8.6 and 8.606, so 8.66 is largest.

8.606 has more thousandths than 8.6, so 8.606 is next largest.

In ascending order, the numbers are: 8.6, 8.606, 8.66.

EXERCISE 7C

1 Use $>$, $<$, or $=$ to complete:

a 0.339 ...... 0.393 **b** 5.05 ...... 0.55 **c** 0.6 ...... 0.60
d 2.62 ...... 2.6 **e** 0.39 ...... 0.4 **f** 12.121 ...... 21.121
g 0.123 ...... 0.132 **h** $\frac{150}{1000}$ ...... 0.15 **i** 2.4 ...... 2.400
j 0.902 ...... 0.209 **k** 0.008 76 ...... 0.0876 **l** 3.20 ...... 3.201

2 Write in ascending order:

a 1.36, 1.3, 1.036 **b** 8.76, 8.67, 8.6
c 0.5, 0.495, 0.052 **d** 32.7, 32.71, 33.17
e 8.066, 7.999, 8.1 **f** 6.304, 6.043, 6.403, 6.34
g 9.1, 9.09, 9.2, 9.009 **h** 0.9, 0.09, 0.99, 0.099

3 Write in descending order:

a 4.4, 4.44, 4.2 **b** 7.03, 7.1, 6.97
c 0.6, 0.56, 0.576 **d** 23.5, 23.59, 32.51
e 16.05, 16.4, 16.053 **f** 10.4, 9.49, 10.046
g 3.11, 3.111, 3.101, 3.011 **h** 5.19, 5.901, 5.911, 5.9

4 Patricia's best four times for the 100 m sprint are 16.98 seconds, 16.91 seconds, 17.19 seconds, and 17.1 seconds. Place these times in order from fastest to slowest.

5 This table shows the value of the Euro in Australian dollars (AUD) each day during a particular week.
On which day was the exchange rate:

a highest **b** lowest?

Day	*Euro value in AUD*
Monday	1.632
Tuesday	1.603
Wednesday	1.62
Thursday	1.637
Friday	1.63

D ROUNDING DECIMAL NUMBERS

We are often given measurements as decimal numbers. We can **approximate** decimal numbers by **rounding** to a certain number of **decimal places** or to a certain number of **significant figures**.

RULES FOR ROUNDING DECIMAL NUMBERS

When rounding decimal numbers, we use the same rule as when rounding whole numbers:

To round to a particular place value, look at the digit in the place value to the right of it.

- If this digit is 0, 1, 2, 3, or 4, we round down.
- If this digit is 5, 6, 7, 8, or 9, we round up.

Example 7 Self Tutor

Round 17.4639 to: **a** 2 decimal places **b** 3 significant figures.

a The digit in the 3rd decimal place is 3, so we round down.
$\therefore$ $17.4639 \approx 17.46$ {to 2 decimal places}

b The 4th significant figure is 6, so we round up.
$\therefore$ $17.4639 \approx 17.5$ {to 3 significant figures}

EXERCISE 7D

1 Round to 1 decimal place:
a 3.42 **b** 5.19 **c** 26.84 **d** 36.35
e 7.838 **f** 9.064 **g** 41.703 **h** 52.955

2 Round to 2 decimal places:
a 6.763 **b** 0.277 **c** 4.104 **d** 11.965
e 8.0637 **f** 24.1752 **g** 0.0679 **h** 108.6098

3 Round to 3 decimal places:
a 5.1829 **b** 7.0762 **c** 0.2666 **d** 3.5005
e 40.0967 **f** 63.4899 **g** 8.115 62 **h** 0.080 09

4 Round to the nearest whole number:
a 56.28 **b** 105.74 **c** 243.392 **d** 429.504

5 Round to 2 significant figures:
a 2.54 **b** 6.06 **c** 0.741
d 18.06 **e** 0.043 75 **f** 8.4562

6 Round to 3 significant figures:
a 37.13 **b** 5.207 **c** 0.1085
d 68.628 **e** 0.758 91 **f** 0.030 457

7 Round 23.0599 to:
a the nearest whole number **b** 2 decimal places
c 3 significant figures.

8 Round 8.042 39 to:
a 1 significant figure **b** the nearest tenth
c 4 decimal places.

9 The speed of Latiyah's baseball pitch was recorded as 83.054 km/h. Round this to 1 decimal place.

10 Rhys scores an average of 5.233 goals per game for his waterpolo team. Round this to 2 significant figures.

11 Vicki calculates the interest due on her savings account to be \$57.2894. Round this to the nearest cent.

12 Use your calculator to write these fractions and square roots as decimal numbers. Round each answer to 3 significant figures.

a $\frac{8}{23}$ **b** $\frac{10}{7}$ **c** $\frac{4}{9}$ **d** $\frac{613}{32}$

e $\sqrt{3}$ **f** $\sqrt{43}$ **g** $\sqrt{74}$ **h** $\sqrt{106}$

E CONVERTING DECIMALS TO FRACTIONS

To convert decimals to fractions, we first write the decimal as a fraction where the denominator is a power of 10. We then look to write the fraction in lowest terms.

Example 8 — Self Tutor

Write as a fraction or mixed number in lowest terms:

a 0.8 **b** 3.88 **c** -0.375

a $0.8 = \frac{8 \div 2}{10 \div 2}$
$= \frac{4}{5}$

b $3.88 = 3 + \frac{88 \div 4}{100 \div 4}$
$= 3\frac{22}{25}$

c $-0.375 = -\frac{375 \div 125}{1000 \div 125}$
$= -\frac{3}{8}$

EXERCISE 7E

1 Write as a fraction or mixed number in lowest terms:

a 0.7 **b** 0.4 **c** 1.1 **d** 2.6
e 0.19 **f** 0.29 **g** 0.25 **h** 0.16
i 0.85 **j** 0.96 **k** 0.15 **l** 0.05
m 0.07 **n** 3.13 **o** 5.08 **p** 7.55

2 Write as a fraction or mixed number in lowest terms:

a 0.101 **b** 0.046 **c** 0.205 **d** 0.125
e 0.146 **f** 0.875 **g** 1.375 **h** 4.076

3 Write as a fraction or mixed number in lowest terms:

a -0.6 **b** -0.9 **c** -0.35 **d** -1.2
e -0.24 **f** -2.25 **g** -0.062 **h** -3.375

F CONVERTING FRACTIONS TO DECIMALS

We have already seen how to convert fractions with denominators 10, 100, and 1000 into decimal numbers.

For example:

- $\frac{3}{10} = 0.3$
- $\frac{47}{100} = 0.47$
- $\frac{209}{1000} = 0.209$

Many other fractions can be written as a decimal by first writing it as an equal fraction whose denominator is a power of 10.

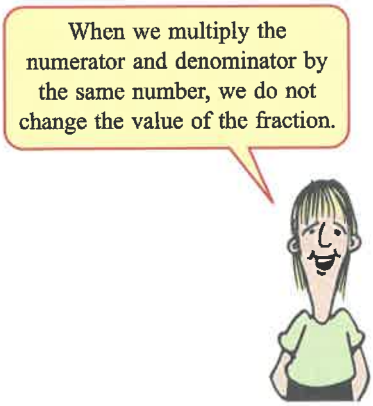
Example 9 Self Tutor

Write as a decimal:

a $\frac{13}{20}$ b $\frac{41}{250}$

a $\frac{13}{20} = \frac{13 \times 5}{20 \times 5}$
$= \frac{65}{100}$
$= 0.65$

b $\frac{41}{250} = \frac{41 \times 4}{250 \times 4}$
$= \frac{164}{1000}$
$= 0.164$



EXERCISE 7F

1 Write as a decimal:

a $\frac{1}{2}$ b $\frac{3}{5}$ c $\frac{11}{20}$ d $\frac{1}{4}$

e $\frac{17}{50}$ f $1\frac{9}{20}$ g $\frac{23}{25}$ h $\frac{1}{50}$

i $\frac{31}{250}$ j $\frac{103}{200}$ k $2\frac{14}{25}$ l $\frac{207}{500}$

m $\frac{19}{20}$ n $\frac{2}{125}$ o $\frac{5}{8}$ p $3\frac{7}{40}$

2 a Write the fractions $\frac{1}{25}$, $\frac{19}{500}$, and $\frac{1}{40}$ as decimals.

b Hence write 0.03, $\frac{1}{25}$, $\frac{19}{500}$, 0.044, and $\frac{1}{40}$ in descending order.

3 Write as a decimal:

a $-\frac{4}{5}$ b $-\frac{3}{4}$ c $-\frac{9}{20}$ d $-\frac{21}{25}$

e $-\frac{67}{100}$ f $-1\frac{1}{4}$ g $-\frac{37}{50}$ h $-2\frac{7}{8}$

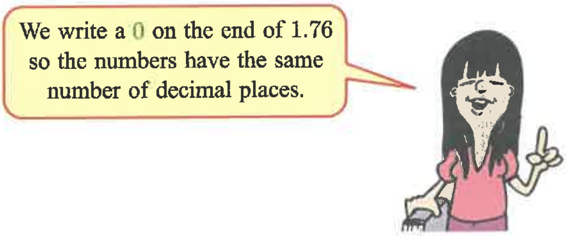
G ADDING AND SUBTRACTING DECIMAL NUMBERS

To **add** or **subtract** decimal numbers, we write the numbers under one another so the decimal points and the place values line up. We then add or subtract each column, working from right to left, in the same way as for whole numbers.

Example 10 — Self Tutor

Find: $1.76 + 0.961$

$$\begin{array}{rcccc} & 1 & . & 7\ 6 & 0 \\ + & {}_1 0 & . & {}_1 9\ 6 & 1 \\ \hline & 2 & . & 7\ 2 & 1 \end{array}$$



EXERCISE 7G

1 Find:

- **a** $0.2 + 0.7$
- **b** $0.6 + 0.33$
- **c** $0.18 + 1.57$
- **d** $0.9 + 0.23$
- **e** $13.56 + 6.073$
- **f** $19.795 + 0.015$
- **g** $0.071 + 0.477$
- **h** $0.0048 + 0.659$
- **i** $0.23 + 0.78 + 3$
- **j** $0.27 + 3.18 + 1.79$
- **k** $1.52 + 0.816 + 3.93$
- **l** $0.104 + 12.78 + 7.379$

Example 11 — Self Tutor

Find: **a** $3.652 - 2.584$ **b** $6 - 0.637$

a
$$\begin{array}{rcccccc} & 3 & . & {}^5\cancel{6} & {}^{14}\cancel{5} & {}^{12}\cancel{2} \\ - & 2 & . & 5 & 8 & 4 \\ \hline & 1 & . & 0 & 6 & 8 \end{array}$$

b
$$\begin{array}{rcccccc} & {}^5\cancel{6} & . & {}^9\cancel{0} & {}^9\cancel{0} & {}^{10}\cancel{0} \\ - & 0 & . & 6 & 3 & 7 \\ \hline & 5 & . & 3 & 6 & 3 \end{array}$$

2 Find:

- **a** $2.8 - 0.5$
- **b** $3.29 - 1.16$
- **c** $7.53 - 2.4$
- **d** $1.6 - 0.9$
- **e** $5 - 0.8$
- **f** $1 - 0.99$
- **g** $3.27 - 1.98$
- **h** $1.01 - 0.002$
- **i** $7.2 - 0.65$
- **j** $0.083 - 0.0091$
- **k** $7.21 - 0.756$
- **l** $1.1 - 0.1234$

3 Find the sum of:

- **a** 9.63, 7.98, and 2.45
- **b** 17.55, 37.2, and 49.892
- **c** 21.38, 279.34, and 10.629
- **d** 9.77, 11.7, 108.54, and 0.285

4 Subtract:

- **a** 7.5 from 19.4
- **b** 15.87 from 21.31
- **c** 4.35 from 16.297
- **d** 8.135 from 57.2

5 A cat jumped 1.8 m from the ground onto a fence, a further 0.95 m onto the garage roof, then another 1.52 m onto the house roof. How high is the house roof above the ground?

6 Each morning, Karina travels 6.3 km by train and 4.75 km by bus to school.

- **a** How far does she travel in total?
- **b** How much farther does Karina travel by train than by bus?

7 The table below shows the results of the 400 m women's final at the 2016 Olympic Games. Shaunae Miller was the winner. Her time is given, and the other results show how much longer it took each runner to finish. For example, Phyllis Francis finished 0.97 seconds after Shaunae Miller.

	Runner	*Time* (s)
1	Shaunae Miller (BAH)	49.44
2	Allyson Felix (USA)	+0.07
3	Shericka Jackson (JAM)	+0.41
4	Natasha Hastings (USA)	+0.90
5	Phyllis Francis (USA)	+0.97
6	Stephenie Ann McPherson (JAM)	+1.53
7	Libania Grenot (ITA)	+1.81
8	Olha Zemlyak (UKR)	DSQ

a What was Stephenie Ann McPherson's time?

b By how much did:

  i Shericka Jackson finish ahead of Natasha Hastings

  ii Allyson Felix finish ahead of Libania Grenot?

c Shaunae Miller's time was 1.19 seconds slower than the Olympic record set by Marie-José Pérec in 1996. What was the Olympic record time?

8 Each week Claire is paid \$1356.28 less the deductions given in the table. How much pay does Claire keep each week?

Taxation	\$507.90
Student loan repayment	\$93.40
Private Health Cover	\$44.62
Union Fees	\$14.82

ACTIVITY

In this Activity, you will try to *estimate* the cost of a selection of items at a supermarket.

What to do:

1 Working in pairs, choose 10 items you would like to buy from the supermarket.

2 Each of you should write down your estimates for the price of each item.

3 Use a supermarket's website to record the actual price of each item on your list.

4 Find the difference between your estimate and the actual price for each item. Compare your results with your partner. Who was closer to the actual price for each item?

5 **a** Find the total of the actual prices for the items, and the total of your estimated prices.

  b Find the difference between the totals.

  c Compare your result with your partner. Who was closer to the actual total price?

6 Repeat this Activity for another shop such as an electrical or hardware store.

H MULTIPLYING BY POWERS OF 10

When we multiply a number by 10, each digit has 10 times its previous value and therefore moves one place to the left.

hundreds	tens	units		tenths	hundredths	thousandths
	8	7	.	4	3	5
8	7	4	.	3	5	

$87.435 \times 10 = 874.35$

This has the effect of moving the decimal point one place to the *right*.

> When multiplying by 10, we move the decimal point one place to the right.

In general:

> When multiplying by a power of 10, the power tells us how many places to move the decimal point to the right.

If necessary, we use a zero to fill an empty place value.

Example 12 Self Tutor

Find: **a** 9.8×10 **b** 0.0751×100 **c** $13.026 \times 10\,000$

a 9.8×10 $\{10 = 10^1\}$
$= 98$

b 0.0751×100 $\{100 = 10^2\}$
$= 7.51$

c $13.0260 \times 10\,000$ $\{10\,000 = 10^4\}$
$= 130\,260$

EXERCISE 7H

1 Multiply 3.271 by:

a 10 **b** 100 **c** 1000 **d** 10 000

2 Multiply 7.6 by:

a 10 **b** 1000 **c** 10^4 **d** 10^6

3 Evaluate:

a 27.1×10 **b** 4.86×100 **c** 2.25×10 **d** 0.034×100
e 16.4×1000 **f** 0.2×10 **g** 0.7941×100 **h** 39.26×10
i 8.1×100 **j** 0.905×10^3 **k** 0.0017×100 **l** 1.67×10^4
m 0.1008×10^2 **n** 0.036×10^3 **o** $0.007\,61 \times 10^4$ **p** $0.338 \times 100\,000$

4 Find:

a -0.48×10 **b** -0.0057×100 **c** $-0.060\,32 \times 1000$

I DIVIDING BY POWERS OF 10

When we divide a number by 10, each digit has $\frac{1}{10}$ of its previous value and therefore moves one place to the right.

hundreds	tens	units		tenths	hundredths	thousandths
1	0	4	.	2	6	
	1	0	.	4	2	6

$104.26 \div 10 = 10.426$

This has the effect of moving the decimal point one place to the *left*.

When dividing by 10, we move the decimal point one place to the left.

In general:

When dividing by a power of 10, the power tells us how many places to move the decimal point to the left.

Example 13 **Self Tutor**

Find: **a** $0.4 \div 10$ **b** $0.18 \div 1000$

a $0.4 \div 10$ $\{10 = 10^1\}$
$= 0.04$

b $0.18 \div 1000$
$= 000.18 \div 1000$ $\{1000 = 10^3\}$
$= 0.000\,18$

EXERCISE 7I

1 Divide 84.6 by:

a 10 **b** 100 **c** 1000 **d** 10 000

2 Divide 0.7 by:

a 10 **b** 10^3 **c** 10 000 **d** 10^5

3 Find:

a $6 \div 10$ **b** $92 \div 10$ **c** $529 \div 10$ **d** $529 \div 100$
e $529 \div 1000$ **f** $529 \div 10\,000$ **g** $0.3 \div 10$ **h** $0.3 \div 1000$
i $0.97 \div 100$ **j** $0.06 \div 10$ **k** $0.06 \div 100$ **l** $0.022 \div 1000$
m $7.7 \div 10\,000$ **n** $0.2963 \div 100$ **o** $0.035 \div 10^3$ **p** $51.6 \div 10^4$

4 Find:

a $-12.5 \div 10$ **b** $-0.44 \div 100$ **c** $-0.0793 \div 1000$

J MULTIPLYING DECIMAL NUMBERS

We have previously used column multiplication to multiply whole numbers.

For example:

$$\begin{array}{rrrr} & & 3 & 4 \\ \times & & 2 & 7 \\ \hline & 2 & {}^{2}3 & 8 \\ + & {}_{1}6 & 8 & 0 \\ \hline & 9 & 1 & 8 \end{array}$$

So, $34 \times 27 = 918$.

If we multiply a decimal number by a whole number, we can still use column multiplication so long as we are careful about the placement of the decimal point.

However, this process gets more confusing when we multiply two decimal numbers.

So, to perform multiplication with decimals, we first convert the decimal numbers to fractions with denominators that are powers of 10. This will allow us to multiply the fractions, and to do our column multiplication with whole numbers.

For example:

- 3.4×27
  $= \frac{34}{10} \times 27$
  $= \frac{34 \times 27}{10}$
  $= \frac{918}{10}$ {using the column multiplication above}
  $= 91.8$

- 3.4×0.27
  $= \frac{34}{10} \times \frac{27}{100}$
  $= \frac{34 \times 27}{1000}$
  $= \frac{918}{1000}$ {using the column multiplication above}
  $= 0.918$

Example 14 **Self Tutor**

Find: **a** 4×0.03 **b** 0.19×0.8

a 4×0.03
$= 4 \times \frac{3}{100}$
$= \frac{4 \times 3}{100}$
$= \frac{12}{100}$
$= 0.12$

b 0.19×0.8
$= \frac{19}{100} \times \frac{8}{10}$
$= \frac{19 \times 8}{1000}$
$= \frac{152}{1000}$
$= 0.152$

$$\begin{array}{rrr} & 1 & 9 \\ \times & {}_{7} & 8 \\ \hline 1 & 5 & 2 \end{array}$$

EXERCISE 7J

1 Find:

a 7×0.4 **b** 0.8×9 **c** 6×0.5 **d** 0.03×8

e 30×0.6 **f** 0.006×40 **g** 300×0.07 **h** 0.009×2000

2 Find:

a 0.2×0.4	**b** 0.8×0.7	**c** 0.6×0.09	**d** 0.03×0.5
e 0.06×0.04	**f** 1.2×0.09	**g** 0.14×0.8	**h** 0.7×0.021
i 2.2×0.05	**j** 0.006×3.9	**k** 0.19×0.13	**l** 2.7×0.15

3 Find:

a $0.4 \times 0.5 \times 0.2$	**b** $0.3 \times 0.7 \times 0.5$	**c** $1.2 \times 0.2 \times 0.06$

4 Given that $22 \times 471 = 10\,362$, evaluate:

a 2.2×471	**b** 2.2×4.71	**c** 2.2×47.1
d 22×0.471	**e** 0.22×0.471	**f** 2.2×0.471
g 0.22×4.71	**h** 220×0.471	**i** $2.2 \times 0.004\,71$

Example 15 — Self Tutor

Find 7.9×3.2, and check your answer using a one figure estimate.

$$\begin{aligned} & 7.9 \times 3.2 \\ = & \frac{79}{10} \times \frac{32}{10} \\ = & \frac{79 \times 32}{100} \\ = & \frac{2528}{100} \\ = & 25.28 \end{aligned}$$

$$\begin{array}{r} 7\;9 \\ \times \quad 3\;2 \\ \hline 1\;{}^{1}5\;8 \\ +\; 2\;{}^{2}_{1}3\;7\;0 \\ \hline 2\;5\;2\;8 \end{array}$$

Check: $7.9 \times 3.2 \approx 8 \times 3 \approx 24$

So, the answer appears reasonable.

5 Evaluate each product and check your answer using a one figure estimate:

a 2×1.8	**b** 3.1×1.9	**c** 8.9×4.2
d 7.3×9.2	**e** 38.6×7.1	**f** 6.23×4.9

6 Find:

a -0.9×7	**b** -0.6×0.8	**c** 0.04×-0.15	**d** -0.12×-0.11

7 Min loads 7 bags of soil improver into her trailer. Each bag has mass 4.5 kg. Find the total mass of the bags.

8 Find the total cost of 2.9 m of chiffon fabric at \$5.79 per metre.

9 I am about to bake biscuits for the school fundraiser. I buy 6 kg of oats at \$7.50 per kg, and 2.5 kg of muesli at \$6.20 per kg. How much money have I spent in total?

10 When travelling on country roads, William's car travels 12.5 km on each litre of petrol. Yesterday William went for a country drive and used 9.3 litres of petrol. How far did he drive?

K DIVIDING DECIMAL NUMBERS

We divide a decimal number by a whole number using the same method as we have used previously for whole numbers:

- We divide each place value in turn, starting with the largest place value.
- If a digit is too small to be divided on its own, we exchange it in the next column.
- We place the decimal point in the answer directly above the decimal point in the question.

Sometimes we may need to write extra zeros at the end of the decimal number we are dividing into.

Example 16 Self Tutor

Find:

a $32.5 \div 5$ **b** $0.417 \div 4$

a

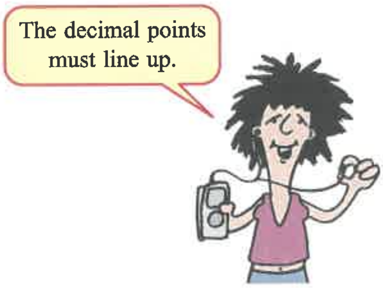
$$\begin{array}{r|l} & 6\,.\,5 \\ \hline 5 & 3\,2\,.\,{}^{2}5 \end{array}$$

$32.5 \div 5 = 6.5$

b

$$\begin{array}{r|l} & 0\,.\,1\,0\,4\,2\,5 \\ \hline 4 & 0\,.\,4\,1\,{}^{1}7\,{}^{1}0\,{}^{2}0 \end{array}$$

$0.417 \div 4 = 0.10425$



To divide a decimal number by **another decimal number**, we:

- write the division as a fraction
- multiply the numerator and denominator by the same power of 10 so the denominator becomes a whole number
- perform the division.

Example 17 Self Tutor

Find: **a** $1.8 \div 0.06$ **b** $0.027 \div 1.2$

a $1.8 \div 0.06$

$= \dfrac{1.8 \times 100}{0.06 \times 100}$

$= \dfrac{180}{6}$

$= 30$

b $0.027 \div 1.2$

$= \dfrac{0.027 \times 10}{1.2 \times 10}$

$= \dfrac{0.27}{12}$

$= 0.0225$

$$\begin{array}{r|l} & 0\,.\,0\,2\,2\,5 \\ \hline 12 & 0\,.\,2\,{}^{2}7\,{}^{3}0\,{}^{6}0 \end{array}$$

EXERCISE 7K

1 Find:

a $7.2 \div 2$ **b** $32.7 \div 3$ **c** $80.4 \div 4$ **d** $0.45 \div 5$

e $0.225 \div 9$ **f** $1.82 \div 7$ **g** $46.2 \div 6$ **h** $0.649 \div 11$

i $0.73 \div 5$ **j** $0.0411 \div 2$ **k** $3.83 \div 4$ **l** $0.399 \div 8$

2 Find:

a $0.8 \div 0.4$ **b** $6.3 \div 0.9$ **c** $0.25 \div 0.05$ **d** $0.96 \div 0.12$

e $0.3 \div 0.15$ **f** $12.1 \div 1.1$ **g** $12 \div 0.4$ **h** $3.6 \div 0.012$

i $0.088 \div 0.008$ **j** $2.4 \div 0.003$ **k** $0.45 \div 0.005$ **l** $84 \div 0.07$

3 Find:

a $0.35 \div 0.7$ **b** $0.06 \div 0.3$ **c** $1.71 \div 0.9$ **d** $2.2 \div 0.05$

e $0.0232 \div 0.8$ **f** $0.438 \div 0.06$ **g** $4.196 \div 0.005$ **h** $0.0694 \div 0.008$

4 Find:

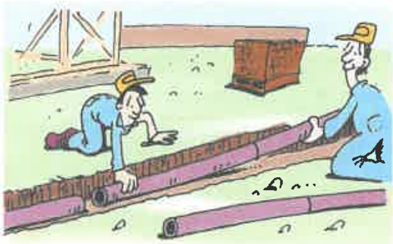
a $-0.96 \div 6$ **b** $-0.8 \div 0.08$ **c** $0.668 \div -0.4$ **d** $-4.41 \div -0.07$

5 A 7.8 L pot of soup is shared equally between 5 people. How much soup does each person receive?

6 Erica is painting a wall which has area 10.4 m^2. She can paint an area of 0.4 m^2 each minute. How long will it take her to complete the job?

7 How many \$0.90 cans of tuna can be bought for \$22.50?

8 Determine the number of 1.2 m lengths of pipe required to construct a 660 m drain.



GLOBAL CONTEXT — LEAP YEARS

Global context: Scientific and technical innovation

Statement of inquiry: Decimal numbers are useful for describing natural occurrences.

Criterion: Applying mathematics in real-life contexts

GLOBAL CONTEXT

MULTIPLE CHOICE QUIZ

QUICK QUIZ

REVIEW SET 7A

1 State the value of the digit:

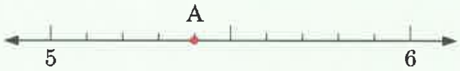
a 5 in 0.5271 **b** 6 in 47.0461

2 Write:

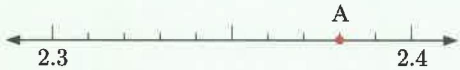
a 2.1023 in expanded form **b** 0.004 as a fraction in lowest terms.

3 What decimal number is at A?

a



b



4 Round:

a 28.549 to 2 decimal places

b 0.4824 to 1 decimal place

c 45.613 to the nearest whole number

d 0.538 52 to 3 significant figures.

5 Write in ascending order:

a 2.5, 2.05, 2.55

b 4.3, 4.403, 4.34, 4.304

6 Write as a decimal:

a $\frac{4}{5}$

b $-\frac{9}{25}$

c $\frac{13}{200}$

7 Find:

a $0.71 + 0.296$

b $9.27 - 3.08$

c $14.2 + 8.93$

8 Multiply 8.59 by:

a 100

b 1000

c 100 000

9 Find:

a 8×0.08

b 0.6×0.8

c 1.7×0.05

d $8.7 \div 3$

e $-2.8 \div 0.4$

f $0.952 \div 0.07$

10 Evaluate 6.7×2.2, and check your answer using a one figure estimate.

11 A race track is 0.9 km long. How many laps are needed to complete a 36 km race?

12 In one day a truck delivered 48 tonnes of sand to a building site. The first three loads measured 11.25 tonnes, 13.76 tonnes, and 12.82 tonnes.
How much sand was delivered in the fourth and final load?

13 Jane and Claire have gone shopping for new clothes. The item prices are:


T-shirt
$22.50


Trousers
$35.25


Singlet
$14.95


Skirt
$29.98


Socks
$5.25

a Find the cost of 6 pairs of socks.

b How much more expensive is the T-shirt than the singlet?

c Claire only has $100 to spend. Can she afford to buy a T-shirt, two pairs of trousers, and two pairs of socks? Explain your answer.

REVIEW SET 7B

1 Write as a decimal:

a $\frac{4}{10}+\frac{3}{100}$ **b** $\frac{7}{10}+\frac{1}{1000}$ **c** $\frac{2}{100}+\frac{8}{10\,000}$

2 **a** Place the decimal numbers 0.6, 0.53, 0.48, 0.62, and 0.5 on a number line.

b Hence write the numbers in descending order.

3 Use $<$, $>$, or $=$ to complete:

a 3.03 ...... 3.303 **b** 0.514 ...... 0.541 **c** 2.404 ...... 2.044

4 Scott averages 3.28 steals per game for his hockey team. Round this to 1 decimal place.

5 Write as a fraction or mixed number in lowest terms:

a 0.86 **b** 2.24 **c** -0.045

6 Evaluate:

a $42.8+19.74$ **b** $11-4.13$ **c** $0.039 \div 100$

7 Given that $28 \times 17 = 476$, evaluate:

a 2.8×0.17 **b** 0.0028×1.7 **c** $0.000\,28 \times 170$

8 Divide 67.4 by:

a 10 **b** 1000 **c** 10 000

9 Find:

a $0.091 \div 7$ **b** -0.06×0.012 **c** $0.352 \div 1.1$

10 A thermos contains 3.2 litres of tea. How many 0.4 litre cups of tea can be poured from the thermos?

11 A man is 1.3 times as tall as his daughter, who is 136 cm tall. Determine the height of the man.

12 Alice earns £12 per hour as a casual checkout attendant. She had shifts of 3 hours, 4.5 hours, 6.25 hours, and 5.2 hours last week.

a How many hours in total did she work?

b How much money did Alice earn last week?

13 Answer the **Opening Problem** on page **134**.

Chapter 8

Algebra

Contents:

OPENING PROBLEM

Elaine is visiting her friend whose pet dog has just had a litter of puppies. When Elaine arrives, she sees a dog kennel, but does not know how many puppies are in it.

Things to think about:

a Suppose there are 5 puppies in the kennel.
  i If one puppy leaves the kennel, how many are left?
  ii If instead, two puppies enter the kennel, how many are in the kennel in total?

b Suppose there are d puppies in the kennel.
  i If one puppy leaves the kennel, how many are left?
  ii If instead, two puppies enter the kennel, how many are in the kennel in total?

In this Chapter we begin our study of **algebra**.

In algebra, we use letters or symbols to represent unknown numbers. Since the unknown number could take many values, we call it a **variable**.

For example, in the **Opening Problem**, since Elaine did not know the number of puppies in the kennel, we used the variable d to represent the number.

As Elaine watches puppies leave or enter the kennel, she can write **algebraic expressions** involving d for the number of puppies in the kennel now.

A BUILDING EXPRESSIONS

We choose operations to perform with variables in the same way that we do with numbers. We can do this because the variables *represent* numbers.

It is often useful to build expressions by considering an example with numbers first.

Example 1 Self Tutor

Suppose we have 2 punnets of strawberries, plus 4 strawberries left over.

Write an expression for the total number of strawberries if each punnet contains:

a 6 strawberries **b** 9 strawberries
c s strawberries.

a If each punnet contains 6 strawberries, then there are $2 \times 6 + 4$ strawberries in total.
b If each punnet contains 9 strawberries, then there are $2 \times 9 + 4$ strawberries in total.
c If each punnet contains s strawberries, then there are $2 \times s + 4$ strawberries in total.

EXERCISE 8A

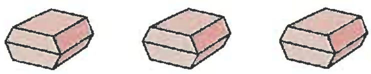
1. Answer the **Opening Problem**.
2. Suppose we have 3 punnets of blueberries, with 7 blueberries left over.
   Write an expression for the number of blueberries if each punnet contains:
   a 8 blueberries **b** 12 blueberries **c** b blueberries.
3. A farm has 6 paddocks of horses, plus 5 stables with one horse in each.
   Write an expression for the number of horses on the farm if each paddock contains:
   a 2 horses **b** 4 horses **c** h horses.

B PRODUCT NOTATION

Product notation is used to represent the *sum* of identical terms.

For example, just as $7+7+7=3\times 7=21$ {3 lots of 7}
we can write $p+p+p=3\times p=3p$ {3 lots of p}

So, if there are p strawberries in each of 3 punnets, the total number of strawberries is $p+p+p$ or $3p$.



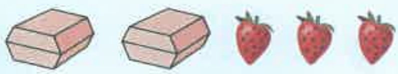
Notice that in product notation:

- we leave out $\times$ signs between multiplied quantities
- we write the number before the letter, so we write $3p$ rather than $p3$.

Example 2 Self Tutor

Suppose each punnet contains p strawberries.

Write an algebraic expression for the total number of strawberries.

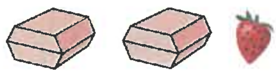


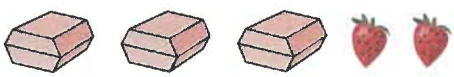
The total number of strawberries is $p+p+3$
$= 2p+3.$

EXERCISE 8B

1. Suppose each punnet contains p strawberries. Write an algebraic expression for the total number of strawberries in:

   a 

   b 

   c 

   d 

2 Write using product notation:

- **a** $a+a$
- **b** $x+x+x$
- **c** $p+p+p+p+p$
- **d** $y+y+3$
- **e** $x+2+x$
- **f** $n+n+3+n$

3 Write using product notation:

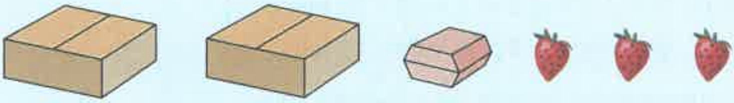
- **a** $2\times a$
- **b** $a\times 2$
- **c** $5\times x$
- **d** $x\times 5$
- **e** $6\times n$
- **f** $3\times 2a$
- **g** $4c\times 5$
- **h** $2\times 5d$

Example 3 **Self Tutor**

Let  represent p strawberries in a punnet

and  represent b strawberries in a box.

Write an algebraic expression for the total number of strawberries in:



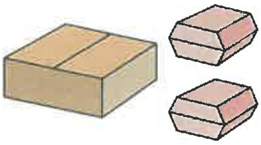
There are 2 boxes, 1 punnet, and 3 more strawberries.

The total number of strawberries is therefore $2b+p+3$.

4 Suppose there are p strawberries in each punnet and b strawberries in each box.
Write an algebraic expression for the total number of strawberries in:

a

b



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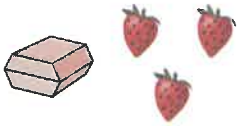
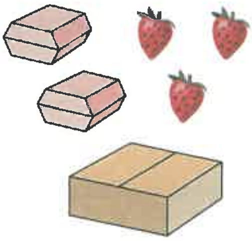
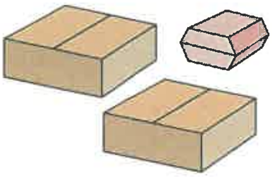


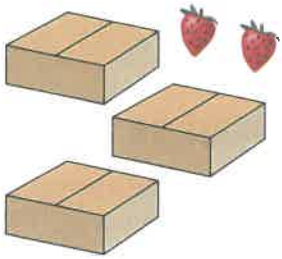
d



e



f



5 Write using product notation:

- **a** $b+b+c$
- **b** $x+y+2+y$
- **c** $x+y+y+x+x$
- **d** $m+1+m+n$
- **e** $n+x+x+3+x$
- **f** $p+q+p+4+q$

Example 4 Self Tutor

Write using product notation:

a $x+x-n$ **b** $p+p+p-(q+q)$

a $x+x-n$
$= 2x-n$

b $p+p+p-(q+q)$
$= 3p-2q$

6 Write using product notation:

a $a+a-b$ **b** $a-c+a+a$
c $x-2+4+x$ **d** $x+x-3+x$

7 Write using product notation:

a $6-(t+t)$ **b** $x-(y+y)$ **c** $s+s-(t+t+t)$
d $n-(m+m+m+m)$ **e** $p+p+p-(q+q+q)$ **f** $a+a+a+a-(b+b)$

Example 5 Self Tutor

Write using product notation: $2\times a+b\times 3$

$2\times a+b\times 3$
$= 2a+3b$

8 Write using product notation:

a $3\times x+2\times y$ **b** $a\times 3+2\times b$ **c** $m\times 5+n\times 2$
d $u-w\times 7$ **e** $c\times 9-4\times d$ **f** $6\times a-b\times 5$

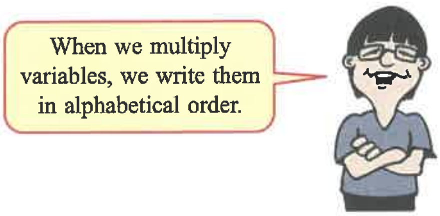
Example 6 Self Tutor

Write using product notation:

a $a\times b$ **b** $b\times 3a$

a $a\times b$
$= ab$

b $b\times 3a$
$= 3ab$



9 Write using product notation:

a $m\times n$ **b** $n\times m$ **c** $e\times d$
d $b\times 2\times a$ **e** $c\times 4d$ **f** $4c\times 4d$
g $x\times y\times 9$ **h** $x\times y\times z$ **i** $k\times b\times h$

10 Write using product notation:

a $z\times x+y\times 2$ **b** $5-4\times x\times y$ **c** $2\times a+3\times b\times c$

C EXPONENT NOTATION

Exponent notation is used to represent the *product* of identical terms.

Just as $2 \times 2 \times 2 \times 2 = 2^4$, we write $a \times a \times a \times a = a^4$.

In this case, a is the base and 4 is the exponent, index, or power.

Example 7 Self Tutor

Write using exponent notation:

a $2 \times a \times a \times a \times b \times b$ **b** $m \times m - 5 \times n \times n$

a $2 \times a \times a \times a \times b \times b$
$= 2a^3b^2$

b $m \times m - 5 \times n \times n$
$= m^2 - 5n^2$

An expression is in **expanded form** if each product in it is written out in full.

Example 8 Self Tutor

Write in expanded form:

a $4mn^2$ **b** $3x^2y + y^2$

a $4mn^2$
$= 4 \times m \times n \times n$

b $3x^2y + y^2$
$= 3 \times x \times x \times y + y \times y$

EXERCISE 8C

1 Write using exponent notation:

a $x \times x \times x$ **b** $2 \times x \times x$ **c** $p \times p \times p \times p$
d $b \times b \times b \times 5$ **e** $3 \times a \times b \times b$ **f** $f \times f \times g \times g \times g \times h$
g $2 \times t \times 2 \times t$ **h** $3 \times y \times 3 \times y \times y$ **i** $b \times a \times 2 \times b \times a \times b$

2 Write using exponent notation:

a $c \times c + d$ **b** $3 + a \times a$ **c** $f \times f - f$
d $w \times w \times w + 7$ **e** $e \times e \times e - 2 \times e \times e$ **f** $5 \times a \times a \times a + b \times b$
g $4 \times x \times y \times y + z \times z$ **h** $x \times x - 3 \times y \times x$ **i** $6 \times x \times y - x + x \times x$

3 Write in expanded form:

a x^2 **b** y^3 **c** $3x^2$ **d** $4m^3$
e $8x^3y$ **f** $5pq^2$ **g** $c^2 + 4d^3$ **h** $3v^2 - 5w^2$
i $2xy^2 + x^2$ **j** $5 - 3x^3y$ **k** $ab^2 + a^2b$ **l** $2p - 3p^2q$

Example 9 — Self Tutor

Write in simplest form: $3x \times 2x$

$$\begin{aligned} & 3x \times 2x \\ &= 3 \times x \times 2 \times x && \{\text{expanded form}\} \\ &= 3 \times 2 \times x \times x && \{\text{changing order}\} \\ &= 6x^2 \end{aligned}$$

4 Write in simplest form:

a $3x \times x$ **b** $x \times 3x$ **c** $4x \times 5x$ **d** $x^2 \times 4x$

e $2x \times 3x^2$ **f** $x^2 \times 2x^2$ **g** $2xy \times x$ **h** $3y \times 2x^2y$

D READING EXPRESSIONS

In order to *read* algebraic expressions, we need to describe mathematical operations using words.

ADDITION

When we **add**, we find a **sum** or **total**.

Here are some ways of reading the expression $x + 5$:

- "x plus 5"
- "the sum of x and 5"
- "5 more than x"
- "x add 5"
- "the total of x and 5".

SUBTRACTION

When we **subtract**, we use the words **minus** or **less than**.

Here are some ways of reading the expression $3 - x$:

- "3 minus x"
- "x less than 3"
- "3 subtract x"
- "3 take x".

MULTIPLICATION

When we **multiply**, we use the words **times** or **product**.

Here are some ways of reading the expression $2x$:

- "the product of 2 and x"
- "twice x"
- "2 times x".

DIVISION

When we **divide**, we use the words **divided by** or **quotient**.

Here are some ways of reading the expression $\frac{x}{3}$:

- "x divided by 3"
- "x over 3"
- "the quotient of x and 3"
- "x on 3".

SQUARES

x^2 can be read as:

- "x squared"
- "the square of x".

Example 10 — Self Tutor

a Write the meaning of $xy - 5$ in words.
b Write as an algebraic expression: The sum of a divided by 3, and b.

a $xy - 5$ means "the product of x and y, minus 5".
b a divided by 3 is $\frac{a}{3}$.
So, the sum of a divided by 3, and b, is $\frac{a}{3} + b$.

EXERCISE 8D

1 Write in words:

a $2 + a$ **b** $a - 2$ **c** $4x$ **d** $\frac{a}{2}$
e $p + q$ **f** $7 - m$ **g** xy **h** $\frac{3}{a}$
i $a + 3b$ **j** $2a - c$ **k** $ab + 4$ **l** $\frac{a}{2b}$

2 Write as an algebraic expression:

a the sum of p and 1
b the product of 2 and q
c p minus q
d 3 divided by p
e 4 subtract the product of p and q
f the sum of 3 times p and 4 times q
g the quotient of p and q.

Example 11 — Self Tutor

a Write the meaning of $3a^2 + b$ in words.
b Write as an algebraic expression: 1 minus b squared.

a $3a^2 + b$ means "the product of 3 and a squared, plus b".
b 1 minus b squared is $1 - b^2$.

3 Write in words:

a $5a^2$ **b** $3 + b^2$ **c** $a^2 - b^2$ **d** $\frac{a^2}{4}$

4 Write as an algebraic expression:

a p plus the square of q
b q squared divided by p
c the product of 3 and the square of q
d the quotient of q and p squared.

5 Match the expression with its meaning in words:

a $3+a$	**A** a plus b squared
b $5-2a$	**B** the sum of 3 and a
c $1-a^2$	**C** 5 minus 2 times a
d $2a^2+3$	**D** the quotient of a squared and 3
e $a+b^2$	**E** two times a squared, plus 3
f $\frac{a^2}{3}$	**F** 1 minus a squared

E TERMS AND COEFFICIENTS

In algebra, we use these key words to help us describe expressions:

- The **terms** of an expression are algebraic forms which are separated by $+$ and $-$ signs, the signs being included.

  For example: The terms of $3x+2y+8$ are $3x$, $2y$, and 8.
  The terms of $2x-3y-5$ are $2x$, $-3y$, and -5.

- A term is **constant** if it does not contain a variable.

  For example: In $5x+6$, the constant is 6.
  In $-7+3x^2$, the constant is -7.

- The **coefficient** of any term is its numerical part, including its sign.

  For example: The coefficient of p in $2p+4$ is 2.
  The coefficient of r in $7-6r$ is -6.
  The coefficient of x^2 in x^2+x is 1 since x^2 is $1\times x^2$.

- **Like terms** are terms with exactly the same variable form. They have the same variables to the same powers.

  For example: $3x$ and $7x$ are like terms.
  8 and 7 are like terms.
  $7x$ and 7 are not like terms.
  x and x^2 are not like terms.

Example 12 **Self Tutor**

Consider the expression $2x-y+3x+2$.

a List the terms of the expression.
b State the constant.
c State the coefficient of y.
d Identify the like terms.

a The terms are $2x$, $-y$, $3x$, and 2.
b The constant is 2.
c The coefficient of y is -1.
d The like terms are $2x$ and $3x$.

EXERCISE 8E

1. List the terms of:
   - **a** $x-3+2y$
   - **b** $2x-y+4$
   - **c** $-x+3y+2x+1$

2. State the constant in:
   - **a** $4-x^2$
   - **b** $2x+3-y$
   - **c** $5x+y-2$

3. Write down the coefficient of x in:
   - **a** $5x$
   - **b** $3+4x$
   - **c** $6y-5+8x$
   - **d** $4x-y+2$
   - **e** $5+2y+x$
   - **f** $-2x+3y-6$
   - **g** $x-7$
   - **h** $9-x$
   - **i** $10-7x$

4. Consider the expression $3x+5y-7-2y$.
   - **a** How many terms are in this expression?
   - **b** Which term is a constant?
   - **c** What is the coefficient of the fourth term?
   - **d** Identify the like terms in the expression.

5. State the like terms in:
   - **a** $3+2x-3x$
   - **b** $2x+3+5x+5$
   - **c** $x+y+5x-y$
   - **d** $2x+y^2+3x+2y+4$
   - **e** $3+q^2+7+4q^2$
   - **f** $2-2b+a+3b$

6. Decide whether each statement is true or false:
   - **a** $2+a-3b$ has 3 terms.
   - **b** $3x^2$ and $2x$ are like terms.
   - **c** The coefficient of x in $3x+2$ is 3.
   - **d** In x^2+3-4x, the constant is 3.
   - **e** In $3x+4y-2$, the constant is 2.
   - **f** x and $-x$ are not like terms.

7. Are $3x$ and xy like terms? Explain your answer.

ACTIVITY 1 — KEY WORD JUMBLE

ACTIVITY

Click on the icon and print the activity sheet. It is divided into three sections: key words, definitions, and examples.

Cut out the boxes and match each key word with its definition and example. Glue the results into your exercise book.

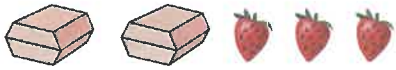
F EQUAL EXPRESSIONS

Expressions are **equal** if they represent the same total.

Expressions that are equal can be connected with an $=$ sign.

Suppose there are p strawberries in a punnet.

Marcel has 2 punnets and 3 more strawberries, so in total he has $2p+3$ strawberries.



His sister Michelle has one punnet and 3 more strawberries, so in total she has $p+3$ strawberries.

Between the two children, there are 3 punnets and 6 more strawberries, so the combined total is $3p+6$ strawberries.

We can conclude that: $\underbrace{2p+3}_{\text{Marcel's total}} + \underbrace{p+3}_{\text{Michelle's total}} = \underbrace{3p+6}_{\text{combined total}}$

Example 13 **Self Tutor**

Suppose there are m mushrooms in a box.

a Use the pictures below to find the total number of mushrooms carried by:

i Eric **ii** Erin **iii** the two children combined.

b Connect expressions for the total number of mushrooms with an $=$ sign.

a **i** Eric is carrying 2 boxes and 4 more mushrooms, so the total is $2m+4$ mushrooms.

ii Erin is carrying 3 boxes and 2 more mushrooms, so the total is $3m+2$ mushrooms.

iii Between the two children, they are carrying 5 boxes and 6 more mushrooms, so the total is $5m+6$ mushrooms.

b $\underbrace{2m+4}_{\text{Eric's total}} + \underbrace{3m+2}_{\text{Erin's total}} = \underbrace{5m+6}_{\text{combined total}}$

EXERCISE 8F

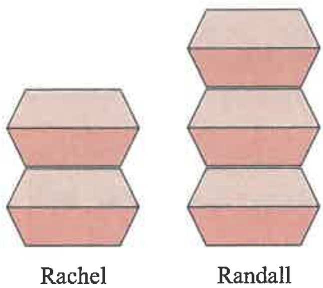
1 Suppose there are b blueberries in a container.

a Find the number of blueberries belonging to:

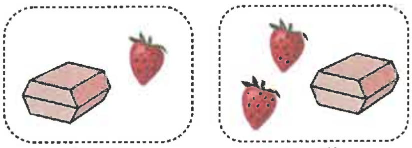
i Rachel **ii** Randall

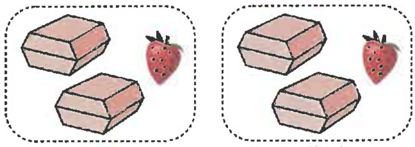
iii the two children combined.

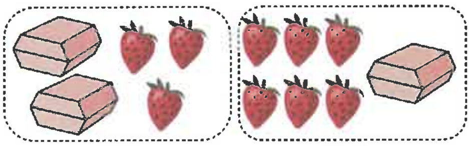
b Connect expressions for the total number of blueberries with an $=$ sign.

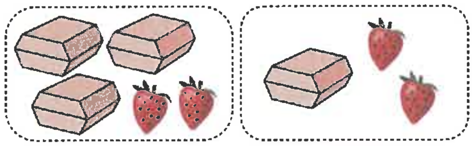


2 Suppose there are p strawberries in a punnet.
For each set of diagrams below, write *two* equal expressions for the total number of strawberries. Connect them with an $=$ sign.

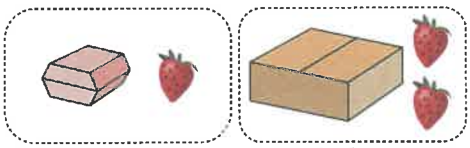
a

Aaron Amelia

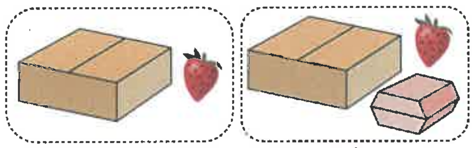
b

Bob Belinda

c

Carl Cindy

d

Daniel Diana

3 Suppose there are p strawberries in a punnet and b strawberries in a box.
For each set of diagrams below, write *two* equal expressions for the total number of strawberries. Connect them with an $=$ sign.

a

Amy Andrew

b

Briana Brian

Example 14 Self Tutor

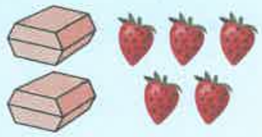
Illustrate each algebraic expression, and hence decide whether the expressions are equal:

$p+2+p+4$ and $2p+5$

Let  represent p strawberries in a punnet.

$p+2+p+4$ is:

$2p+5$ is:



The total number of strawberries is different, so the expressions are *not* equal.

4 Illustrate each algebraic expression, and hence decide whether the expressions are equal:

a $p+2+2p+1$ and $3p+3$

b $2p+3+2+p$ and $p+4+3p+1$

c $5+p+3p$ and $2p+3+2p+1$

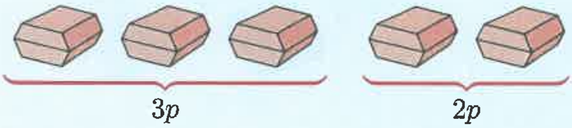
Example 15 — Self Tutor

With the aid of diagrams, write an equal algebraic expression which has fewer terms:

a $3p+2p$ **b** $2p+1+4+p$

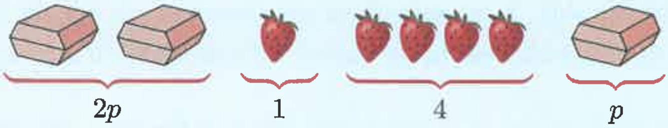
Let  represent p strawberries in a punnet.

a



So, $3p+2p=5p$.

b



So, $2p+1+4+p=3p+5$.

5 With the aid of diagrams, write an equal algebraic expression which has fewer terms:

a $3p+p$ **b** $4p+2p$ **c** $p+p+3$

d $3p+1+p+4$ **e** $p+2+p+5$ **f** $3+2p+1+3p$

Example 16 — Self Tutor

With the aid of diagrams, write an equal algebraic expression which has fewer terms:

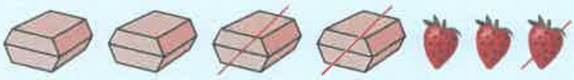
a $5p-2p$ **b** $4p+3-2p-1$

Let  represent p strawberries in a punnet.

a

So, $5p-2p=3p$.

b



So, $4p+3-2p-1=2p+2$.

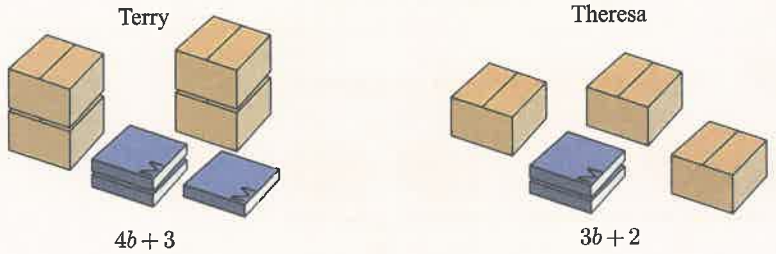
6 With the aid of diagrams, write an equal algebraic expression which has fewer terms:

a $5p-3p$ **b** $4p-p$ **c** $3p-3p$

d $4p+3-2p$ **e** $3p+5-p-1$ **f** $4p+5-2p-3$

DISCUSSION

Terry and Theresa have mathematics textbooks for their classes. There are b books in each box.



In total there are 7 boxes and 5 more books, so $4b+3+3b+2=7b+5$.

1. Compare the *constants* on each side. Is the sum of the constants on each side the same?
2. Compare the *coefficients* of b on each side. Is the sum of the coefficients of b on each side the same?
3. Look back at other equal expressions you found in the previous **Exercise**. Do the same principles you saw in **1** and **2** always apply?
4. How can we add or subtract like terms to write an equal expression with fewer terms?

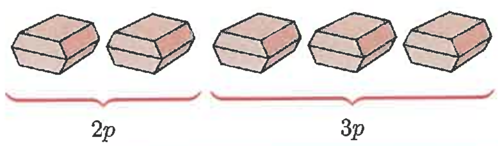
G COLLECTING LIKE TERMS

Algebraic expressions can often be simplified by adding like terms. This process is called **collecting like terms**.

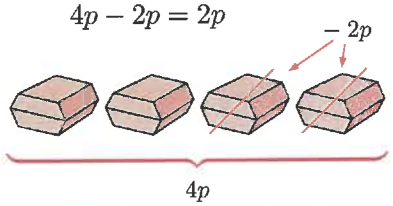
When we add like terms, we add the coefficients of the terms.

For example:

- Since $2+3=5$,
  $2p+3p=5p$



- Since $4-2=2$,
  $4p-2p=2p$



Example 17 — Self Tutor

If possible, simplify by collecting like terms:

a $4x+3x$ **b** $3x-2x$ **c** $8x-x$ **d** $3x+2$

a $4x+3x=7x$ $\{4x$ and $3x$ are like terms$\}$

b $3x-2x=1x=x$ $\{3x$ and $-2x$ are like terms$\}$

c $8x-x=7x$ $\{8x-x$ is really $8x-1x\}$

d $3x+2$ cannot be simplified as $3x$ and 2 are not like terms.

EXERCISE 8G

1 Simplify by collecting like terms:

a $a+a$ **b** $b+b+b$ **c** $a+a+b+b+b$

d $3+x+x+y$ **e** $f+f+2+f+1$ **f** $3-a+2+a$

g $q+q-q+q$ **h** $x+x-x+x-x-2$ **i** $2+m-m+n+3+n$

2 If possible, simplify by collecting like terms:

a $a+3a$ **b** $2y+2y$ **c** $x+y$ **d** $2x+x+y$

e $5b+7b$ **f** $9r-6r$ **g** $4x-5$ **h** $8n-2n$

i $7p-5p$ **j** $3z-3$ **k** $3c-3c$ **l** $4e+4f$

m $10w-5w$ **n** $12x-6$ **o** $2x+5x+4x$

Example 18 **Self Tutor**

If possible, simplify by collecting like terms:

a $2x-4x$ **b** $-3x-x$ **c** $3x+x-7x$

a $\begin{aligned} &2x-4x \\ &= -2x \end{aligned}$

b $\begin{aligned} &-3x-x \\ &= -4x \end{aligned}$

c $\begin{aligned} &3x+x-7x \\ &= 4x-7x \\ &= -3x \end{aligned}$

3 If possible, simplify by collecting like terms:

a $3z-5z$ **b** $2b-6b$ **c** $3c-3$

d $m-3m$ **e** $x+x-4x$ **f** $-f-f-f-f$

g $-3y-y-2y$ **h** $4k-8k$ **i** $4k-8k+4$

j $-15r-5$ **k** $-15r-5r$ **l** $-t-t-2t$

m $2v-3v$

Example 19 **Self Tutor**

Simplify by collecting like terms:

a $3a+b+4a+2b+1$ **b** $3a+6b-a-2b$

a $\begin{aligned} &3a+b+4a+2b+1 \\ &= 3a+4a+b+2b+1 \\ &= 7a+3b+1 \end{aligned}$

b $\begin{aligned} &3a+6b-a-2b \\ &= 3a-a+6b-2b \\ &= 2a+4b \end{aligned}$

4 If possible, simplify by collecting like terms:

a $6x+2x+5y+y$ **b** $p+2p+5q+3q$ **c** $4a+3b+3a+3b$

d $c+d-c+d$ **e** $4v+3-5v-7$ **f** $5x+4z-x-2z$

g $g+2h-g+3h$ **h** $5r+5+t-r$ **i** $8x-12y+4x-y$

5 Pat says that $4-(p+p+p)$ is the same as $4-p+p+p$.
Explain why Pat is incorrect.

H ALGEBRAIC SUBSTITUTION

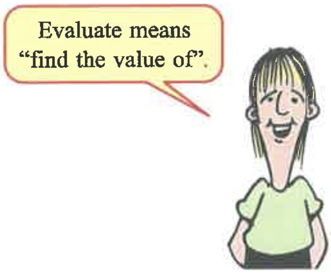
If we know the values of the variables in an expression, we can **substitute** the values for the variables. This allows us to **evaluate** the expression.

When we **evaluate** a mathematical expression, we calculate its value for particular numerical values of the variables or unknowns.

Example 20 Self Tutor

Evaluate $3c + 7$ when $c = 4$.

If $c = 4$ then $3c + 7 = 3 \times 4 + 7$
$= 12 + 7$
$= 19$



EXERCISE 8H.1

1 Evaluate:

a $2z + 7$ when $z = 1$
b $3y - 1$ when $y = 4$
c $2x + 1$ when $x = 4$
d $2 + 9d$ when $d = 2$
e $19 - 3q$ when $q = 3$
f $12 - 4a$ when $a = 2$
g $3k + 3$ when $k = 6$
h $5 \times (j - 2)$ when $j = 7$
i $x^2 - 2$ when $x = 3$
j $2a - a^2$ when $a = 4$

2 Suppose we have 3 bags each containing p potatoes, and 10 potatoes left over.

a Write an expression for the total number of potatoes.
b Find the total number of potatoes if:
i $p = 5$ **ii** $p = 12$ **iii** $p = 25$

Example 21 Self Tutor

If $x = 3$ and $y = 2$, evaluate:
a $2x - y$ **b** $3(3y + 5x)$

a $2x - y$
$= 2 \times 3 - 2$
$= 6 - 2$
$= 4$

b $3(3y + 5x)$
$= 3(3 \times 2 + 5 \times 3)$
$= 3(6 + 15)$
$= 3 \times 21$
$= 63$

3 If $x = 3$ and $y = 4$, find the value of:

a $2x$	**b** $x + 3y$	**c** $4(x + y)$	**d** $6x - 4y + 2$
e $5y - 3x$	**f** $3(x - y)$	**g** $7 + 2y - 6x$	**h** $2(5x - 3y)$
i $\frac{2x}{y}$	**j** $\frac{x+5}{2y}$	**k** $5 - (2x + y)$	**l** $x - (2 + 3y)$

4 If $p = 4$, $q = 2$, and $r = 5$, find the value of:

a $q + r$	**b** pqr	**c** p^2	**d** pq
e $r - q$	**f** $3q - p$	**g** $2r - 3p$	**h** $3q^2$
i $pq + qr$	**j** $3(p + 3q)$	**k** $9(2q - r)$	**l** $p(qr + 2)$

5 If $a = 2$, $b = 0$, and $c = 3$, evaluate:

a $a + 4c$	**b** $4a + 5c$	**c** $4(a - c)$	**d** c^2
e $4c^2$	**f** $7b - 6$	**g** $ab + bc$	**h** abc
i $3ac^2 - a$	**j** $2(a + 7c)$	**k** $(2a - c)^2$	**l** $6(c - 3a)$

NEGATIVE SUBSTITUTION

Variables do not always take positive values. They can also be assigned negative values. To avoid confusion with signs, we write negative substitutions in brackets.

Example 22 **Self Tutor**

If $x = 5$ and $y = -4$, find the value of:

a $2x + 3y$ **b** $x^2 - xy$

a
$$\begin{aligned} & 2x + 3y \\ &= 2 \times 5 + 3 \times (-4) \\ &= 10 + -12 \\ &= 10 - 12 \\ &= -2 \end{aligned}$$

b
$$\begin{aligned} & x^2 - xy \\ &= 5^2 - 5 \times (-4) \\ &= 25 - (-20) \\ &= 25 + 20 \\ &= 45 \end{aligned}$$

EXERCISE 8H.2

1 Given $w = -1$, $x = 2$, and $y = -4$, evaluate:

a $x - w$	**b** $4xy$	**c** $y - w$	**d** $3y + 3w$
e wxy	**f** $w + y - x$	**g** $y - x^2$	**h** $wx + xy$
i $3(x - 2w)$	**j** $x + wy$	**k** $2(x + y)$	**l** $2w^2 - 3y$

2 If $f = 2$, $g = 8$, and $h = -5$, find the value of:

a $f - 2g$	**b** $\frac{gh}{f}$	**c** $f^2 - h$	**d** $3h + 5f$
e $4(g - h)$	**f** $f(g + h)$	**g** $h^2 - fg$	**h** $g - (f - h)$
i $(g - 2h)^2$	**j** $3(2g - 3h)$	**k** $g(h - 3f)$	**l** $(fgh)^2$

INVESTIGATION — COMMUTATIVE AND ASSOCIATIVE LAWS

PRINTABLE WORKSHEET

In this Investigation we study some properties of the mathematical operations addition, subtraction, and multiplication.

Click on the icon to obtain this printable worksheet.

ACTIVITY 2 — "THINK OF A NUMBER" GAMES

What to do:

- Play in pairs - player A and player B.
- Player A chooses a number while player B calls out the steps in each game.
- Choose a few different numbers in each game and take it in turns to be player A or B.
- Discuss the results. Letting the number be x, write an algebraic expression to describe the steps in each game.

Game 1

Step 1: Think of a number.
Step 2: Double it.
Step 3: Add 7.
Step 4: Take away 1.
Step 5: Divide by 2.
Step 6: Subtract the original number.
Step 7: State the output.

Game 2

Step 1: Think of a number.
Step 2: Add nine.
Step 3: Multiply by two.
Step 4: Subtract eighteen.
Step 5: Divide by two.
Step 6: Subtract the original number.
Step 7: State the output.

I FORMULAE

In **Example 1** part **c** we have 2 punnets which each contain s strawberries, plus 4 strawberries left over.

If we let N be the total number of strawberries, then $N = 2s + 4$.

This equation is a *formula* connecting the total number of strawberries N, with the number of strawberries in each punnet, s.

We say that N is the *subject* of the formula, and that N is written *in terms of* s.

A **formula** is an equation that connects two or more variables.

The plural of formula is *formulae* or *formulas*.

The **subject** of a formula is the variable which appears by itself on one side of the equation.

We can evaluate the subject of a formula by substituting particular values of the remaining variables.

Example 23 Self Tutor

Copy and complete the table by substituting into the formula $W = 3t + 2$.

t	1	3	6	15
W				

When $t = 1$, $W = 3 \times 1 + 2$
$= 3 + 2$
$= 5$

When $t = 3$, $W = 3 \times 3 + 2$
$= 9 + 2$
$= 11$

When $t = 6$, $W = 3 \times 6 + 2$
$= 18 + 2$
$= 20$

When $t = 15$, $W = 3 \times 15 + 2$
$= 45 + 2$
$= 47$

We can now complete the table:

t	1	3	6	15
W	5	11	20	47

EXERCISE 8I

1 Complete each table by substituting into the given formula:

a $S = n + 3$

n	1	2	3	4	5
S					

b $L = 4b$

b	1	3	5	7	9
L					

c $C = 2d + 7$

d	-1	1	4	8	10
C					

d $P = 3t - 4$

t	-3	0	2	5	9
P					

e $A = 3x - 6$

x	2	5	8	12
A				

f $A = x^2 + 3x$

x	1	2	4	6
A				

2 Let $y = 5x - 3$. Find the value of y when:
a $x = 0$ **b** $x = 1$ **c** $x = 5$ **d** $x = 10$

3 Let $y = 25 - 2x$. Find the value of y when:
a $x = 5$ **b** $x = 10$ **c** $x = -2$ **d** $x = -5$

4 Let $A = 16 - x^2$. Find the value of A when:
a $x = 0$ **b** $x = 2$ **c** $x = 3$ **d** $x = 4$

5 Let $V = x^2(x + 2)$. Find the value of V when:
a $x = 1$ **b** $x = 2$ **c** $x = 3$ **d** $x = 4$

Example 24 — Self Tutor

The cost of hiring a tennis court is given by $C = 5h + 8$ dollars, where h is the number of hours the court is hired for.

Find the cost of hiring the tennis court for:

a 4 hours **b** 10 hours.

The formula is $C = 5h + 8$.

a If $h = 4$, $C = 5 \times 4 + 8$
$= 20 + 8$
$= 28$

So, it costs $28 for 4 hours.

b If $h = 10$, $C = 5 \times 10 + 8$
$= 50 + 8$
$= 58$

So, it costs $58 for 10 hours.

6 The cost of hiring a campervan is given by $C = 65d + 100$ dollars, where d is the number of days the campervan is hired.
Find the cost of hiring the campervan for:

a 2 days **b** 5 days **c** 2 weeks.

7 The distance travelled by a car is given by $D = st$ km, where s is the car's speed in km/h, and t is the time in hours.
Find the distance travelled by a car travelling at:

a 50 km/h for 3 hours **b** 80 km/h for 7 hours.

8 The recommended dose of medicine for a child aged a years is $D = \dfrac{50 \times a}{a + 12}$ mL.

Find the recommended dose for a child aged:

a 4 years **b** 8 years **c** 12 years.

Example 25 — Self Tutor

A taxi company charges $3 "flagfall" plus $1.80 for each kilometre travelled.
Suppose the total charge is $\$C$ for travelling n kilometres. Find:

a the cost for just travelling the distance n kilometres

b the formula connecting C and n

c the total charge for travelling 21.6 km.

a The charge for each kilometre is $1.80, so the cost for just the distance n kilometres is $\$1.80n$.

b To find the total charge C we need to add on the $3 "flagfall".
So, $C = 1.80n + 3$.

c If $n = 21.6$, $C = 1.80 \times 21.6 + 3$
$= 41.88$

So, the total charge for travelling 21.6 km is $41.88.

9 Maya is raising money for charity by reading books. Her mother donates \$2 for each book she reads.
Suppose Maya reads b books and her mother donates \$$R$.

a Write a formula for R in terms of b.

b Find the total money Maya raises for reading:

i 10 books **ii** 25 books.

10 A gardener charges a \$20 callout fee plus \$40 for every hour he spends on the job.
Suppose the total charge is \$$C$ for a job taking h hours.

a What is the charge for doing h hours work, excluding the callout fee?

b Find the formula connecting C and h.

c Find the total charge for a job taking:

i 1 hour **ii** 4 hours **iii** $1\frac{3}{4}$ hours.

11 An online craft shop sells balls of wool for \$7 each. There is a \$9 charge for postage and handling no matter how big or small the order.

a Write a formula for the total cost \$$C$ of buying n balls of wool. Include postage and handling in your answer.

b Use your formula to copy and complete the table alongside.

Number of wool balls purchased	*Total cost* (\$)
1	
2	
3	
4	
5	
6	
10	
15	

12 This table of fees shows what an electrician charges for jobs of different length:

Number of hours (h)	1	2	3	4	5
Cost of job (\$$C$)	80	130	180	230	280

a By finding the difference between successive costs in the table, find the amount the electrician charges for each hour spent on the job.

b Using **a** and the cost of a job taking 1 hour, find the amount the electrician charges as a callout fee.

c Hence write a formula connecting C and h.

d Find the cost of a job that takes:

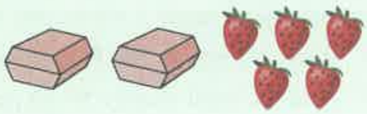
i 8 hours **ii** 12 hours.

MULTIPLE CHOICE QUIZ

QUICK QUIZ

REVIEW SET 8A

1 Suppose we have 2 punnets of strawberries, plus 5 strawberries left over.
How many strawberries are there if each punnet contains:



a 4 strawberries **b** 7 strawberries **c** s strawberries?

2 Write using product notation:

a $m+m+m+m$ **b** $k \times 7$ **c** $4y \times 5$ **d** $3q \times 6p$

3 Write using exponent notation:

a $4 \times c \times c \times c$ **b** $x \times x - 3 \times y \times y$ **c** $2x \times 4x$

4 Write in words:

a $r+s$ **b** $\frac{5}{m}$ **c** $xy-3$ **d** z^2-2y

5 How many terms are in the expression $4x^3+6x^2+4x+1$?

6 State the coefficient of y in $2x^2+3xy-7y$.

7 State the like terms in:

a $2x^2-4x+3+3x^2$ **b** $5a-3b+a-2b+1$
c $3c-c^2+3c+2$ **d** $2e+ef-4e+2ef+2f$

8 If possible, simplify by collecting like terms:

a $7x+5x+5$ **b** $2p+1+3p-3$
c $3x-5y+5x+5y$ **d** $-7c+5d+cd+4c$

9 Evaluate:

a $5x-3$ when $x=6$ **b** $1-2a$ when $a=3$
c $4n-8$ when $n=5$ **d** x^2-3x when $x=2$

10 If $a=-2$, $b=5$, and $c=-3$, evaluate:

a abc **b** $5a-7b$ **c** $6c-9b$ **d** $a(b-c)$

11 Complete each table by substituting into the given formula:

a $P=4k+5$

k	1	2	3	4	5
P					

b $N=2m-3$

m	1	4	6	10	15
N					

c $A=\frac{3a+5}{2}$

a	1	2	3	4	5
A					

d $M=x^2-x$

x	1	2	3	5	8
M					

12 The time required to milk c cows each day is given by $T=2.25c+35$ minutes. Find the time required to milk a herd of:

a 40 cows **b** 72 cows.

13 Will, Jerry, Mike, and Scott collect trading cards.

Will has 20 loose cards.
Mike has 1 box of cards and 3 loose cards.
Jerry has 2 boxes of cards.
Scott has 2 boxes of cards and 5 loose cards.

a How many loose cards are there?

b How many boxes of cards are there?

c If there are c cards in a box, write an expression for the total number of cards owned by the four friends.

d Now suppose each box contains 30 cards. Find how many cards the four friends have between them.

REVIEW SET 8B

1 There are 3 cages which contain some birds. How many birds are present in total if each cage contains:

a 3 birds, and there are 2 flying nearby

b b birds, and there are 2 flying nearby

c b birds, and there are f flying nearby?

2 Write using product notation:

a $3x \times 7$ **b** $a + b + b + a + a$ **c** $2 \times 2x + 2 \times y$

3 Write using exponent notation:

a $k \times k \times k \times k$ **b** $c \times c + 5 \times c \times c \times c$ **c** $2m^2 \times 5m$

4 Write in expanded form:

a p^4 **b** $2q^3$ **c** $3x^3 - 4y^2$

5 Write as an algebraic expression:

a the product of a and the square of d **b** 7 less than the quotient of p and q.

6 For the expression $6x^2 - 2xy + 2y - 3y^2$, state the coefficient of y^2.

7 State the like terms in $6x - 6y + 2x + y + 1$.

8 State the constant in:

a $5x + 4 - 2y$ **b** $1 - x + 2x^2$ **c** $3ab - a + 2 + 2b$

9 Simplify by collecting like terms:

a $2x + 3y + 5x$ **b** $-q + 2 - 5q$ **c** $2a - 3b + 8a + 4b$

10 If $a = -2$, $b = -3$, and $c = 5$, find the value of:

a $3a^2 + 4b - c$ **b** $\dfrac{c - b}{2a}$ **c** $b(2a + c)$

11 The cost of hiring a suit for d days is given by $C = 30 + 50d$ dollars. Find the cost of hiring the suit for:

a 1 day **b** 3 days.

12 A company stores printers at Warehouses A and B.
Warehouse A has 10 crates of printers, and 5 spare printers.
Warehouse B has 8 crates of printers, and 15 spare printers.
Suppose there are c printers in each crate.

a Write an expression for the number of printers:

i in Warehouse A **ii** in Warehouse B **iii** in total.

b If each crate contains 5 printers, show that there are the same number of printers at each warehouse.

c Which warehouse has more printers if each crate contains:

i 4 printers **ii** 8 printers?

13 Peter orders coffee beans online. The coffee company charges \$35 for each kg of coffee, plus a \$15 delivery fee.

a What does the company charge for b kg of beans, excluding delivery?

b Write a formula for the total cost $\$C$ of an order for b kg of coffee beans.

c Find the total cost of an order for:

i 1.5 kg of coffee beans

ii 4 kg of coffee beans.

Chapter 9

Percentage

Contents:

OPENING PROBLEM

A department store buys jackets from a manufacturer for \$42.

It "marks up" the price so the customer is normally expected to pay \$79.80 for a jacket.

Things to think about:

a By what percentage did the store "mark up" the price?

b On Black Friday, the store offers a 40% discount off all products.

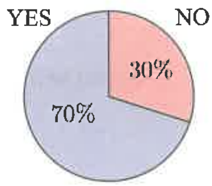
  i How much is the discount offered on a jacket?

  ii What is the selling price that a customer will pay?

  iii Will the store make a profit when selling a jacket at the discounted price?

Percentages are used every day around us, so it is important to understand what they mean and how we use them.

The symbol % indicates a **percentage**.



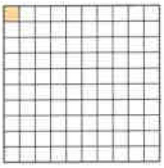
A PERCENTAGE

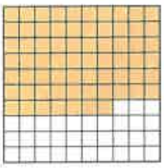
A **percentage** is used to compare a portion with a whole amount.

The whole amount is represented by 100%, which has the value 1.

% reads **percent**, which means "in every hundred".

If an object is divided into one hundred equal parts, then each part is called 1 percent or 1%.

$\frac{1}{100} = 1\%$ 

$\frac{67}{100} = 67\%$ 

$\frac{100}{100} = 100\%$ 

So, a percentage is like a fraction which has denominator 100.

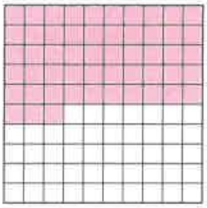
$$x\% = \frac{x}{100}$$

EXERCISE 9A

1 What percentage is represented by the following shaded diagrams?

a

b

c

d


2 Write as a fraction with denominator 100:

a 13% **b** 37% **c** 6% **d** 92% **e** 79%

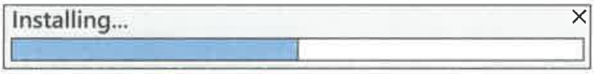
3 Write as a percentage:

a $\frac{17}{100}$ **b** $\frac{38}{100}$ **c** $\frac{90}{100}$ **d** $\frac{125}{100}$ **e** $\frac{1}{100}$

4 Which progress bar shows the installation to be:

a 50% complete
b 7% complete
c 40% complete
d 95% complete?

A


B


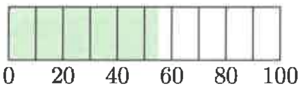
C

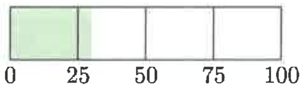

D


5 The members of the Brown family each have their own shampoo bottle. Whose bottle is:

a 75% full **b** 20% full
c 5% full **d** 90% full?

6 Estimate the percentage of each diagram which is shaded:

a


b


c


7 This table gives the products that can be made from petroleum.

a What percentage of petroleum products is:
  i jet fuel **ii** heavy fuel oil or lubricants?

b Write the product categories in order from greatest to least.

c Find the sum of the percentages. Explain your answer.

Petroleum product	%
gasoline	46%
jet fuel	9%
diesel and other fuel	26%
asphalt	3%
heavy fuel oil	4%
other products	11%
lubricants	1%

DISCUSSION

The word percent means "in every hundred".

For each of the following statements, discuss:

- what the percentage means in the real-world context
- why this information may be useful.

a Sarah has improved by 18% in her times tables test.
b Our netball goal shooter has a 68% accuracy rate.
c The company sales reduced by 9% this year.
d 27% of school students do not eat sufficient fruit and vegetables.
e 76% of the population has been vaccinated against the SARS-COV-2 coronavirus.

DISCUSSION

Is it possible to have a percentage greater than 100%?

What would a percentage greater than 100% *mean*?

In what situations might it be reasonable to have a percentage more than 100%?

B CONVERTING PERCENTAGES INTO DECIMALS AND FRACTIONS

To convert a percentage into a decimal or a fraction, we divide by 100%.

$100\% = \frac{100}{100} = 1$, so dividing by 100% does not change the value of the number.

Example 1 Self Tutor

Write as a decimal:

a 31% **b** 150%

a 31%
$= 31.\ \%$
$= 0.31$

b 150%
$= 150.\ \%$
$= 1.5$

To divide by 100, move the decimal point 2 places to the left.

When we convert a percentage into a fraction, we write the result in lowest terms.

Example 2 Self Tutor

Write as a fraction in lowest terms:

a 76% **b** 120%

a $76\% = \frac{76}{100} = \frac{76 \div 4}{100 \div 4}$ {HCF is 4} $= \frac{19}{25}$

b $120\% = \frac{120}{100} = \frac{120 \div 20}{100 \div 20}$ {HCF is 20} $= \frac{6}{5}$

EXERCISE 9B

1 Write as a decimal:

a 60% **b** 35% **c** 18% **d** 8%
e 46% **f** 89% **g** 125% **h** 200%
i 49.5% **j** 16.7% **k** 37.5% **l** 77.7%
m 38.01% **n** 0.1% **o** 129.8% **p** 0.002%

2 Write as a whole number or as a fraction in lowest terms:

a 40% **b** 20% **c** 100% **d** 15%
e 6% **f** 55% **g** 150% **h** 4%
i 88% **j** 175% **k** 62% **l** 120%
m 300% **n** 245% **o** 184% **p** 12.5%

3 84% of the students at a school are right-handed. What fraction of the students are:

a right-handed **b** left-handed?

4 The gift shop at a zoo sells several different soft toy animals from Africa.

a Copy and complete this table to include the decimal and fraction of sales for each animal.

Animal	*Sales %*	*Decimal*	*Fraction*
Lion	13%		
Giraffe	24%		
Cheetah	14%		
Meerkat	32%		
Zebra	17%		

b List the animals in order from least to most popular.

C CONVERTING DECIMALS AND FRACTIONS INTO PERCENTAGES

To convert a decimal or a fraction into a percentage, we multiply by 100%.

Example 3 Self Tutor

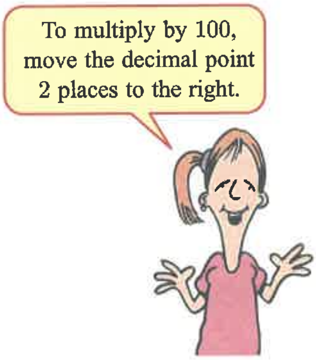
Write as a percentage:

a 0.28 **b** 0.064 **c** 2.7

a 0.28
$= 0.28 \times 100\%$
$= 28\%$

b 0.064
$= 0.064 \times 100\%$
$= 6.4\%$

c 2.7
$= 2.70 \times 100\%$
$= 270\%$



Many fractions can be converted into a percentage by first writing the fraction with denominator 100.

Example 4 Self Tutor

Write as a percentage: **a** $\frac{3}{5}$ **b** $\frac{71}{200}$

a $\frac{3}{5} = \frac{3 \times 20}{5 \times 20}$
$= \frac{60}{100}$
$= 60\%$

b $\frac{71}{200} = \frac{71 \div 2}{200 \div 2}$
$= \frac{35.5}{100}$
$= 35.5\%$

For fractions which cannot be easily written with denominator 100, we multiply by 100% immediately.

Example 5 Self Tutor

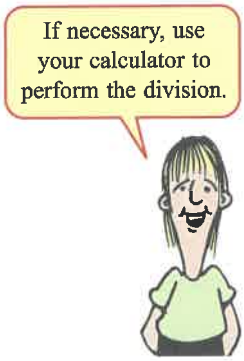
Write as a percentage by multiplying by 100%:

a $\frac{3}{40}$ **b** $\frac{7}{8}$ **c** $\frac{2}{3}$

a $\frac{3}{40}$
$= \frac{3}{40} \times 100\%$
$= \frac{30\cancel{0}}{4\cancel{0}}\%$
$= 7.5\%$

b $\frac{7}{8}$
$= \frac{7}{8} \times 100\%$
$= \frac{700}{8}\%$
$= 87.5\%$

c $\frac{2}{3}$
$= \frac{2}{3} \times 100\%$
$= \frac{200}{3}\%$
$= 66\frac{2}{3}\%$



EXERCISE 9C

1 Write as a percentage:

- **a** 0.38 **b** 0.93 **c** 0.15 **d** 0.317
- **e** 0.546 **f** 0.802 **g** 0.07 **h** 1.58
- **i** 0.9 **j** 0.004 **k** 0.059 **l** 0.4073
- **m** 1.6 **n** 4.2 **o** 3 **p** 0.0026

2 Copy and complete:

a

Fraction	*Percentage*	*Decimal*
$\frac{1}{4}$		
$\frac{2}{4}$		
$\frac{3}{4}$		
$\frac{4}{4}$		

b

Fraction	*Percentage*	*Decimal*
$\frac{1}{5}$		
$\frac{2}{5}$		
$\frac{3}{5}$		
$\frac{4}{5}$		

3 Write as a percentage, by first writing the fraction with denominator 100:

- **a** $\frac{7}{10}$ **b** $\frac{9}{25}$ **c** $\frac{11}{20}$ **d** $\frac{17}{25}$
- **e** $\frac{33}{50}$ **f** $\frac{41}{200}$ **g** $\frac{341}{1000}$ **h** $\frac{709}{1000}$

4 Copy and complete, using your calculator and rounding to 3 decimal places if necessary:

a

Fraction	*Percentage*	*Decimal*
$\frac{1}{3}$		
$\frac{2}{3}$		
$\frac{3}{3}$		

b

Fraction	*Percentage*	*Decimal*
$\frac{1}{8}$		
$\frac{2}{8}$		
$\frac{3}{8}$		
$\frac{4}{8}$		
$\frac{5}{8}$		
$\frac{6}{8}$		
$\frac{7}{8}$		

c

Fraction	*Percentage*	*Decimal*
$\frac{1}{6}$		
$\frac{2}{6}$		
$\frac{3}{6}$		
$\frac{4}{6}$		
$\frac{5}{6}$		

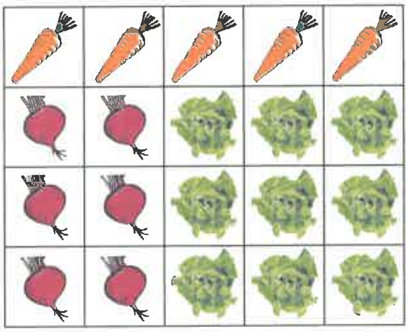
5 Write as a percentage, using your calculator to perform the division:

- **a** $\frac{9}{40}$ **b** $\frac{3}{16}$ **c** $\frac{7}{80}$ **d** $\frac{9}{16}$ **e** $\frac{27}{40}$ **f** $\frac{21}{250}$

6 Write as a percentage, rounding your answers to 1 decimal place:

a $\frac{1}{9}$ **b** $\frac{5}{7}$ **c** $\frac{4}{9}$ **d** $\frac{5}{12}$ **e** $\frac{10}{13}$ **f** $\frac{18}{37}$

7 A garden plot is planted with beetroot, carrots, and lettuces as shown.



a What fraction of the plot has lettuces?

b What percentage of the plot has carrots?

c What percentage of the plot *does not* have beetroot?

8 A restaurant has 80 seats. 50 are occupied by diners.

a What fraction of the seats are occupied?

b What percentage of the seats are occupied?

c What percentage of the seats are *not* occupied?

D EXPRESSING ONE QUANTITY AS A PERCENTAGE OF ANOTHER

Percentages are often used to *compare* quantities, so it is useful to express one quantity as a percentage of another.

It is only meaningful to compare **like with like**. For example:

- 5 bicycles as a percentage of 45 cars is meaningless, because a bicycle is not a car.
- 5 bicycles as a percentage of 45 vehicles does have meaning, because a bicycle is a type of vehicle.

$$\begin{aligned}\frac{5 \text{ bicycles}}{45 \text{ vehicles}} &= \frac{1}{9} \times 100\% \\ &= \frac{100}{9}\% \\ &\approx 11.1\%\end{aligned}$$

We must also make sure that the quantities are compared in the **same units**.

For example, if we are asked to express 280 g as a percentage of 7 kg, we would first convert the larger unit to the smaller one:

$$\begin{aligned}\frac{280 \text{ g}}{7 \text{ kg}} &= \frac{280 \text{ g}}{7000 \text{ g}} \\ &= \frac{280 \div 70}{7000 \div 70} \\ &= \frac{4}{100} \\ &= 4\%\end{aligned}$$

To express one quantity as a percentage of another, we write them as a fraction, then convert the fraction to a percentage.

Example 6 **Self Tutor**

Express as a percentage:

a A test mark of 17 out of a possible 25.

b Out of 1600 vehicles sold in a month, 250 of them were vans.

a $\dfrac{17 \text{ marks}}{25 \text{ marks}} = \dfrac{17 \times 4}{25 \times 4}$

$= \dfrac{68}{100}$

$= 68\%$

b $\dfrac{250 \text{ vans}}{1600 \text{ vehicles}} = \dfrac{250}{1600} \times 100\%$

$= \dfrac{250}{16}\%$

$= 15.625\%$

EXERCISE 9D

1 Express as a percentage:

a 2 cm out of 5 cm
b 13 kg out of 20 kg
c 1 L out of 10 L
d \$3 out of \$6
e 4 km out of 10 km
f 6 m out of 8 m
g 36 minutes out of 40 minutes
h 200 kg out of 250 kg
i 75 mL out of 375 mL
j 43 marks out of 50 marks
k 49 points out of 70 points
l \$18 out of \$300
m 470 chocolate bars sold out of 1000 made
n 160 hybrid vehicles out of 400 cars sold
o 75 marks out of 120 marks
p 135 points out of 180 points.

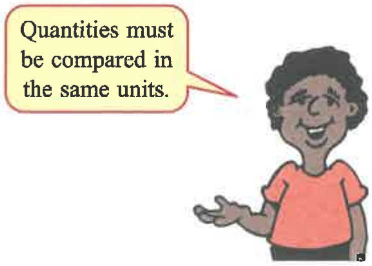
Example 7 **Self Tutor**

Express 45 minutes as a percentage of 3 hours.

$\dfrac{45 \text{ minutes}}{3 \text{ hours}} = \dfrac{45 \text{ minutes}}{3 \times 60 \text{ minutes}}$

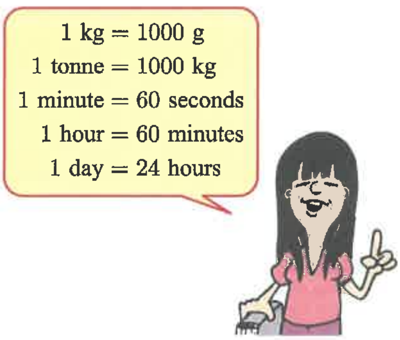
$= \dfrac{45}{180} \times 100\%$

$= 25\%$



2 Express the first quantity as a percentage of the second:

a 24 minutes, 1 hour
b 450 g, 1 kg
c 800 m, 2 km
d 3 months, 1 year
e 25 cm, 0.5 m
f 800 mL, 4 L
g 300 m, 1.5 km
h 70 cents, \$4.20
i 14 days, 3 weeks
j 440 mL, 2 L
k 84 cm, 2 m
l 550 g, 1 kg
m 21 hours, 1 day
n 250 g, 2 tonnes
o 1.8 seconds, 3 hours



3 The 25 students of a class were each given a calculator at the start of the year. By the end of the year, six students had lost their calculators, and five students had broken theirs. What percentage of the class had:

a lost their calculators
b broken their calculators
c lost *or* broken their calculators?

4 A television station is required to show at least 55% local programming between 6 am and midnight each day.
If the station averages 10 hours of local programming during these times, has the station met the requirement?

5 Of the 1500 trains leaving a train station one day, 1290 left within 5 minutes of the scheduled time.
What percentage of the trains left:

a within 5 minutes of the scheduled time
b at least 5 minutes late?

6 The table below shows the forest area and total land area of various countries.

	Forest area (km^2)	*Land area* (km^2)	*Forest as % of land area*
Bangladesh	14 238	130 170	10.9%
Colombia	584 483	1 038 700	56.3%
Finland	222 180	303 815	
Indonesia	896 412	1 811 570	
Madagascar	124 410	581 540	
Niger	11 172	1 266 700	
Philippines	85 200	298 170	
Spain	184 861	498 980	

a Copy and complete the table, rounding the percentages to 1 decimal place.
b Which country has the highest percentage of its land covered by forest?

PRINTABLE TABLE

ACTIVITY 1 SCORING SPREADSHEET

A spreadsheet is a fast and efficient tool for calculating percentages.

Click on the icon to obtain this online Activity.

ACTIVITY

ACTIVITY 2 PERCENTAGES IN THE WORLD AROUND US

In this Activity you will calculate percentages of various things at home and at school.

What to do:

1 In your home, what percentage of:

a rooms have tiled floors
b electrical goods are used every day
c groceries in your fridge were grown or made in your own country?

2 What percentage of your classmates:

a were born overseas **b** have at least one parent born overseas?

3 In a normal school week, what percentage of lesson time is spent on each subject?

4 **a** Of the students in your class, what percentage regularly buy their lunch from the school canteen?

b Is the percentage you calculated for your class likely to be a good estimate of the percentage of all students in your school who regularly buy their lunch from the canteen?

E FINDING A PERCENTAGE OF A QUANTITY

To find a **percentage of a quantity**, we first convert the percentage to a **decimal**. We then **multiply** to find the required amount.

Example 8 Self Tutor

Find:

a 35% of \$5000
b 12.4% of 6 m, giving your answer in cm.

a 35% of \$5000
$= 0.35 \times \$5000$
$= \$1750$

b 12.4% of 6 m
$= 0.124 \times 600$ cm
$= 74.4$ cm

EXERCISE 9E

1 Find:

a 25% of 36
b 10% of 70
c 20% of 45
d 36% of £4200
e 125% of \$600
f 95% of 5 tonnes
g 3.8% of 100 m
h 112% of 5000 mL
i $33\frac{1}{3}\%$ of $360°$

2 Find:

a 15% of 1 hour, giving your answer in minutes

b 40% of 1.8 km, giving your answer in metres

c 29% of 2 kg, giving your answer in grams

d 5.6% of 8 L, giving your answer in mL.

3 Carly has 20 sweets. She gives 35% of them to her sister. How many sweets is Carly giving away?

4 On average, 88% of seeds in a packet are expected to germinate. If a packet contains 150 seeds, how many are expected to germinate?

5 In Formula One racing, a driver must complete at least 90% of the race distance in order to be classified. How many laps of a 60 lap race must a driver complete to be classified?

6 In a class of 20 students, 70% are boys, 85% are 12 years old, 10% are 13 years old, and 35% catch the bus to school. Calculate the number of students who:

a are boys

b do not catch the bus to school

c are 12 years old

d are either 12 or 13 years old.

7 A kitten will reach 70% of its adult body weight after six months. If a cat weighs 4.5 kg when fully grown, estimate how much it weighed when it was six months old.

8 Fran's Fruit Juice contains 60% orange juice. How much orange juice is contained in a:

a 300 mL glass

b 2 litre bottle of Fran's Fruit Juice?

9

An orchard produces 150 tonnes of apples in one season. 30 tonnes of the apples are second grade, 8% are unfit for sale, and the rest are first grade. First grade apples sell for $1640 per tonne, and second grade for $1250 per tonne. Find:

a the weight of apples unfit for sale

b the weight of first grade apples

c the total value of the apple harvest.

F PERCENTAGE INCREASE OR DECREASE

There are many situations where quantities are either increased or decreased by a certain percentage.

For example:

- the attendance at football games this year has increased by 5% from last year
- the price of an electric drill is *discounted* by 15% in a sale
- a store manager *marks up* the price of a sound system by 45% in order to make a profit.

To apply a percentage increase or decrease, we can:

- find the *size* of the increase or decrease, then
- *apply* this change to the original quantity by addition or subtraction.

Example 9 — Self Tutor

Last year, a total of 382 000 spectators attended a club's football games. This year, the attendance has increased by 5%. Find:

a the number of extra spectators who have attended this year

b the total number of spectators who have attended this year.

a 5% of $382\,000 = 0.05 \times 382\,000$
$= 19\,100$ spectators

b The total number of spectators this year
$= 382\,000 + 19\,100$
$= 401\,100$ spectators

EXERCISE 9F

1 Last year, a tree was measured to be 2.6 m high. Since then, it has grown by 20%.

a By how much has the tree grown in the last year?

b Find the height of the tree now.

2 Lizzie has deposited €4000 in a bank account. This year it will earn 2% interest.

a How much interest will Lizzie earn this year?

b How much will Lizzie's investment be worth at the end of the year?

3 A department store buys a sound system for \$240. The store manager increases the price by 45% in order to make a profit.

a By how much has the store manager increased the price?

b For what price will the store sell the sound system?

4 Increase:

a 18 m by 30%
b 120 L by 20%
c \$320 by 15%
d 840 g by 6%
e €4200 by 1.8%
f 64.2 kg by 2.6%.

Example 10 — Self Tutor

Finn's backpack had mass 8 kg on the way to school. After putting some books in his locker, the mass of the backpack decreased by 15%.

a By how much did the mass of Finn's backpack decrease?

b Find the new mass of the backpack.

a 15% of $8\text{ kg} = 0.15 \times 8\text{ kg}$
$= 1.2\text{ kg}$

b New mass of the backpack
$= 8\text{ kg} - 1.2\text{ kg}$
$= 6.8\text{ kg}$

5 The shares in a company were worth \$27.60 each. However, the value of the shares fell by 10% in a stock market crash.

a By how much did the price of the shares fall?

b Find the price of the shares after the crash.

6 An 800 g block of ice is left out in the sun. After 15 minutes, 25% of the block has melted.

- a How much ice has melted after 15 minutes?
- b How much ice is remaining after 15 minutes?

7 Last year, a town had population 6000. In the past year, the population has decreased by 8%.

- a By how many people has the population decreased?
- b Find the population of the town now.

8 Decrease:

- a 120 cm by 20%
- b 800 mL by 40%
- c \$2000 by 12%
- d 164 ha by 5%
- e £390 by 4.7%
- f 118 kg by 8.5%.

G DISCOUNT

In order to attract customers to buy items, stores often reduce the price of an item. This is called a **discount**.

The discount offered by the store is often advertised as a percentage of the usual price of the item.

The usual price is called the **marked price** because it is the price marked on the label.

The actual price the customer pays is called the **selling price**.

selling price = marked price − discount

Example 11 Self Tutor

The marked price of a wetsuit is \$200. A discount of 25% is offered in a sale.

- a Find the amount of discount.
- b Find the actual selling price of the wetsuit.

a Discount = 25% of the marked price
= 25% of \$200
= $0.25 \times$ \$200
= \$50

b Selling price = marked price − discount
= \$200 − \$50
= \$150

EXERCISE 9G

1 At an electronics store, the marked price of a game console is \$240. Georgina receives a 10% discount because she works at the store.

- a How much discount would Georgina receive?
- b How much would Georgina pay for the game console?

2 The marked price of an exercise bike is \$490. A 50% discount is offered because a new model has just been released.

- a Find the amount of discount.
- b Find the actual selling price of the exercise bike.

3 The regular price of a wooden cabinet is £750. In a sale, the price is discounted by 30%.

- **a** By how much is the price of the cabinet discounted?
- **b** Find the discounted price of the cabinet.

4 A clothing store is having a "25% off" sale. Travis wants to buy a tie with marked price \$24, and a jacket with marked price \$120. He has \$110 available to spend.

- **a** Find the total of the marked prices of the items Travis wants to buy.
- **b** Find the discount that would be applied to this total.
- **c** Can Travis afford to buy both items? Explain your answer.

5 Find the selling price of:

- **a** a kettle with marked price \$39.40 if 25% discount is given
- **b** a car with marked price \$14 950 if 10% discount is given
- **c** a pair of shoes with marked price \$115.80 if 20% discount is given.

6 An electrician has quoted €1600 plus 17.5% service tax to install some lights. Seeing that his customers are pensioners, he offers them a 10% discount.

- **a** Find the total amount charged by applying:
  - **i** the tax first and then the discount to the result
  - **ii** the discount first and then the tax to the result.
- **b** If you are applying several percentage changes, does the order in which they are applied matter?

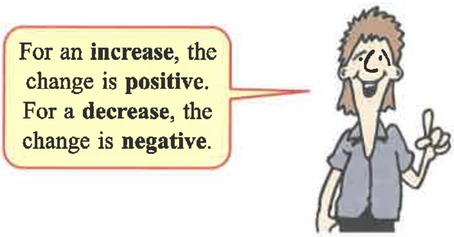
H FINDING A PERCENTAGE CHANGE

If we increase or decrease a quantity, the change in the quantity is given by:

$$\textbf{change} = \textbf{final amount} - \textbf{original amount}$$

For example:

- If 30 kg increases to 35 kg, the change is 35 kg − 30 kg = 5 kg.
- If 60 kg decreases to 48 kg, the change is 48 kg − 60 kg = −12 kg.



PERCENTAGE CHANGE

We often use a percentage to compare the change in a quantity with the original amount.

$$\textbf{percentage change} = \frac{\textbf{change}}{\textbf{original amount}} \times 100\%$$

Example 12 Self Tutor

Find the percentage increase or decrease in the change from:

a 50 m to 60 m

b 600 mL to 450 mL.

a change
$= \text{final amount} - \text{original amount}$
$= 60 \text{ m} - 50 \text{ m}$
$= 10 \text{ m}$

$\therefore$ percentage change

$$= \frac{\text{change}}{\text{original amount}} \times 100\%$$

$$= \frac{10}{50} \times 100\%$$

$$= 20\%$$

$\therefore$ there is a 20% increase.

b change
$= \text{final amount} - \text{original amount}$
$= 450 \text{ mL} - 600 \text{ mL}$
$= -150 \text{ mL}$

$\therefore$ percentage change

$$= \frac{\text{change}}{\text{original amount}} \times 100\%$$

$$= \frac{-150}{600} \times 100\%$$

$$= -25\%$$

$\therefore$ there is a 25% decrease.

EXERCISE 9H

1 Describe the change if:
  a 18 m increases to 21 m
  b 40 kg decreases to 32 kg
  c £28 increases to £34
  d 500 mL decreases to 265 mL
  e I improve my best time from 28 minutes to 25 minutes
  f the airline ticket price rose from \$212 to \$238.

2 Find the percentage increase or decrease in the change from:
  a 10 cm to 13 cm
  b 90 m to 72 m
  c 44 g to 66 g
  d 80 L to 48 L
  e \$20 to \$32
  f 120 min to 95 min
  g 6 m to 8.4 m
  h 3.6 kg to 2.7 kg
  i 7.6 s to 6.9 s.

3 In 5 minutes, an ice cube melted from 38 g to 16 g. Find the percentage reduction in mass.

4 In a clearance sale, the price of a washing machine was reduced from \$660 to \$540. What was the percentage discount?

5 The table alongside shows the heights in cm of two children at some of their birthdays.
  a Who grew by the greater percentage over this time?
  b In which year was there the greatest percentage increase in:
    i Lucas' height
    ii Annie's height?

Age	*Lucas*	*Annie*
12	122	119
13	129	139
14	141	153
15	162	163
16	175	167

6 Answer the **Opening Problem** on page **178**.

INVESTIGATION — USING DECIMALS FOR PERCENTAGE INCREASE OR DECREASE

We have seen that to apply a percentage increase or decrease, we can:

- find the *size* of the increase or decrease, then
- *apply* this change to the original quantity by addition or subtraction.

In this Investigation we will look for a quicker way to apply a percentage increase or decrease.

What to do:

1 The original quantity we start with is 100%.

 a If we increase the quantity by 20%, what percentage will we have *in total*?

 b Write this percentage as a decimal.

 c To find this percentage of the original quantity, what operation do we need to perform with this decimal?

2 Describe how you could use a decimal number to increase a quantity by:

 a 30% **b** 15% **c** 7% **d** 3.8%.

3 Use a decimal number to increase:

 a \$130 by 10% **b** 44 kg by 25% **c** 250 mL by 6%

 d 140 cm by 1.5% **e** €165 by 18% **f** 38.2 m^2 by 3.6%.

4 **a** If we *decrease* a quantity by 20%, what percentage will we have *remaining*?

 b Write this percentage as a decimal.

 c To find this percentage of the original quantity, what operation do we need to perform with this decimal?

5 Describe how you could use a decimal number to decrease a quantity by:

 a 30% **b** 15% **c** 7% **d** 3.8%.

6 Use a decimal number to decrease:

 a \$130 by 10% **b** 44 kg by 25% **c** 250 mL by 6%

 d 140 cm by 1.5% **e** €165 by 18% **f** 38.2 m^2 by 3.6%.

7 Describe how you could use a decimal number to find the percentage by which a quantity has been increased or decreased.

GLOBAL CONTEXT — TREE CENSUS

Global context: Globalisation and sustainability

Statement of inquiry: Providing accurate data about our planet can help society make informed decisions about environmental issues.

Criterion: Applying mathematics in real-life contexts

QUICK QUIZ

MULTIPLE CHOICE QUIZ

REVIEW SET 9A

1 Which pie portion represents:

- **a** 75% of the pie
- **b** 20% of the pie
- **c** $33\frac{1}{3}\%$ of the pie?

A  **B**  **C** 

2 Write as a decimal:

a 47% **b** 6% **c** 92.7% **d** 165%

3 Write as a fraction in lowest terms:

a 29% **b** 74% **c** 45% **d** 190%

4 Write as a percentage:

a 0.56 **b** $\frac{17}{25}$ **c** 2.6 **d** $\frac{55}{200}$

5 Express as a percentage:

a 27 cm out of 50 cm **b** 6 hours out of one day **c** 409 mL out of 1 L.

6 13 of the 20 houses on a street have burglar alarms. What percentage of the houses have burglar alarms?

7 In a school of 800 students, 67% have brown hair, 24% have black hair, 2% have red hair, and the remainder have fair hair. Find:

a the percentage of fair-haired students **b** the number of brown-haired students.

8 A fishing trawler catches 3200 fish in its nets. 6% of the fish are too small to sell. Find the number of fish:

a which are too small to sell **b** which can be sold.

9 Last week, Monica scored 140 points in a game of ten-pin bowling. This week, she increased her score by 20%.

- **a** By how many points did Monica increase her score?
- **b** How many points did she score this week?

10 **a** Increase \$500 by 25%. **b** Decrease 40 km by 10%.

11 The marked price of a calculator is \$238. In a back-to-school sale, a discount of 10% is offered.

a Find the amount of discount.

b Find the actual selling price of the calculator.

12 Find the percentage increase or decrease in the change from:

a 24 m to 42 m **b** 8.4 kg to 6.7 kg.

13 A nursery has 250 plants for sale, 40 of which are tall trees. 48% of the plants are seedlings, and the rest are other mature plants. Trees sell for \$40 each, seedlings for \$4 each, and the mature plants for \$11 each. Find:

a the number of seedlings for sale **b** the number of mature plants for sale

c the total value of the nursery's plants.

14 In the town of Fernanda there are three schools. At Fernanda East, there are 400 students and 80% of them are girls. At Fernanda West, there are 500 students and 60% of them are girls. At Fernanda Lake, there are 800 students and 25% of them are girls.

a How many girls go to Fernanda East?

b How many boys go to Fernanda West?

c Altogether, what percentage of students in Fernanda are boys?

REVIEW SET 9B

1 A fuel gauge reads as shown:

a What fraction of the tank is full?

b What percentage of the tank is full?

2 Write as a percentage:

a 0.239 **b** 0.0022 **c** $\frac{7}{40}$ **d** $\frac{1}{8}$

3 Find:

a 60% of £300 **b** 28% of 350 m

c 10% of 4 hours, giving your answer in minutes.

4 Write as a decimal number: **a** 72.5% **b** 238%

5 Write as a fraction in lowest terms: **a** 15% **b** 180% **c** $\frac{2}{3}\%$

6 42 out of 96 players in a hockey club are female. Find the percentage of female players.

7 15% of the crowd at an international rugby tournament supported Kenya. If there were 81 000 spectators at the tournament, how many supported Kenya?

8 Student council meetings can only proceed if 70% of the council is present. The council consists of 40 students, but only 25 students are present. Will the meeting proceed?

9 A fruit punch contains 35% orange juice, 25% mango juice, 35% soda water, and the rest is passionfruit pulp.

- **a** What percentage of the fruit punch is passionfruit pulp?
- **b** In a 5 L bowl of punch, how much is: **i** soda water **ii** mango juice?
- **c** Express the amount of soda water as a percentage of the amount of passionfruit pulp.

10 Increase: **a** 150 cm by 3% **b** 45 L by 70%.

11 Describe the percentage change if:

- **a** my speed increases from 40 km/h to 50 km/h
- **b** the amount of water in my glass decreases from 160 mL to 130 mL.

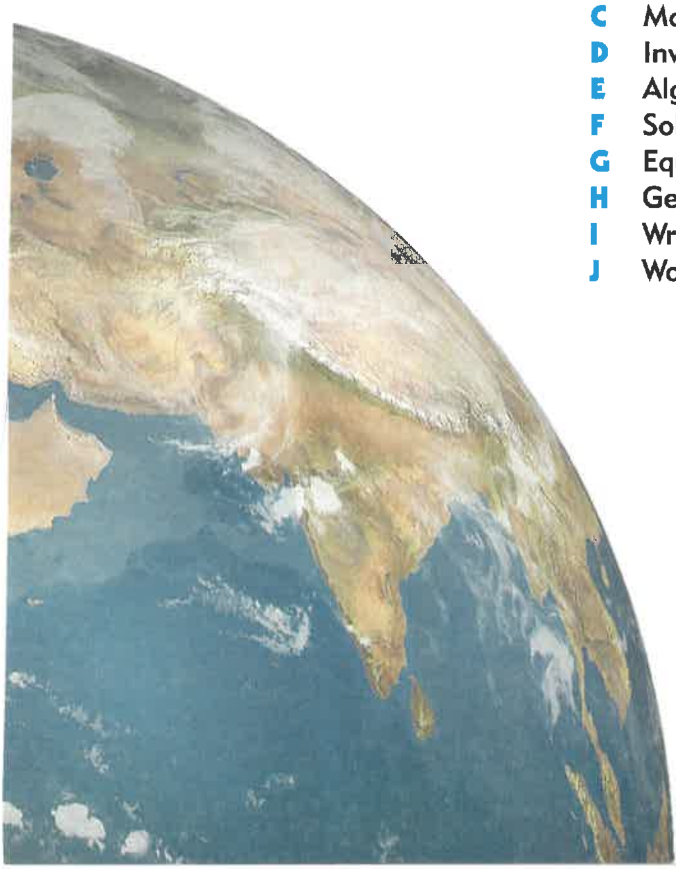
12 In a sale, the price of a toaster was reduced from \$60 to \$39. Find the percentage discount.

13 A furniture store is having a "12% off" sale.

- **a** Write 12% as a:
  - **i** decimal **ii** fraction in lowest terms.
- **b** A wardrobe has a marked price of \$260.
  - **i** Find the amount of discount offered on the wardrobe.
  - **ii** Find the actual selling price of the wardrobe.
- **c** A bookshelf has a marked price of \$330. Find the actual selling price of the bookshelf.

Chapter 10

Equations



Contents:

OPENING PROBLEM

Ray is at a carnival with five good friends. They decide to all go on the MegaTwister, one of the new rides at the carnival. It costs \$15 per ride. Ray's mother has given them \$30 to help pay for one ride each, but they will each have to pay some extra money as well.

Suppose each of the six friends pays $\$x$ extra.

Things to think about:

a Explain why the total amount paid by the six friends is $(6x + 30)$ dollars.

b Explain why $6x + 30 = 6 \times 15$.

c Given the equation $6x + 30 = 90$, how can we find the exact value of x?

d How much does each of the friends pay?

A EQUATIONS

An **equation** is a mathematical statement which indicates that two expressions have the same value.

The expressions are connected by an **equal sign** $=$.

The **left hand side** (LHS) is on the left of the $=$ sign.

The **right hand side** (RHS) is on the right of the $=$ sign.

For example: $\underbrace{3 \times 5}_{\text{LHS}} \underset{\uparrow \atop \text{equals}}{=} \underbrace{7+8}_{\text{RHS}}$.

The LHS $= 3 \times 5 = 15$ and the RHS $= 7 + 8 = 15$. Since LHS $=$ RHS, the equation is true.

Example 1 — Self Tutor

Decide whether each equation is true or false:

a $4 + 5 = 11 - 2$ **b** $2 \times 4 = 20 \div 2$

a LHS $= 4 + 5 = 9$
RHS $= 11 - 2 = 9$
Since LHS $=$ RHS, the equation is true.

b LHS $= 2 \times 4 = 8$
RHS $= 20 \div 2 = 10$
Since LHS $\neq$ RHS, the equation is false.

Most of the equations we will look at contain an **unknown** such as x.

An **algebraic equation** contains an unknown or variable.

For example, $6x + 30 = 48$ is an algebraic equation.

If we replace the x with a number, the resulting equation will either be true or false.

When we **solve** an equation, we find the value of the unknown which makes the equation true.

This value is called the **solution** of the equation.

If we replace x in the equation $6x + 30 = 48$ with different numbers, most of them will make the equation false.

For example:

If $x = 1$, LHS $= 6 \times 1 + 30 = 6 + 30 = 36$ so LHS $\neq$ RHS. ✗
If $x = 5$, LHS $= 6 \times 5 + 30 = 30 + 30 = 60$ so LHS $\neq$ RHS. ✗
If $x = 3$, LHS $= 6 \times 3 + 30 = 18 + 30 = 48$ so LHS $=$ RHS. ✓

So, $x = 3$ is the *solution* of the equation $6x + 30 = 48$.

Example 2 **Self Tutor**

The solution of $2x + 9 = 23$ is one of the numbers $\{5, 6, 7, 8\}$.
Find the solution.

If $x = 5$, LHS $= 2 \times 5 + 9 = 10 + 9 = 19$ so LHS $\neq$ RHS. ✗
If $x = 6$, LHS $= 2 \times 6 + 9 = 12 + 9 = 21$ so LHS $\neq$ RHS. ✗
If $x = 7$, LHS $= 2 \times 7 + 9 = 14 + 9 = 23$ so LHS $=$ RHS. ✓

So, the solution of the equation $2x + 9 = 23$ is $x = 7$.

EXERCISE 10A

1 Decide whether each equation is true or false:

a $5 \times 3 = 15$ **b** $28 \div 4 = 6$ **c** $7 + 5 = 2 \times 6$
d $22 - 11 = 5 \times 2$ **e** $24 \div 3 = 17 - 4$ **f** $13 + 19 = 4 \times 8$

2 Decide whether the given value of x makes the equation true or false:

a $x \div 3 = 15 - 8$ when $x = 21$
b $x + 9 = 20 \div 2$ when $x = 4$
c $16 - 7 = x + 2$ when $x = 5$
d $14 \div 2 = 23 - x$ when $x = 14$
e $3x = 11 + 7$ when $x = 6$
f $30 \div 6 = x - 6$ when $x = 11$

3 One of the numbers in the brackets is the solution of the equation. Find the solution.

a $x + 7 = 13$ $\{4, 5, 6, 7\}$
b $21 - x = 9$ $\{10, 12, 14, 16\}$
c $2x + 5 = 17$ $\{2, 4, 6, 8\}$
d $3x - 2 = 25$ $\{8, 9, 10, 11\}$
e $16 - 2x = 6$ $\{2, 3, 4, 5\}$
f $5x + 3 = 38$ $\{6, 7, 8, 9\}$

4 Match each equation to its solution:

a $3x - 4 = 11$ **b** $7 - x = 11$
c $2 + 5x = 17$ **d** $19 - 4x = 3$
e $2x + 24 = 18$ **f** $18 - x = 18$
g $3 + 9x = 12$ **h** $6x + 15 = 3$

A $x = 3$	**B** $x = 0$
C $x = 5$	**D** $x = -3$
E $x = -2$	**F** $x = 1$
G $x = -4$	**H** $x = 4$

ACTIVITY 1 GUESS, CHECK, AND IMPROVE

One method of solving simple equations is to try different values for the unknown until the correct solution is found.

If we only used trial and error, this could take a very long time. So, instead, each time we **guess** a solution, we **check** whether our guess is correct. If it is not, we try to **improve** our guess so that the LHS is at least getting *closer* to the RHS.

For example, suppose we want to solve $4x - 13 = 35$.

We start with the initial guess $x = 5$.

x	$4x - 13$	
5	7	much too low
9	23	getting closer
13	39	too far
12	35	LHS = RHS ✓

So, the solution is $x = 12$.

What to do:

1 Solve using *guess, check, and improve*:

a $2x + 3 = 17$ **b** $5x - 4 = 41$ **c** $3x - 15 = 18$
d $4 + 11x = 81$ **e** $33 - 2x = 19$ **f** $50 - 3x = 26$

2 Try to solve using *guess, check, and improve*:

a $2x - 8 = 13$ **b** $3x + 5 = 9$

3 Discuss in a small group:

- What strategies did you use to decide on your initial guess?
- How did you know whether you needed to increase or decrease your guess?
- What are the problems with using guess, check, and improve to solve equations?

DISCUSSION

1 How can we write an equation whose solution is a particular value?

2 As a class, write down some equations which have the solution:

- $x = 2$ • $x = -1$ • $x = 5$ • $x = -3$

3 Can you write down an equation which has more than one solution?

B SOLVING BY INSPECTION

Some simple equations can be solved by **inspection**.

This means we look at the equation and compare it with a known number fact, in order to find the solution.

Example 3 Self Tutor

Solve by inspection:

a $x+6=11$ b $\frac{x}{3}=8$ c $14-x=8$ d $7x=49$

a $x+6=11$

We know $5+6=11$

$\therefore\ x=5$

b $\frac{x}{3}=8$

We know $\frac{24}{3}=8$

$\therefore\ x=24$

c $14-x=8$

We know $14-6=8$

$\therefore\ x=6$

d $7x=49$

We know $7\times 7=49$

$\therefore\ x=7$

EXERCISE 10B

1 Solve by inspection:

a $6+x=9$ b $x-3=5$ c $21-x=12$

d $2x=18$ e $11-x=4$ f $\frac{x}{10}=3$

g $x+7=16$ h $4x=28$ i $\frac{14}{x}=2$

j $5-x=-1$ k $11+x=23$ l $8x=0$

2 Solve by inspection:

a $3x=-6$ b $-2+x=1$ c $5-x=-7$

d $\frac{x}{5}=-1$ e $x+9=5$ f $5x=-10$

g $\frac{-12}{x}=3$ h $3+x=-2$ i $x+13=0$

DISCUSSION

- What strategies did you use to help solve equations by inspection?
- If we cannot see how to solve an equation by inspection, is there something else we could do?

C MAINTAINING BALANCE

The **balance** of an equation is like the balance of a set of scales.

Changing one side of the equation without doing the same thing to the other side will upset the balance.

For example, the scales opposite represent the equation $3 = 3$.

- If 2 is added to both sides, we obtain another true equation:

$$3 + 2 = 3 + 2$$

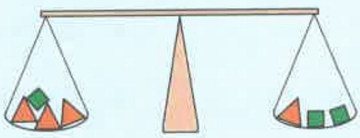
- If 2 was added to one side only, the scales would be unbalanced and the equation would become false:

$$3 + 2 \neq 3$$

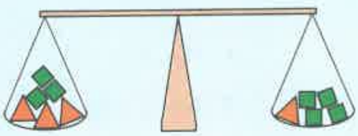
Example 4 **Self Tutor**

This set of scales is perfectly balanced.
Illustrate the result if we:

a add ■ ■ to both sides
b double both sides.



a



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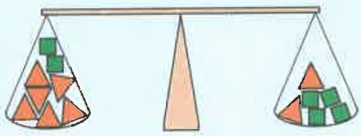
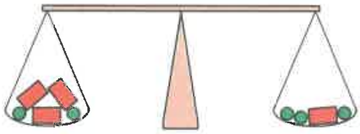


The balance of an equation will be maintained if we:
- add the same amount to both sides
- subtract the same amount from both sides
- multiply both sides by the same amount
- divide both sides by the same amount.

By maintaining the balance of an equation, we will not change its solutions.

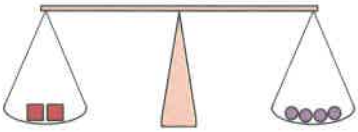
EXERCISE 10C

1 This set of scales is perfectly balanced.
Illustrate the result if we:



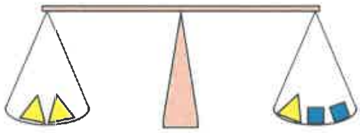
- **a** add to both sides
- **b** subtract from both sides.

2 This set of scales is perfectly balanced.
Illustrate the result if we:



- **a** add to both sides
- **b** divide both sides by 2.

3 This set of scales is perfectly balanced.
Illustrate the result if we:



- **a** subtract from both sides
- **b** multiply both sides by 2.

Example 5 **Self Tutor**

Consider the equation $x+5=10$.

What equation results when we perform the following on both sides of the equation:

- **a** add 3
- **b** subtract 3
- **c** divide by 2
- **d** multiply by 4?

a
$$x+5=10$$
$$\therefore\ x+5+3=10+3$$
$$\therefore\ x+8=13$$

b
$$x+5=10$$
$$\therefore\ x+5-3=10-3$$
$$\therefore\ x+2=7$$

c
$$x+5=10$$
$$\therefore\ \frac{x+5}{2}=\frac{10}{2}$$
$$\therefore\ \frac{x+5}{2}=5$$

d
$$x+5=10$$
$$\therefore\ 4(x+5)=4\times 10$$
$$\therefore\ 4(x+5)=40$$

By performing the same operation on *both* sides, the balance of the equation is maintained.

4 Find the equation which results when we add:

- **a** 2 to both sides of $x+1=5$
- **b** 3 to both sides of $x+2=6$
- **c** 4 to both sides of $x-4=7$
- **d** 6 to both sides of $2x-6=8$
- **e** 6 to both sides of $1-x=2$
- **f** 5 to both sides of $3-2x=1$

5 Find the equation which results when we subtract:

- **a** 3 from both sides of $x+4=12$
- **b** 4 from both sides of $x+4=7$
- **c** 2 from both sides of $2-x=11$
- **d** 5 from both sides of $2x+5=3$
- **e** 4 from both sides of $5x+4=2$
- **f** 3 from both sides of $2x-1=5$

6 Find the equation which results when we multiply both sides of:

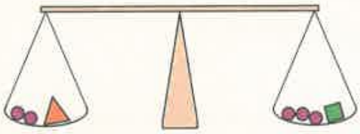
a $x = 2$ by 2
b $2x = 3$ by 4
c $\frac{x}{3} = 6$ by 3
d $x + 1 = 3$ by 2
e $\frac{x-1}{3} = 2$ by 3
f $\frac{2-x}{2} = -3$ by 2

7 Find the equation which results when we divide both sides of:

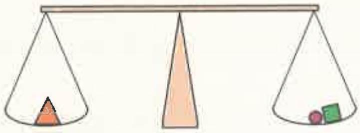
a $2x = 4$ by 2
b $4x = 20$ by 4
c $2(x + 1) = 8$ by 2
d $3(x - 2) = 18$ by 3
e $5x = 8$ by 5
f $6x = 13$ by 6
g $-x = 7$ by -1
h $-2(1 - x) = -12$ by -2

DISCUSSION

These scales are perfectly balanced:



By taking 2 ● from both sides, we get:

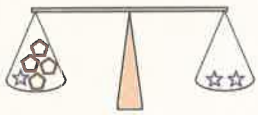


The simplest relationship between the objects is: ▲ = ● + ■

Discuss what relationships between the objects you can deduce from each set of scales:

a

b



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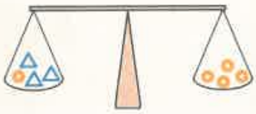

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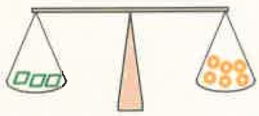

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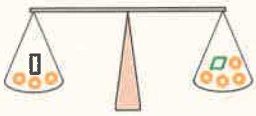

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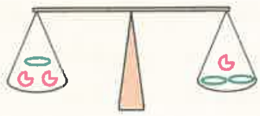

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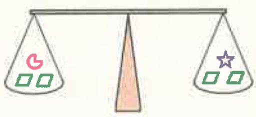

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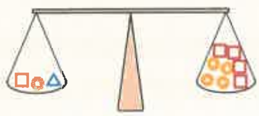


D INVERSE OPERATIONS

Imagine starting with \$50 in your pocket. You find \$5, and then buy a drink for \$5. You are left with the original \$50 you had in your pocket.

\$50 \$55 \$50

Adding 5 and subtracting 5 have the opposite effect. One *undoes* the other.

Addition and subtraction are **inverse operations**.

Now imagine you start with \$50, and your friend gives you the same amount. Your money is now doubled. If you decide to give half to your brother, you will be back to your original \$50.

\$50 \$100 \$50

Multiplying by 2 and dividing by 2 have the opposite effect. One *undoes* the other.

Multiplication and division are **inverse operations**.

Example 6 **Self Tutor**

State the inverse of:

a $\times\ 4$ **b** $\div\ 7$ **c** $+\ 6$ **d** $-\ 3$

a The inverse of $\times\ 4$ is $\div\ 4$.

b The inverse of $\div\ 7$ is $\times\ 7$.

c The inverse of $+\ 6$ is $-\ 6$.

d The inverse of $-\ 3$ is $+\ 3$.

SOLVING EQUATIONS

We can solve simple equations using *inverse operations*. However, we must remember to maintain the *balance* by performing the same operation on *both sides*.

For example, consider $x + 3 = 7$, where 3 has been added to x.

$$x + 3 = 7$$

$$\therefore\ x + 3 - 3 = 7 - 3 \quad \{\text{subtracting 3 is the inverse of adding 3}\}$$

$$\therefore\ x = 4 \quad \{\text{simplifying}\}$$

By performing the correct inverse operation, we are left with x by itself on one side of the equation. The correct value of x is on the other side.

EXERCISE 10D

1 State the inverse of each operation:

a $+\,4$ **b** $\times\,2$ **c** $\div\,3$ **d** $-\,1$ **e** $+\,0.7$

f $-\,\frac{1}{3}$ **g** $\times\,\frac{1}{4}$ **h** $-\,11$ **i** $+\,\frac{3}{4}$ **j** $\times\,-1$

k $\times\,7$ **l** $\div\,-1$ **m** $-\,0.6$ **n** $\div\,\frac{1}{2}$ **o** $\times\,-4$

2 Simplify:

a $x-6+6$ **b** $x\times 2\div 2$ **c** $x+4-4$

d $5x\div 5$ **e** $\frac{3x}{3}$ **f** $\frac{x}{4}\times 4$

g $7+x-7$ **h** $\frac{2}{3}x\div\frac{2}{3}$ **i** $\frac{3x}{2}\times 2$

Example 7 Self Tutor

Solve for x using an inverse operation: $x+5=11$

$x+5=11$

$\therefore\ x+5-5=11-5$ {The inverse of $+\,5$ is $-\,5$, so we take 5 from both sides.}

$\therefore\ x=6$

3 Solve for x using an inverse operation:

a $x+2=9$ **b** $x+6=15$ **c** $x+9=20$

d $x+5=0$ **e** $x+4=1$ **f** $x+7=-3$

g $5+x=13$ **h** $2+x=11$ **i** $x+15=11$

j $x+9=-8$ **k** $x+0.5=0.7$ **l** $\frac{1}{3}+x=1$

Example 8 Self Tutor

Solve for y using an inverse operation: $y-6=-2$

$y-6=-2$

$\therefore\ y-6+6=-2+6$ {The inverse of $-\,6$ is $+\,6$, so we add 6 to both sides.}

$\therefore\ y=4$

4 Solve for y using an inverse operation:

a $y-4=5$ **b** $y-8=17$ **c** $y-7=0$

d $y-12=-2$ **e** $y-6=-13$ **f** $y-3=-3$

g $y-8=25$ **h** $y-1.3=2.6$ **i** $y-\frac{1}{2}=4$

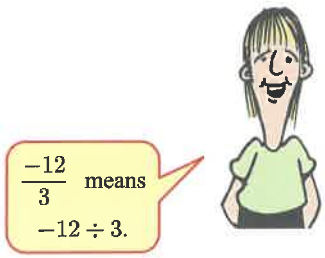
Example 9 **Self Tutor**

Solve for t using an inverse operation: $3t = -12$

$$3t = -12$$

$$\therefore \frac{3t}{3} = \frac{-12}{3} \quad \{\text{The inverse of } \times 3 \text{ is } \div 3, \text{ so we divide both sides by 3.}\}$$

$$\therefore t = -4$$



5 Solve for a using an inverse operation:

a $2a = 10$ **b** $3a = 12$ **c** $2a = 16$
d $5a = 20$ **e** $4a = -16$ **f** $6a = 30$
g $5a = -35$ **h** $8a = 64$ **i** $10a = 7$

Example 10 **Self Tutor**

Solve for d using an inverse operation: $\frac{d}{7} = 8$

$$\frac{d}{7} = 8$$

$$\therefore \frac{d}{7} \times 7 = 8 \times 7 \quad \{\text{The inverse of } \div 7 \text{ is } \times 7, \text{ so we multiply both sides by 7.}\}$$

$$\therefore d = 56$$

6 Solve for x using an inverse operation:

a $\frac{x}{3} = 2$ **b** $\frac{x}{2} = 6$ **c** $\frac{x}{4} = 7$
d $\frac{x}{5} = -1$ **e** $\frac{x}{3} = 18$ **f** $\frac{x}{11} = -2$
g $\frac{x}{6} = 2$ **h** $\frac{x}{-2} = 5$ **i** $\frac{x}{-3} = -6$

7 Solve each equation using an inverse operation:

a $3 + a = 11$ **b** $b - 17 = -4$ **c** $8c = 48$
d $d \div 4 = 11$ **e** $e + 19 = 12$ **f** $2f = -10$
g $g - 17 = 24$ **h** $\frac{h}{7} = 3$ **i** $5 + i = -3$
j $6j = -42$ **k** $k - \frac{1}{2} = 1$ **l** $4l = -8$
m $16 + m = 12$ **n** $\frac{n}{-4} = -2$ **o** $o - 9 = -14$
p $p + 0.4 = 0.9$ **q** $\frac{q}{-10} = 5$ **r** $-3r = -15$
s $s - \frac{2}{5} = 0$ **t** $\frac{t}{7} = -4$ **u** $9u = 5$

ACTIVITY 2 INVERSE OPERATIONS

Click on the icon to practise solving equations using an inverse operation.

LEARNING ALGEBRA

E ALGEBRAIC FLOWCHARTS

To solve more complicated equations, we must understand how expressions are **built up**. We can study this using an **algebraic flowchart**.

Example 11 Self Tutor

Complete each flowchart:

a $\boxed{x} \xrightarrow{\times 2} \boxed{} \xrightarrow{+5} \boxed{}$

b $\boxed{x} \xrightarrow{+5} \boxed{} \xrightarrow{\times 2} \boxed{}$

c $\boxed{x} \xrightarrow{-3} \boxed{} \xrightarrow{\div 2} \boxed{}$

d $\boxed{x} \xrightarrow{\div 2} \boxed{} \xrightarrow{-3} \boxed{}$

a $\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+5} \boxed{2x+5}$

b $\boxed{x} \xrightarrow{+5} \boxed{x+5} \xrightarrow{\times 2} \boxed{2(x+5)}$

c $\boxed{x} \xrightarrow{-3} \boxed{x-3} \xrightarrow{\div 2} \boxed{\dfrac{x-3}{2}}$

d $\boxed{x} \xrightarrow{\div 2} \boxed{\dfrac{x}{2}} \xrightarrow{-3} \boxed{\dfrac{x}{2}-3}$

By reversing the flowchart with **inverse operations**, we can *undo* the expression.

The inverse operations must be performed in the **reverse order**.

Example 12 Self Tutor

Use flowcharts to show how to "build up" and "undo" each expression:

a $5x+2$

b $\dfrac{x+3}{2}$

a Build up:

$\boxed{x} \xrightarrow{\times 5} \boxed{5x} \xrightarrow{+2} \boxed{5x+2}$

Undo:

$\boxed{5x+2} \xrightarrow{-2} \boxed{5x} \xrightarrow{\div 5} \boxed{x}$

b Build up:

$\boxed{x} \xrightarrow{+3} \boxed{x+3} \xrightarrow{\div 2} \boxed{\dfrac{x+3}{2}}$

Undo:

$\boxed{\dfrac{x+3}{2}} \xrightarrow{\times 2} \boxed{x+3} \xrightarrow{-3} \boxed{x}$

EXERCISE 10E

1 Copy and complete each flowchart:

a $\boxed{x} \xrightarrow{\times 2} \boxed{} \xrightarrow{-3} \boxed{}$

b $\boxed{x} \xrightarrow{+1} \boxed{} \xrightarrow{\times 4} \boxed{}$

c $\boxed{x} \xrightarrow{-2} \boxed{} \xrightarrow{\div 3} \boxed{}$

d $\boxed{x} \xrightarrow{\div 2} \boxed{} \xrightarrow{+5} \boxed{}$

2 Copy and complete each flowchart by inserting the missing operations:

a $\boxed{x} \xrightarrow{?} \boxed{3x} \xrightarrow{?} \boxed{3x+2}$

b $\boxed{x} \xrightarrow{?} \boxed{\frac{x}{3}} \xrightarrow{?} \boxed{\frac{x}{3}-1}$

c $\boxed{x} \xrightarrow{?} \boxed{x-7} \xrightarrow{?} \boxed{\frac{x-7}{2}}$

d $\boxed{x} \xrightarrow{?} \boxed{x+4} \xrightarrow{?} \boxed{2(x+4)}$

3 Use flowcharts to show how to "build up" and "undo" each expression:

a $2x+4$ **b** $3x-1$

c $4x+3$ **d** $5x-12$

e $\frac{x}{2}+4$ **f** $-x-3$

g $-2x+5$ **h** $\frac{2}{3}x-1$

To undo the expression, perform the **inverse operations** in the **reverse order**.

4 Use flowcharts to show how to "build up" and "undo" each expression:

a $\frac{x}{3}-1$ **b** $\frac{x-1}{3}$ **c** $\frac{x+5}{3}$ **d** $\frac{x}{3}+5$

e $3x+8$ **f** $3(x+8)$ **g** $2(x-6)$ **h** $2x-6$

5 Use flowcharts to show how to "build up" and "undo" each expression:

a $\frac{x-1}{2}+5$ **b** $3-2(x+1)$ **c** $1-\frac{x-1}{4}$

d $\frac{3x-2}{5}$ **e** $3(2-3x)$ **f** $\frac{5-2x}{3}+2$

ACTIVITY 3 EXPRESSION INVADERS

Click on the icon to play a game where you can practise building up and undoing expressions.

EXPRESSION INVADERS

F SOLVING EQUATIONS

When we are given an equation containing a **built up** expression, we use **inverse operations** in the **reverse order** to write the unknown by itself.

We say that we *isolate* the unknown.

Example 13 Self Tutor

Find the value of x: $5x + 7 = 17$

$5x + 7 = 17$

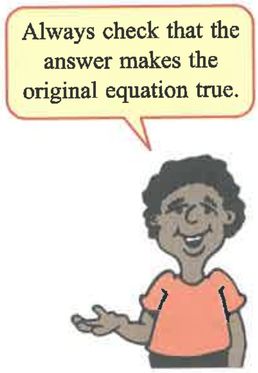
$\therefore\ 5x + 7 - 7 = 17 - 7$ {subtracting 7 from both sides}

$\therefore\ 5x = 10$

$\therefore\ \frac{5x}{5} = \frac{10}{5}$ {dividing both sides by 5}

$\therefore\ x = 2$

Check: LHS $= 5 \times 2 + 7 = 10 + 7 = 17 =$ RHS ✓



EXERCISE 10F

1 Find the value of x:

- **a** $2x - 3 = 7$
- **b** $3x + 5 = 8$
- **c** $5x + 9 = 24$
- **d** $4x - 16 = 0$
- **e** $2x - 4 = -10$
- **f** $3x - 1 = 0$
- **g** $10x - 2 = 18$
- **h** $6x + 5 = 10$
- **i** $2x + 11 = 11$
- **j** $11x - 3 = 41$
- **k** $7x - 5 = -4$
- **l** $4x + 15 = 17$

2 Solve for x:

- **a** $-x + 4 = 3$
- **b** $-x - 2 = -7$
- **c** $-2x + 5 = -3$
- **d** $-2x - 3 = 1$
- **e** $-3x + 4 = 13$
- **f** $-5x - 8 = -13$

Example 14 Self Tutor

Solve for x: $\frac{x}{3} - 4 = -3$

$\frac{x}{3} - 4 = -3$

$\therefore\ \frac{x}{3} - 4 + 4 = -3 + 4$ {adding 4 to both sides}

$\therefore\ \frac{x}{3} = 1$

$\therefore\ \frac{x}{3} \times 3 = 1 \times 3$ {multiplying both sides by 3}

$\therefore\ x = 3$

Check: LHS $= \frac{3}{3} - 4 = 1 - 4 = -3 =$ RHS ✓

3 Solve for x:

a $\frac{x}{2}+1=3$ **b** $\frac{x}{3}-2=5$ **c** $\frac{x}{5}+7=1$ **d** $\frac{x}{4}-1=-6$

e $\frac{x}{2}-3=-3$ **f** $\frac{x}{10}+6=-2$ **g** $\frac{x}{-2}-3=-1$ **h** $\frac{x}{-3}+4=2$

Example 15 Self Tutor

Solve for x: $\frac{x-3}{7}=-3$

$$\frac{x-3}{7}=-3$$

$\therefore\ 7\left(\frac{x-3}{7}\right)=7\times -3$ {multiplying both sides by 7}

$\therefore\ x-3=-21$

$\therefore\ x-3+3=-21+3$ {adding 3 to both sides}

$\therefore\ x=-18$

Check: LHS $=\frac{-18-3}{7}=\frac{-21}{7}=-3=$ RHS ✓

4 Solve for x:

a $\frac{x-2}{2}=5$ **b** $\frac{x+3}{3}=2$ **c** $\frac{x-5}{4}=-1$ **d** $\frac{x+4}{3}=12$

e $\frac{x-8}{-2}=3$ **f** $\frac{x+6}{3}=0$ **g** $\frac{x-7}{-3}=2$ **h** $\frac{x+4}{-4}=-1$

Example 16 Self Tutor

Solve for x: $3(x-4)=39$

$$3(x-4)=39$$

$\therefore\ \frac{3(x-4)}{3}=\frac{39}{3}$ {dividing both sides by 3}

$\therefore\ x-4=13$

$\therefore\ x-4+4=13+4$ {adding 4 to both sides}

$\therefore\ x=17$

Check: LHS $=3(17-4)=3\times 13=39=$ RHS ✓

5 Find the value of x:

a $3(x-1)=12$ **b** $4(x+2)=24$ **c** $5(x-3)=25$

d $2(x+7)=8$ **e** $7(x-1)=0$ **f** $12(x-2)=24$

g $3(x-2)=9$ **h** $5(x+2)=-15$ **i** $10(x+4)=60$

j $6(x+3)=42$ **k** $6(x-1)=-18$ **l** $3(x-2)=2$

6 Solve for x:

a $-2(x+1)=-8$ **b** $-3(x-4)=2$ **c** $-(x+6)=-8$

d $-3(x-2)=-1$ **e** $-2(x+1)=1$ **f** $-3(x-1)=-2$

7 Solve for x:

a $7x-3=18$ **b** $\frac{x}{2}-1=4$ **c** $2(x+5)=20$

d $\frac{x-3}{4}=5$ **e** $\frac{x}{-2}=-4$ **f** $3(x-6)=0$

g $\frac{x}{3}+2=-1$ **h** $3x+7=-5$ **i** $\frac{x+1}{3}=-2$

j $4x-6=14$ **k** $\frac{x+4}{-5}=2$ **l** $-2(x+7)=14$

m $\frac{x+2}{4}=-\frac{1}{2}$ **n** $3x+15=0$ **o** $9(x-2)=-63$

p $\frac{x-11}{-2}=8$ **q** $6(x+1)=-54$ **r** $\frac{x}{5}-1=7$

s $-3x+7=-2$ **t** $-4(x+2)=3$ **u** $2(-x+1)=-1$

8 Solve for x:

a $\frac{3(x-1)}{2}=6$ **b** $-4(1+2x)=-20$ **c** $2-\frac{x+4}{3}=1$

d $3(2x-5)=21$ **e** $\frac{1-3x}{2}=5$ **f** $\frac{-5(2x-4)}{3}=30$

G EQUATIONS WITH A REPEATED VARIABLE

In many equations, the variable appears more than once. In this course we consider some simple cases where the variable appears more than once on the LHS.

To solve these equations, we *collect like terms* and then isolate the unknown.

Example 17 Self Tutor

Solve for x: $3x+2x=65$

$3x+2x=65$

$\therefore\ 5x=65$ {collecting like terms}

$\therefore\ \frac{5x}{5}=\frac{65}{5}$ {dividing both sides by 5}

$\therefore\ x=13$

EXERCISE 10G

1. Solve for x:
   a. $2x + x = 18$
   b. $3x - x = 24$
   c. $5x + 2x = -63$
   d. $4x - x - 24 = 0$
   e. $3x - 4 + 5x = 0$
   f. $9x - 5 - 3x = 13$

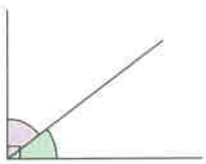
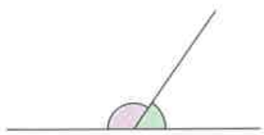
2. Solve for x:
   a. $x + x + 40 = 180$
   b. $x - 50 + x = 90$
   c. $x + x + 30 = 90$
   d. $x + x - 30 + 50 = 180$
   e. $x + x + 20 + x + 40 = 180$
   f. $x + x + 50 + x - 20 = 180$

3. Solve for x:
   a. $2x - 3 = x + 2$
   b. $5x + 1 = 15 - 2x$

H GEOMETRY PROBLEMS

In **Chapter 3** we saw that:

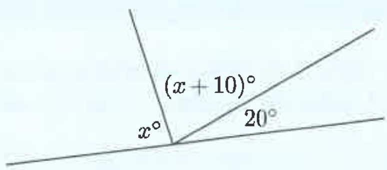
- angles in a right angle add to 90°
- angles on a line add to 180°
- angles at a point add to 360°


We can use these geometric facts to construct equations.

Example 18 Self Tutor

Find the value of x:



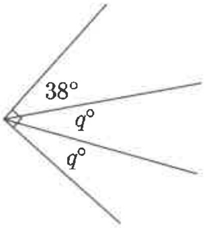
Angles on a line are supplementary.

$$\begin{aligned}
\therefore\ x + x + 10 + 20 &= 180 \\
\therefore\ 2x + 30 &= 180 && \{\text{collecting like terms}\} \\
\therefore\ 2x + 30 - 30 &= 180 - 30 && \{\text{subtracting 30 from both sides}\} \\
\therefore\ 2x &= 150 \\
\therefore\ \frac{2x}{2} &= \frac{150}{2} \\
\therefore\ x &= 75
\end{aligned}$$

EXERCISE 10H

1 Find the value of the unknown:

a



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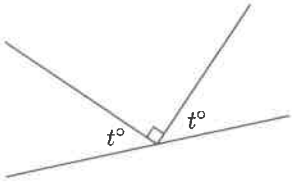

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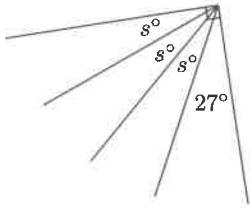

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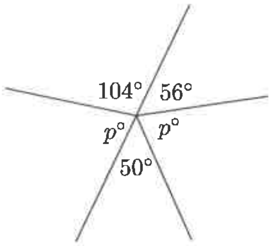
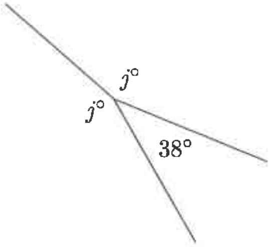
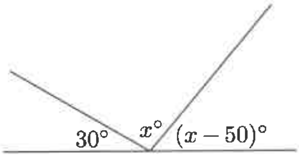


e



2 Solve for x:

a



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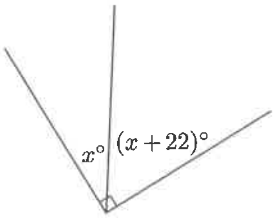

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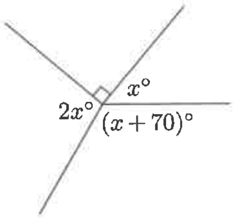


I WRITING EQUATIONS

Most real life problems are described using sentences rather than symbols. Before we can solve these problems, we must first write the problem as an equation.

Example 19 Self Tutor

When a number multiplied by 3 is added to 5, the result is 23.
Write this as an algebraic equation.

Let the unknown number be x.

Start with a number	x
multiply it by 3	$3x$
add 5	$3x+5$
the result is 23	$3x+5=23$

The algebraic equation which represents the problem is $3x+5=23$.

EXERCISE 10I

1. Starting with the number x, write each of the following as an algebraic equation:
   - **a** When a number is multiplied by 5, the result is 30.
   - **b** When 10 is added to a number, the result is 23.
   - **c** When a number is divided by 4, and then 6 is added, the result is 8.
   - **d** When a number is subtracted from 11, and the result is divided by 3, the answer is 2.
2. Summarise the information as an equation:
   - **a** Paul has x coins, then his father gives him 12 more. Paul now has 27 coins.
   - **b** Philippa had $\$x$ in her purse. She spent \$150 at a mall. She now has \$80.
   - **c** Darren is x years old. His daughter Fiona, who is one third his age, is 12 years old.
   - **d** A plant was initially x cm high. In a week it doubled its height, and the next week it grew another 10 cm. It was then 31 cm high.

J WORD PROBLEMS

To solve word problems, we follow these steps:

Step 1: Let x be the unknown quantity to be found.
Step 2: Write an equation using the information in the question.
Step 3: Solve the equation.
Step 4: Write the answer to the question in a sentence.

Example 20 **Self Tutor**

I am thinking of a number. If I subtract 7 from the number, the result is 11. What is the number?

Let x be the number.
When 7 is subtracted from the number, the result is 11.

$$\therefore \quad x - 7 = 11$$
$$\therefore \quad x - 7 + 7 = 11 + 7 \qquad \{\text{adding 7 to both sides}\}$$
$$\therefore \quad x = 18$$

The number is 18.

Check: If I subtract 7 from 18, the result is $18 - 7 = 11$ ✓

EXERCISE 10J

1. I am thinking of a number. When 9 is added to the number, the result is 15. What is the number?
2. I am thinking of a number. When the number is tripled and then 4 is taken away, the result is 11. What is the number?

3 I am thinking of a number. When the number is halved and then 8 is added, the result is 13. What is the number?

Example 21 Self Tutor

Callum has a collection of badges. His aunt gives him 9 more badges. Then, while Callum is on holidays, he collects enough to double his collection. He now has 132 badges in total. How many badges did Callum have to start with?

Let x be the number of badges Callum had to start with.

He is given 9 by his aunt, so the number is now $x + 9$.

Callum doubles his collection, so he now has $2(x + 9)$.

Callum now has 132 badges.

$\therefore \; 2(x+9) = 132$

$\therefore \; \dfrac{2(x+9)}{2} = \dfrac{132}{2}$ {dividing both sides by 2}

$\therefore \; x + 9 = 66$

$\therefore \; x + 9 - 9 = 66 - 9$ {subtracting 9 from both sides}

$\therefore \; x = 57$

Callum had 57 badges to start with.

4 Ethan has a box of chocolates. He ate 7 chocolates and there are now 17 chocolates left. How many chocolates did Ethan have to start with?

5 A school band has 15 musicians and some singers. The members of the band are split into 6 groups of 4 students. How many singers are in the band?

6 At a kitchenware store, Yvonne bought a bowl for \$15, as well as 4 plates. The total cost was \$35. How much did each plate cost?

7 When Nicola went to the state car museum, she saw cars and three motorbikes. In total she saw 134 wheels in the museum. How many cars did Nicola see?

8 At a birthday party, Hessa was given some balloons. Her friend Afra was given 9 balloons. The two girls decided to share their balloons equally, and now they each have 7 balloons. How many balloons was Hessa given?

9 David bought some boxes of ice blocks. There were six ice blocks in every box. When he opened the freezer he realised that his children had found them and had already eaten 3. There are now 21 ice blocks left. How many boxes did David buy?

10 Each day a salesman is paid £180 plus one tenth of the value of the sales he makes for the day. On one day the salesman is paid £300. What value of sales did he make that day?

DISCUSSION

The equations we have solved in this Chapter have exactly one solution.

Discuss the solutions to these equations, and hence match each equation to its number of solutions:

a $x - x = 0$	**A** no solutions
b $x \times x = 1$	**B** one solution
c $x + x = 0$	**C** two solutions
d $x \div x = 2$	**D** infinitely many solutions

MULTIPLE CHOICE QUIZ

QUICK QUIZ

REVIEW SET 10A

1 Decide whether each equation is true or false:

a $4 \times 7 = 30 - 2$ **b** $10 \div 2 = 14 - 7$ **c** $5 + 13 = 6 \times 3$

2 One of the numbers $\{6, 7, 8, 9\}$ is the solution of the equation $3x - 11 = 10$. Find the solution.

3 Solve by inspection:

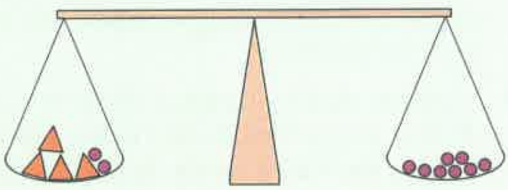
a $x + 10 = 17$ **b** $12 - x = 4$ **c** $4x = -24$

4 This set of scales is perfectly balanced. Illustrate the result if we:

a add △ to both sides

b subtract ● ● from both sides

c divide both sides by 2.



5 State the inverse of each operation:

a $-\ 7$ **b** $\div\ 8$ **c** $+\ 13$ **d** $\times\ \frac{1}{2}$

6 Find x using an inverse operation:

a $x + 4 = 17$ **b** $x - 8 = 2$ **c** $3x = -15$ **d** $\frac{x}{-2} = -12$

7 Copy and complete each flowchart:

a $\boxed{x} \xrightarrow{?} \boxed{2x} \xrightarrow{?} \boxed{2x - 9}$

b $\boxed{x} \xrightarrow{\div 3} \boxed{} \xrightarrow{+4} \boxed{}$

8 Use flowcharts to show how to "build up" and "undo" each expression:

a $3(x-4)$ **b** $\frac{x+2}{3}$

9 Solve for x:

a $2x-9=-5$ **b** $\frac{x+4}{9}=-1$ **c** $\frac{x}{3}-6=3$

d $\frac{x}{5}+12=10$ **e** $2x+7=15$ **f** $2(x-8)=28$

10 Solve for x:

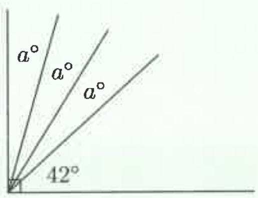
a $-2x+5=-3$ **b** $\frac{x}{-5}-1=2$ **c** $-3(x-1)=-1$

11 Solve for x:

a $5x+x=42$ **b** $8x-11-3x=9$ **c** $x+x-40=90$

12 Find the value of the unknown:

a



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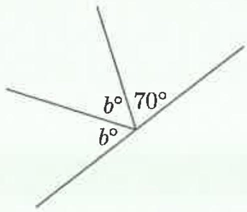

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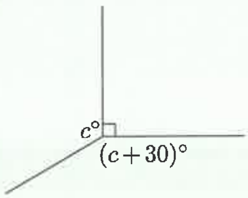


13 Starting with the number x, write each of the following as an algebraic equation:

a When a number is multiplied by 7, the result is 35.

b When a number is added to 6, and the result is divided by 2, the answer is 8.

14 Charlotte saves the same amount of money each week. She starts with \$12, and after 4 weeks she has \$36. How much does she save each week?

15 Emma buys a packet of lollies at a cinema. She eats half of the packet, and her friend eats 10 lollies. If there are now 7 lollies remaining in the packet, how many lollies were in the packet originally?

16 A garden supplies shop has 200 pavers stacked on three pallets. The second pallet has 40 more pavers than the first pallet. The third pallet has 8 less pavers than the first pallet. Suppose there are x pavers on the first pallet.

a Write an expression for the number of pavers on the:

i second pallet **ii** third pallet.

b Write an equation describing the situation.

c Find the number of pavers on each pallet.

REVIEW SET 10B

1 Decide whether the given value of x makes the equation true or false:

a $x+8=19-2$ when $x=9$ **b** $5x=30\div 3$ when $x=3$.

2 Solve by inspection:

a $x - 5 = 17$ **b** $-3x = -9$ **c** $1 + x = -8$

3 Find the equation which results when we:

a add 5 to both sides of $4x - 5 = 11$

b divide both sides of $3(x-2) = 15$ by 3.

4 State the inverse of multiplying by 8.

5 Find x using an inverse operation:

a $3 + x = 7$ **b** $x - 4 = 11$ **c** $5x = -30$ **d** $\frac{x}{6} = -2$

6 Copy and complete each flowchart:

a $\boxed{x} \xrightarrow{?} \boxed{4x} \xrightarrow{?} \boxed{4x-7}$ **b** $\boxed{x} \xrightarrow{+9} \boxed{} \xrightarrow{\times 2} \boxed{}$

7 Use a flowchart to show how to "build up" and "undo" each expression:

a $\frac{x-9}{4}$ **b** $2(x+5)$

8 Solve for x:

a $4x + 5 = 29$ **b** $3x - 7 = -1$ **c** $\frac{x}{4} - 5 = -2$

d $3x + 8 = -25$ **e** $\frac{x-2}{5} = -4$ **f** $7(x+3) = -49$

9 Solve for x:

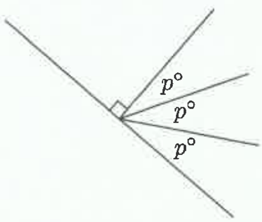
a $-x - 5 = 3$ **b** $\frac{x}{-2} + 4 = 0$ **c** $-6(x+2) = 30$

10 Solve for x:

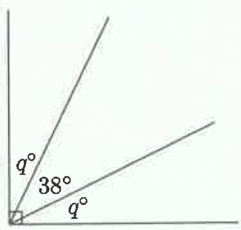
a $2x - 4 + 3x = 41$ **b** $x + x + 40 + x + 65 = 180$

11 Find the value of the unknown:

a



b



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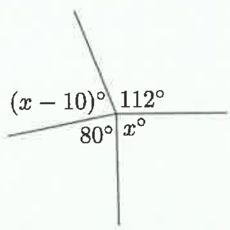


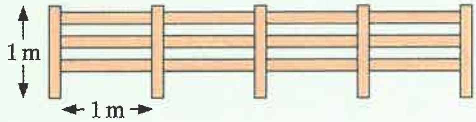
12 There were initially 16 ice cubes in the freezer. After placing 2 ice cubes each into x glasses, only 4 ice cubes remained in the freezer.
Summarise this information as an equation.

13 When a number is divided by four, the result is 9. What is the number?

14 Monica asked each of her friends to buy a charity raffle ticket for \$3. All but 4 of her friends bought a ticket, and she raised \$18. How many friends did Monica ask?

15 Answer the **Opening Problem** on page **198**.

16 Philip runs a fencing company which makes the type of fence shown alongside.



Each fence is made using 1 m lengths of timber. The example above is 4 m long, and uses 17 m of timber.
Philip has worked out that to make a fence l m long, he will need $(4l + 1)$ m of timber.

a How many metres of timber will Philip need to make a 7 m long fence?

b Philip has 61 m of timber, and wants to know how long the resulting fence will be.

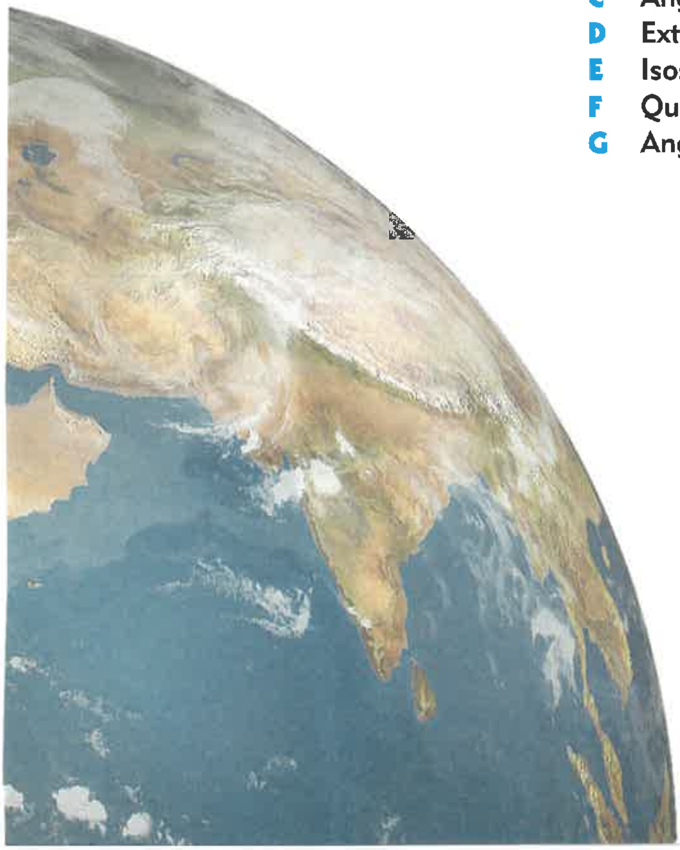
  i Write an equation to describe the situation.

  ii Solve the equation and hence state the length of the fence.

Chapter 11

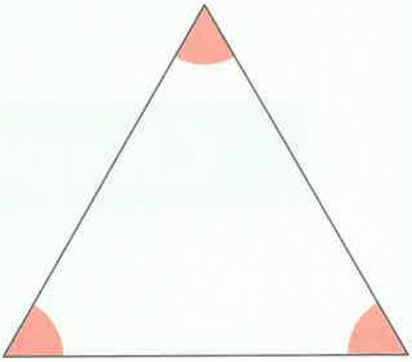
Polygons

Contents:



OPENING PROBLEM

Consider the triangle alongside.



Things to think about:

a Use a ruler to measure each side length.
Use a protractor to measure the size of each angle. Can you *classify* the triangle using your observations of the side lengths and angles?

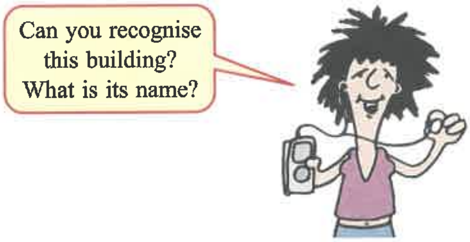
b Find the sum of the angles of this triangle.
Draw a triangle of your own, and find the sum of its angles.
What can you conclude about the sum of the angles of any triangle?

In geometry, a **plane** is a flat surface.

Any shape which is drawn on a plane is called a **plane figure**.

Plane figures are two-dimensional, having height and width.

In this Chapter we study the special group of plane figures called **polygons**.



A POLYGONS

A **polygon** is a closed plane figure which has only straight line sides which do not cross.

Each corner of a polygon is called a **vertex**.

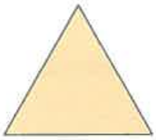
The plural of vertex is *vertices*.

In any polygon, the number of sides equals the number of vertices.

Polygons are named according to the number of sides and vertices they have.

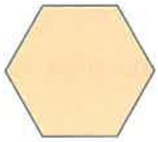
For example, a 9-sided polygon can be called a 9-gon.

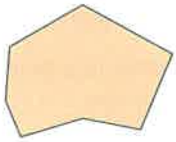
However, many polygons are known by more familiar names:

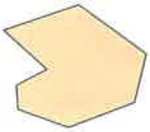

triangle
3 sides


quadrilateral
4 sides


pentagon
5 sides


hexagon
6 sides


heptagon
7 sides


octagon
8 sides

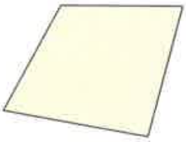

nonagon
9 sides

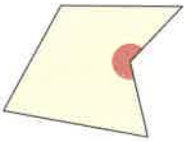

decagon
10 sides


dodecagon
12 sides

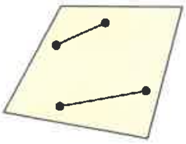
CONVEX POLYGONS

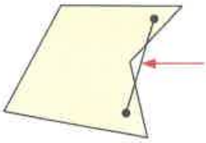
A **convex polygon** is a polygon which has no interior reflex angles.


is a convex quadrilateral.


is a non-convex pentagon.

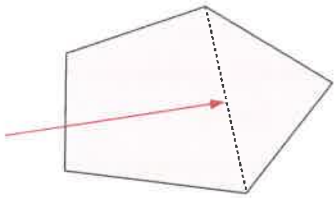
Every pair of points inside a convex polygon can be joined by a straight line segment which remains inside the polygon.


All line segments between points remain inside the quadrilateral, so the quadrilateral is convex.


This line segment goes outside the pentagon, so the pentagon is not convex.

DIAGONALS OF A POLYGON

A **diagonal** of a polygon is a straight line segment within the polygon which joins a pair of vertices.

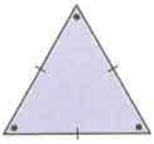
diagonal


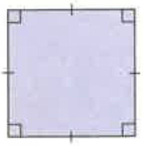
REGULAR POLYGONS

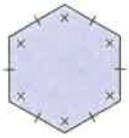
A **regular polygon** is a polygon with all sides the same length and all angles the same size.

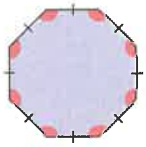
We use tick marks to show sides with the same length, and angle markings to show angles with the same size.

Here are some regular polygons which we commonly see:


equilateral triangle

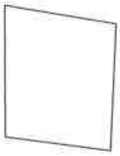
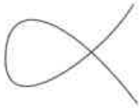
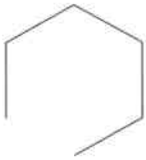
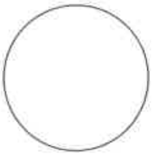
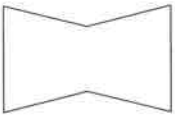
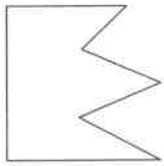

square


regular hexagon

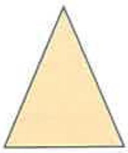
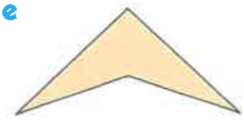
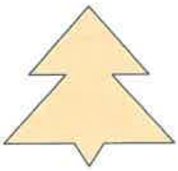
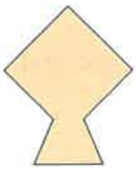

regular octagon

EXERCISE 11A

1 Which of these figures is a polygon? Give a reason if the figure is not a polygon.

a 
b 
c 
d 
e 
f 
g 
h 

2 Name each polygon according to its number of sides and whether it is convex:

a 
b 
c 
d 
e 
f 
g 
h 

3 Write down the name given to a polygon with:

a six sides **b** eight sides **c** twelve sides.

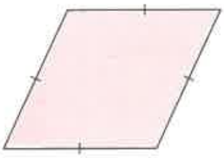
4 Sketch a quadrilateral which is:

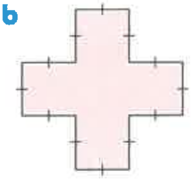
a convex **b** non-convex.

5 Sketch a pentagon which is:

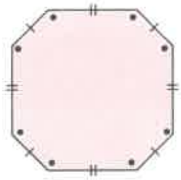
a convex **b** non-convex.

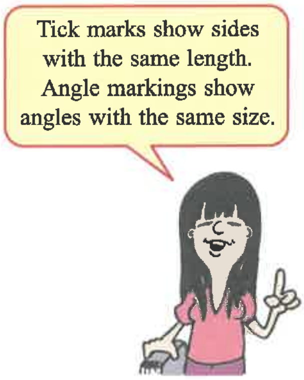
6 Explain why these figures are *not* regular polygons:

a


b


c

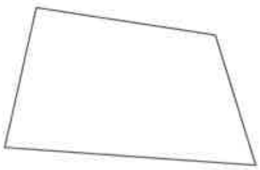

d


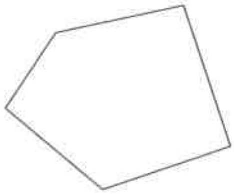


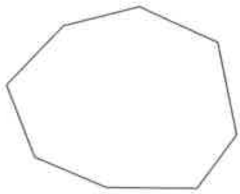
7 Decide whether each statement is true or false:

- **a** Every triangle is convex.
- **b** Any hexagon with six equal sides is regular.
- **c** Every regular polygon is convex.
- **d** Regular polygons cannot have acute angles.
- **e** A triangle has three diagonals.

8 Copy each shape, and draw all of its diagonals:

a


b


c


INVESTIGATION 1 — DIAGONALS

How many diagonals does a convex polygon have?

What to do:

1. Look at your answers to **8** above.
   Count the diagonals you drew in each polygon, and write your answers in a table like this.
2. To complete the *Diagonals* column, draw *convex* polygons of your own, draw in their diagonals, and count the diagonals you draw.
3. Fill in the third column by substituting the number of sides n into the expression $\frac{1}{2}n(n-3)$.

Sides (n)	*Diagonals*	$\frac{1}{2}n(n-3)$
3		
4		
5		
6		
7		
8		
9		
10		

4. Copy and complete:
   A convex polygon with n sides has ...... diagonals.
5. Find the number of diagonals in a convex 20-gon.

ACTIVITY — CREATING POLYGONS

In this online Activity you will use sets of points to create polygons, then explore the properties of the polygons generated.

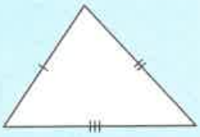
ACTIVITY

B TRIANGLES

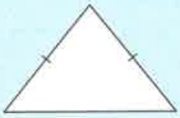
A **triangle** is a polygon with three sides.

We can classify triangles according to the number of sides which are equal in length, or according to their angles.

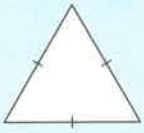
CLASSIFICATION BY SIDE LENGTHS



A **scalene triangle** has no equal sides.

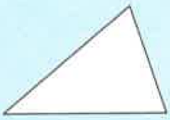


An **isosceles triangle** has at least two equal sides.



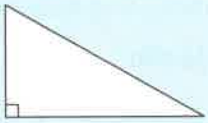
An **equilateral triangle** has three equal sides.

CLASSIFICATION BY ANGLES



An **acute angled triangle** has all acute angles.

An **obtuse angled triangle** has one obtuse angle.

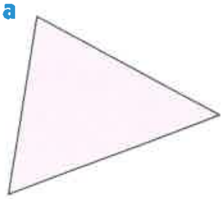


A **right angled triangle** has one right angle.

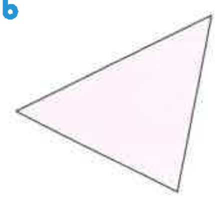
EXERCISE 11B

1 Measure the side lengths of each triangle. Hence classify the triangle as equilateral, isosceles, or scalene.

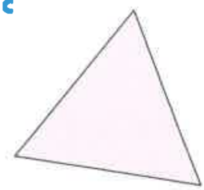
a



b



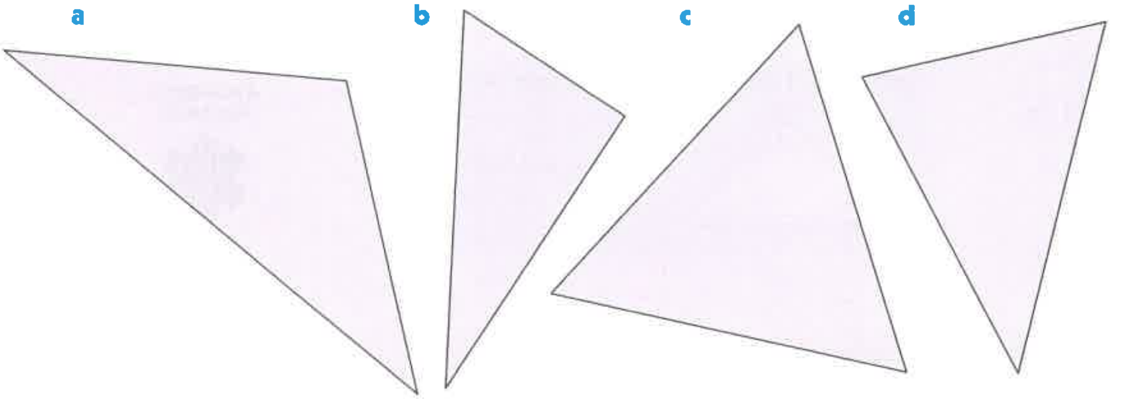
c



d

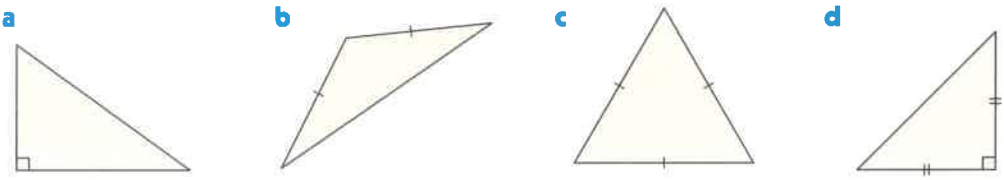
2 Measure the angles of each triangle. Hence classify the triangle as acute, obtuse, or right angled.

a **b** **c** **d**



3 Use the indicated lengths of the sides *and* the sizes of the angles to classify each triangle. Include *two* descriptions for each triangle.

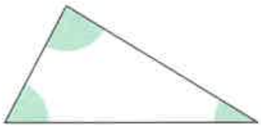
a **b** **c** **d**



4 Is an equilateral triangle also isosceles? Explain your answer.

C ANGLE SUM OF A TRIANGLE

When we talk about the angles of a triangle, we are usually referring to the **interior angles** *inside* the triangle.



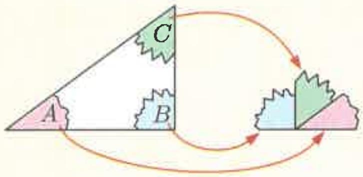
INVESTIGATION 2 ANGLE SUM OF A TRIANGLE

You will need: a large piece of paper, scissors, ruler, and pencil.

What to do:

1 **a** Draw any triangle on a piece of paper. Label the angles A, B, and C on the *inside* of the triangle. Cut out the triangle.

b Tear off each of the 3 angles. Place them next to each other so the vertices all meet and the edges touch. What do you notice?

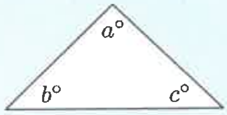


2 Repeat this experiment with other triangles.

3 Summarise your observations.

From **Investigation 2** and the **Opening Problem**, you should have discovered the **angle sum of a triangle theorem**:

The sum of the angles in a triangle is $180°$.



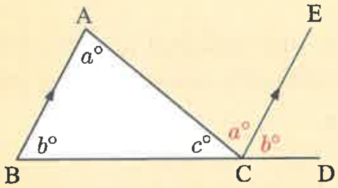
$a + b + c = 180$

GEOMETRY PACKAGE

Proof:

Let triangle ABC have angles $a°$, $b°$, and $c°$ at the vertices A, B, and C respectively.

We extend [BC] to D, and draw [CE] parallel to [BA].



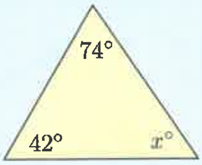
Now	$\widehat{ACE} = \widehat{BAC} = a°$	{equal alternate angles}
and	$\widehat{ECD} = \widehat{ABC} = b°$	{equal corresponding angles}
But	$a + b + c = 180$	{angles on a line}
$\therefore$	$a + b + c = 180$	

When we solve geometry problems, the diagrams are often not drawn to scale. We solve the problem using the side lengths and angles given, and known geometric properties.

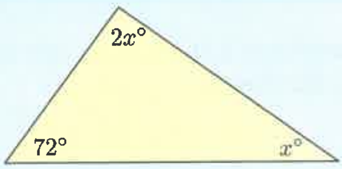
Example 1 — Self Tutor

Find the value of x:

a



b



a

$x + 42 + 74 = 180$ {angle sum of a triangle}

$\therefore \; x + 116 = 180$

$\therefore \; x + 116 - 116 = 180 - 116$ {subtracting 116 from both sides}

$\therefore \; x = 64$

b

$2x + x + 72 = 180$ {angle sum of a triangle}

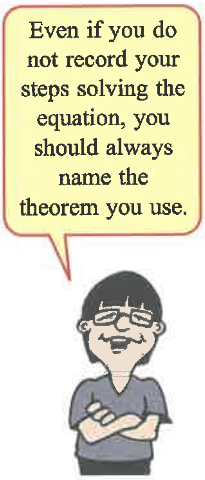
$\therefore \; 3x + 72 = 180$ {collecting like terms}

$\therefore \; 3x + 72 - 72 = 180 - 72$ {subtracting 72 from both sides}

$\therefore \; 3x = 108$

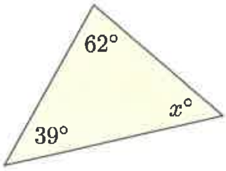
$\therefore \; \dfrac{3x}{3} = \dfrac{108}{3}$ {dividing both sides by 3}

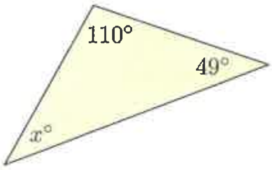
$\therefore \; x = 36$

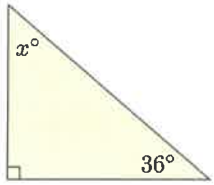


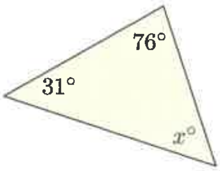
EXERCISE 11C

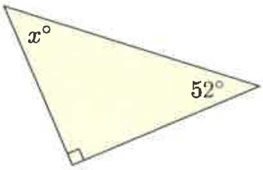
1 Find the value of x:

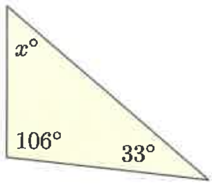
a


b


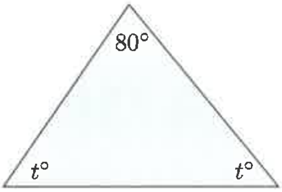
c


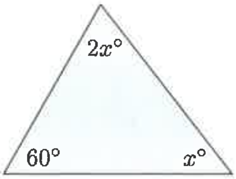
d


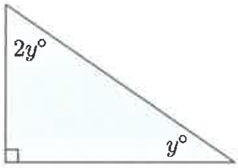
e


f


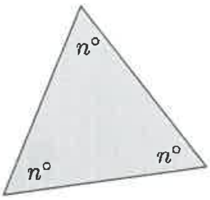
2 Find the value of the unknown:

a


b


c


3 **a** Find the value of n:



b Copy and complete:
The interior angles of any equilateral triangle always measure ....

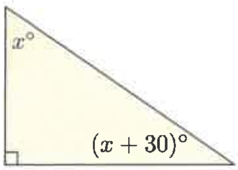
4 Explain why it is not possible to draw a triangle which has:

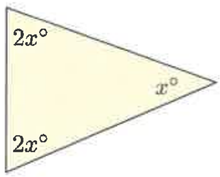
a two obtuse angles

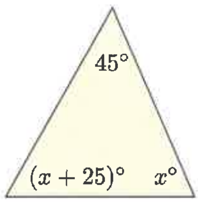
b one obtuse angle and one right angle

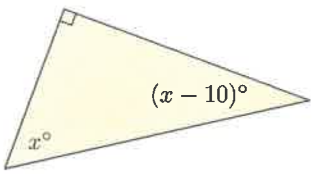
c all angles less than $60°$.

5 Find the value of x:

a


b


c


d


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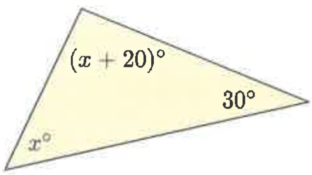

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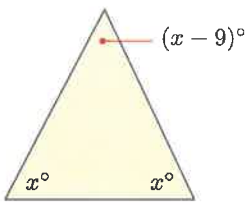


6 The figure alongside can be used to prove the angle sum of a triangle theorem.

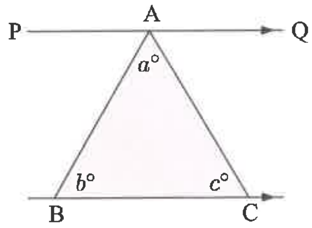
Copy and complete:

$\widehat{QAC} = ...$ {equal alternate angles}

$\widehat{PAB} = ...$ {equal alternate angles}

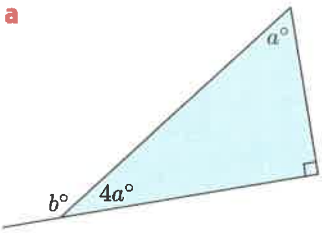
Now $\widehat{PAB} + \widehat{BAC} + \widehat{QAC} = ...$ {angles on a line}

$\therefore\ a + b + c = ...$

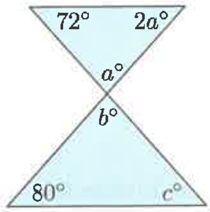


7 Find, in alphabetical order, the values of the unknowns:

a

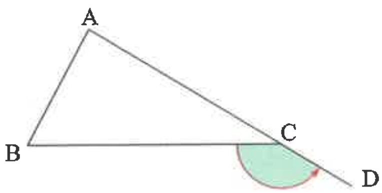


b

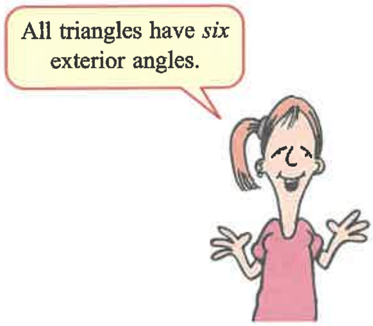


D EXTERIOR ANGLES OF A TRIANGLE

If we extend any side of a triangle, we can create an **exterior angle**.

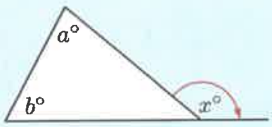


$\widehat{BCD}$ is an *exterior angle* of triangle ABC.



The **exterior angle of a triangle theorem** states that:

Any exterior angle of a triangle is equal in size to the sum of the interior opposite angles.



$x = a + b$

GEOMETRY PACKAGE

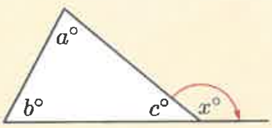
Proof:

Let the third interior angle of the triangle be $c°$.

$\therefore\ a + b + c = 180$ {angle sum of a triangle}

But $x + c = 180$ {angles on a line}

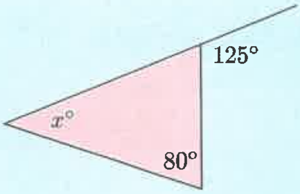
$\therefore\ x = a + b$



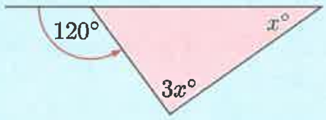
Example 2 — Self Tutor

Find the value of x:

a



b



a $x + 80 = 125$ {exterior angle of a triangle}

$\therefore\ x + 80 - 80 = 125 - 80$ {subtracting 80 from both sides}

$\therefore\ x = 45$

b $3x + x = 120$ {exterior angle of a triangle}

$\therefore\ 4x = 120$ {collecting like terms}

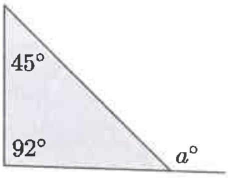
$\therefore\ \dfrac{4x}{4} = \dfrac{120}{4}$ {dividing both sides by 4}

$\therefore\ x = 30$

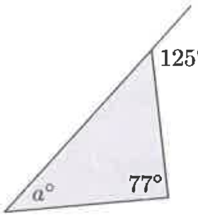
EXERCISE 11D

1 Find the value of a:

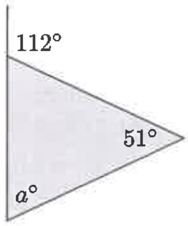
a



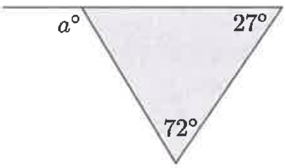
b



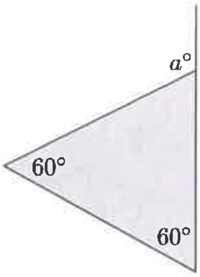
c



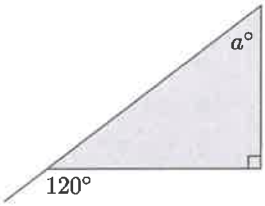
d



e

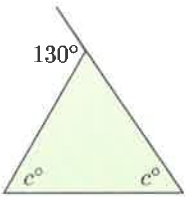


f

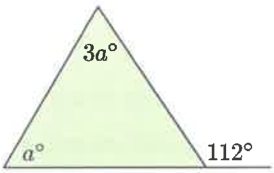


2 Find the values of the unknowns:

a



b



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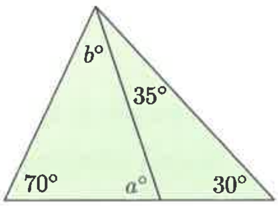
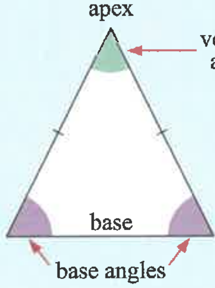


E ISOSCELES TRIANGLES

An **isosceles triangle** is a triangle which has at least two sides equal in length.

Having identified two equal sides of an isosceles triangle, we label the triangle as follows:

- The third side is called the **base**.
- The vertex between the equal sides is called the **apex**.
- The angle at the apex is called the **vertical angle**.
- The angles opposite the equal sides are called the **base angles**.



INVESTIGATION 3 THE ISOSCELES TRIANGLE THEOREM

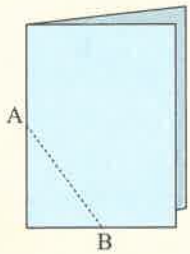
In this Investigation you will make your own isosceles triangle then explore its properties.

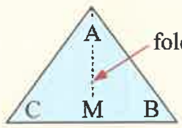
DEMO

You will need: a sheet of paper, scissors, ruler, and pencil.

What to do:

1. Fold the piece of paper exactly in half, then draw a straight line segment [AB] from the fold to the edge as shown.
2. With the sheets pressed tightly together, cut along [AB] through both sheets.
   Keep the triangular piece of paper. Unfold it and label it as shown in the second diagram.





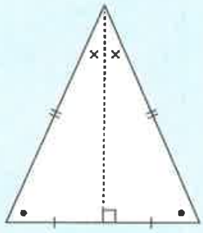
3. Match each of these facts to its reason:
   - **a** AB = AC
   - **b** $A\widehat{C}B = A\widehat{B}C$
   - **c** $C\widehat{A}M = B\widehat{A}M$
   - **d** M is the midpoint of [BC]
   - **e** [AM] $\perp$ [BC]

   - **A** both angles are the angle at which the cut was made from the fold line
   - **B** both lengths are the distance from the end of the cut at B to the fold line
   - **C** the length of the cut was the same across both sheets
   - **D** the fold was made perpendicular to the edge of the page
   - **E** both angles are the angle at which the cut was made from the edge of the page.
4. Which fact in **3** tells you the triangle is isosceles?
5. Discuss what you can deduce from the other facts.

From the **Investigation**, you should have found the properties of isosceles triangles which are stated in the **isosceles triangle theorem**:

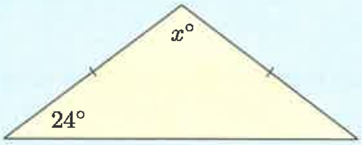
In any isosceles triangle:

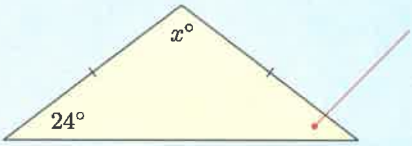
- the base angles are equal
- the line joining the apex to the midpoint of the base bisects the vertical angle and meets the base at right angles.



Example 3 — Self Tutor

Find the value of x:





Since the triangle is isosceles, the base angles are equal in size.

$\therefore$ this angle is also 24°.

$\therefore\ x + 24 + 24 = 180$ {angle sum of a triangle}

$\therefore\ x + 48 = 180$

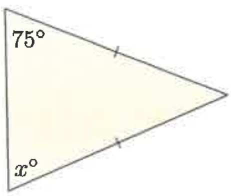
$\therefore\ x + 48 - 48 = 180 - 48$ {subtracting 48 from both sides}

$\therefore\ x = 132$

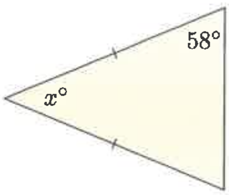
EXERCISE 11E

1 The following diagrams are not drawn to scale, but the information on them is correct. Use the information to find the value of the unknown:

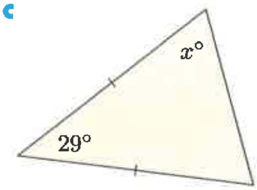
a



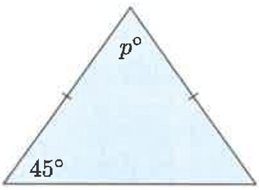
b



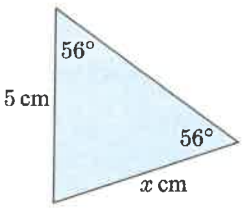
c



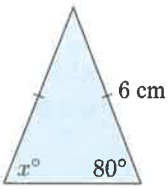
d



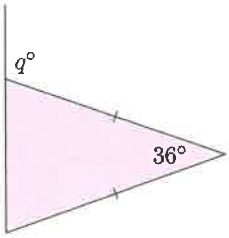
e



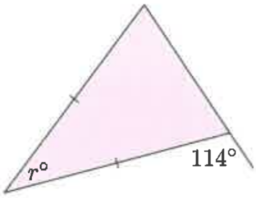
f



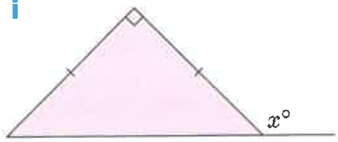
g



h



i



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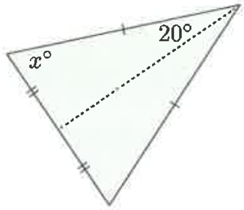
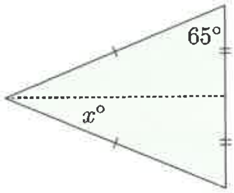
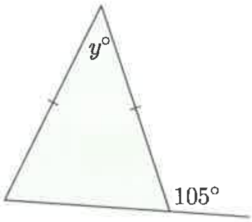


k

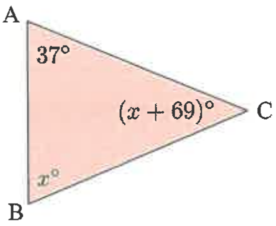


l

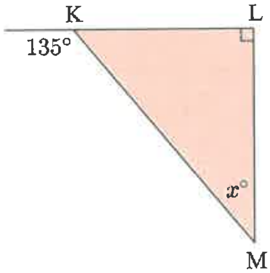


2 Find the value of x in each triangle. Hence deduce something about the triangle.

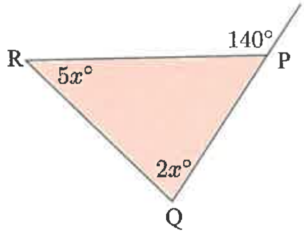
a



b



c



3 Match each set of angle sizes with the corresponding description of a triangle:

a 25°, 90°, 65°
b 50°, 60°, 70°
c 90°, 45°, 45°
d 125°, 35°, 20°
e 60°, 60°, 60°
f 40°, 100°, 40°
g 40°, 70°, 70°

A isosceles and acute angled
B scalene and obtuse angled
C scalene and right angled
D isosceles and obtuse angled
E scalene and acute angled
F equilateral and acute angled
G isosceles and right angled.

F QUADRILATERALS

A **quadrilateral** is a polygon with four sides.

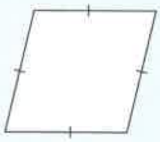
To classify quadrilaterals, we need to consider not only side lengths and angles, but also whether opposite sides are parallel.

There are six special quadrilaterals:

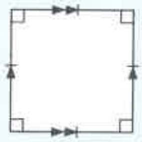
- A **parallelogram** has both pairs of opposite sides parallel.

- A **rectangle** is a parallelogram with right angled corners.

- A **rhombus** is a quadrilateral with all four sides equal in length.

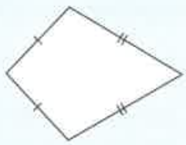


- A **square** is a rectangle with all sides equal in length.



- A **trapezium** has one pair of opposite sides which are parallel.

- A **kite** has two pairs of adjacent sides which are equal in length.



INVESTIGATION 4 PROPERTIES OF QUADRILATERALS

What to do:

PRINTABLE WORKSHEET

1 Print the quadrilaterals by clicking on the icon.

2 For each **parallelogram**:

  a Measure the lengths of the opposite sides. What do you notice?

  b Measure the sizes of the opposite angles. What do you notice?

  c Draw the two diagonals. Measure the distances from each vertex to the point of intersection of the diagonals. What do you notice?

3 For each **rectangle**:

  a Measure the lengths of the opposite sides. What do you notice?

  b Measure the lengths of the diagonals. What do you notice?

  c Copy and complete:

  A rectangle is a parallelogram with diagonals that are ....

4 For each **rhombus**:

  a Determine whether each pair of opposite sides is parallel.

  b Measure the sizes of the opposite angles. What do you notice?

  c Draw the two diagonals.

    i At what angle do the diagonals intersect?

    ii What do the diagonals do to the angles at each vertex?

5 For each **square**:

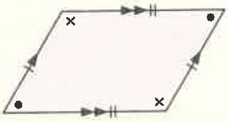
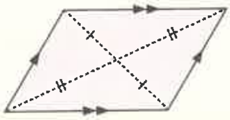
- **a** Check that opposite sides are parallel.
- **b** Draw the two diagonals.
  - **i** Measure the distances from each vertex to the point of intersection of the diagonals. What do you notice?
  - **ii** At what angle do the diagonals intersect?
  - **iii** What do the diagonals do to the angle at each vertex?

6 For the **kite**:

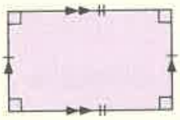
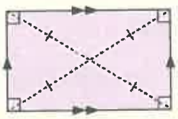
- **a** Measure the sizes of the opposite angles. What do you notice?
- **b** Draw the two diagonals.
  - **i** Measure the distances from each vertex to the point of intersection of the diagonals. What do you notice?
  - **ii** At what angle do the diagonals intersect?

From the **Investigation**, you should have discovered these properties of special quadrilaterals:

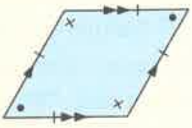
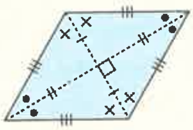
Parallelogram
- opposite sides are equal in length
- opposite angles are equal in size
- diagonals bisect each other

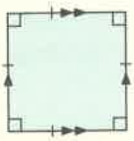
Rectangle
- opposite sides are equal in length
- diagonals are equal in length
- diagonals bisect each other

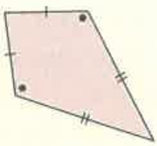
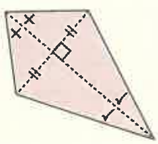
Rhombus
- opposite sides are parallel
- opposite angles are equal in size
- diagonals bisect each other at right angles
- diagonals bisect the angles at each vertex

Square
- opposite sides are parallel
- diagonals bisect each other at right angles
- diagonals bisect the angles at each vertex




Kite
- one pair of opposite angles is equal in size
- diagonals cut each other at right angles
- one diagonal bisects one pair of angles at the vertices

EXERCISE 11F

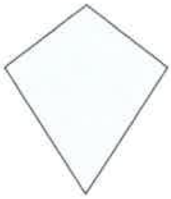
1 Sketch and fully label:

a a parallelogram **b** a rhombus **c** a kite.

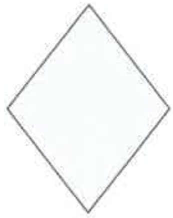
2 Use a ruler to help classify each shape:

a

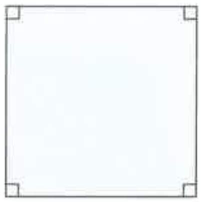
b



c

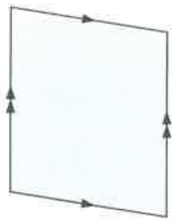


d



e

f

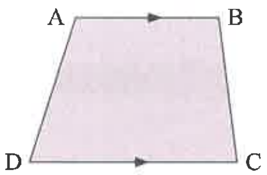


3 True or false?

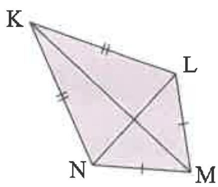
a Every rhombus is a parallelogram.
b Every parallelogram is a rhombus.
c A square is a rhombus with four right angles.
d The diagonals of a kite bisect each other at right angles.
e The opposite angles of a kite are equal.
f A kite can have parallel sides.

4 Use $\parallel$ and $\perp$ to write statements about each figure:

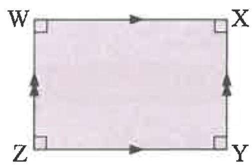
a



b

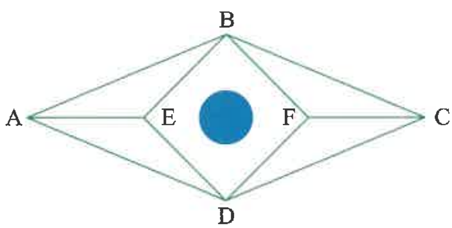


c



5 Stefan has designed this logo for an optometrist. It consists of a circle and four identical isosceles triangles with base angles $22.5°$.
State, with reasons, the shape of quadrilateral:

a ABCD **b** ABFD **c** BEDF.



6 Illustrate accurately:

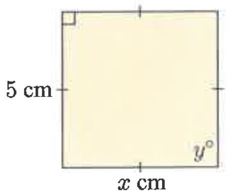
a ABCD is a rhombus with sides 2.5 cm and an interior angle 65°.

b ABCD is a quadrilateral in which [AB] ∥ [DC] and [AD] ∥ [BC].

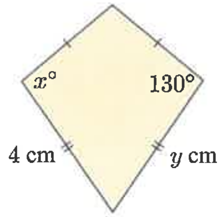
c PQRS is a quadrilateral in which PQ = 3 cm, RS = 4 cm, PS = 2 cm, [PQ] ∥ [SR], and [QP] ⊥ [PS].

7 Find the values of the unknowns, giving brief reasons for your answers:

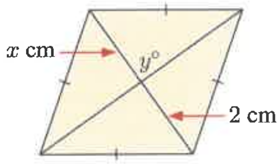
a



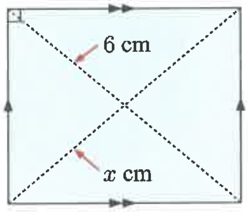
b



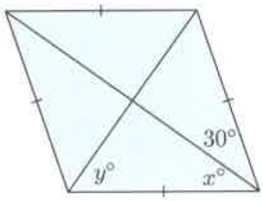
c



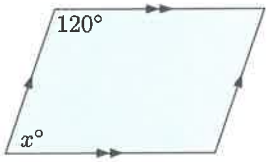
d



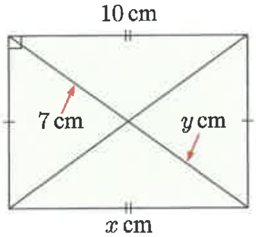
e



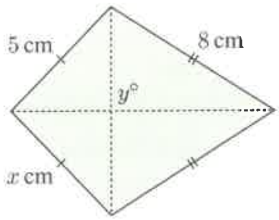
f



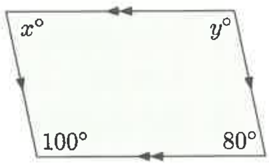
g



h



i



G ANGLE SUM OF A QUADRILATERAL

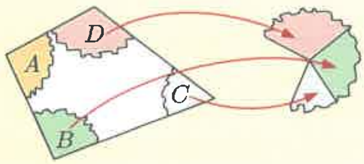
INVESTIGATION 5 ANGLE SUM OF A QUADRILATERAL

You will need: a large piece of paper, scissors, ruler, and pencil.

What to do:

1 **a** Draw any quadrilateral on a piece of paper. Label the angles A, B, C, and D on the *inside* of the quadrilateral. Cut out the quadrilateral.

b Tear off each of the 4 angles. Place them next to each other so the vertices all meet and the edges touch. What do you notice?

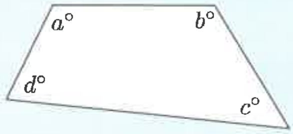


2 Repeat this experiment with other quadrilaterals.

3 Summarise your observations.

From the **Investigation**, you should have discovered that:

The sum of the angles of a quadrilateral is 360°.



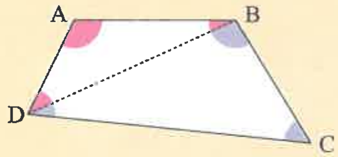
$a + b + c + d = 360$

GEOMETRY PACKAGE

Proof:

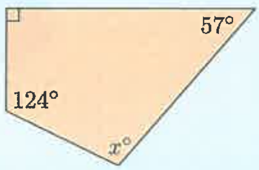
Suppose we divide quadrilateral ABCD into the two triangles ABD and BCD.

The sum of the interior angles of ABCD
$=$ sum of angles of $\triangle$ABD $+$ sum of angles of $\triangle$BCD
$= 180° + 180°$ {angle sum of a triangle}
$= 360°$



Example 4 Self Tutor

Find the value of x:



The sum of the angles of a quadrilateral is 360°.

$\therefore\ x + 57 + 90 + 124 = 360$

$\therefore\ x + 271 = 360$

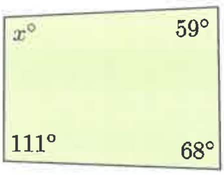
$\therefore\ x + 271 - 271 = 360 - 271$ {subtracting 271 from both sides}

$\therefore\ x = 89$

EXERCISE 11G

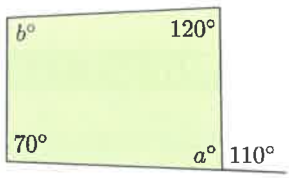
1 Find the values of the unknowns:

a

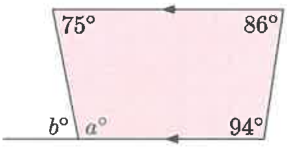


b

c

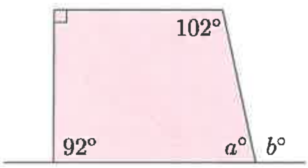


d



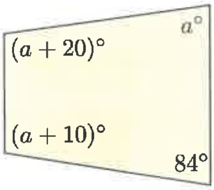
e

f

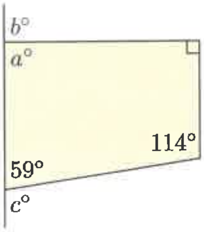


2 Find the values of the unknowns:

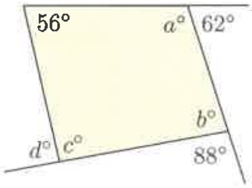
a



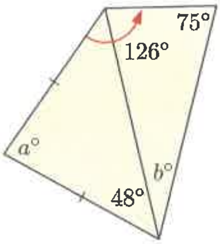
b



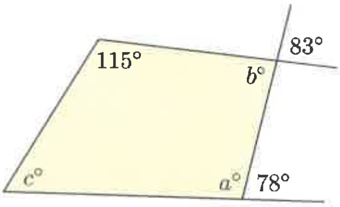
c



d



e



INVESTIGATION 6 EULER'S RULE

Euler's rule is an equation which connects the numbers of edges, vertices, and regions of any plane figure.

Click on the icon to obtain this Investigation.

EULER'S RULE

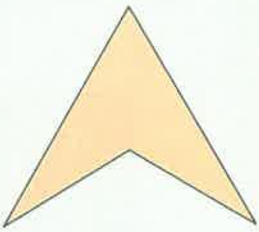
MULTIPLE CHOICE QUIZ

QUICK QUIZ

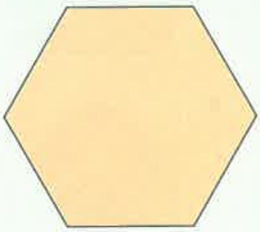
REVIEW SET 11A

1 Name each polygon according to its number of sides and whether it is convex:

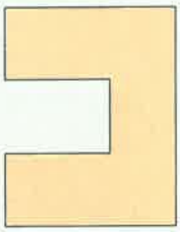
a



b

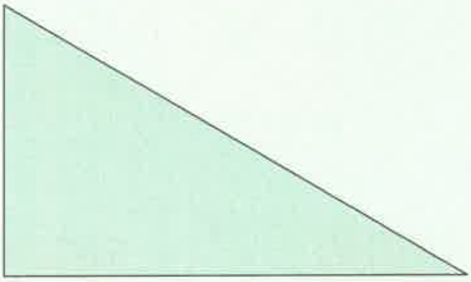


c

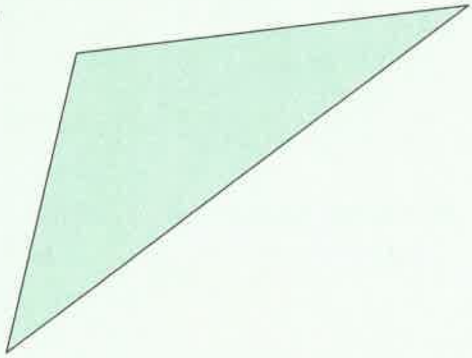


2 Use a protractor to classify each triangle as acute, obtuse, or right angled.

a

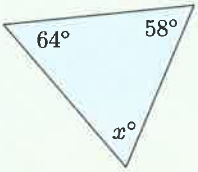


b

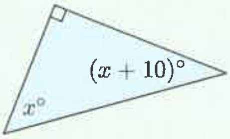


3 Find the value of x:

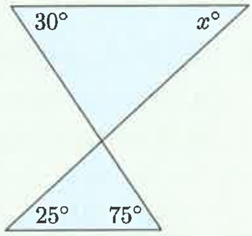
a



b

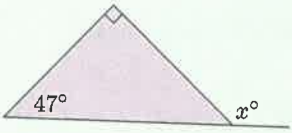


c

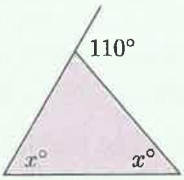


4 Find the value of x:

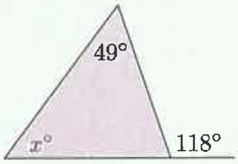
a



b

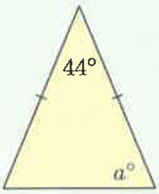


c

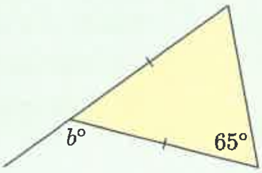


5 Find the value of the unknown:

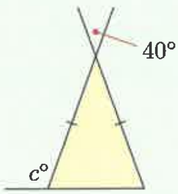
a



b



c



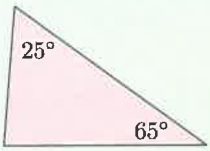
6 True or false?

a Every square is a kite.

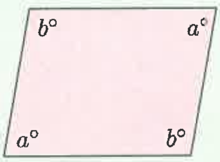
b Every parallelogram is a rectangle.

7 Use the information given to name each figure.
Explain your answers.

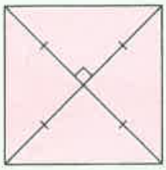
a



b

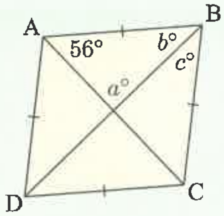


c



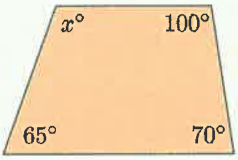
8 ABCD is a rhombus. Find the value of:

a a **b** b **c** c

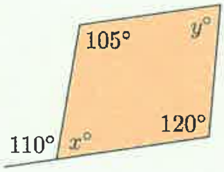


9 Find the values of the unknowns:

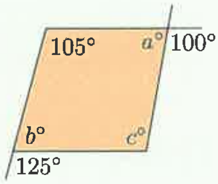
a



b



c



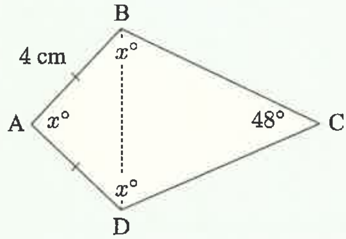
10 Look at the quadrilateral ABCD.

a Find the value of x.

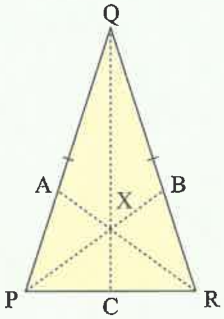
b Find the measure of:

i $A\widehat{B}D$ **ii** $A\widehat{D}B$
iii $C\widehat{B}D$ **iv** $C\widehat{D}B$

c Classify the quadrilateral, giving reasons for your answer.



11



Triangle PQR is isosceles with base angles $Q\widehat{P}R = Q\widehat{R}P = 72°$.

a Find the size of $P\widehat{Q}R$.

b Each angle of triangle PQR is bisected as shown. The angle bisectors all meet at point X, and they divide triangle PQR into six smaller triangles.
Which of these smaller triangles are:

i acute angled **ii** obtuse angled
iii right angled **iv** isosceles?

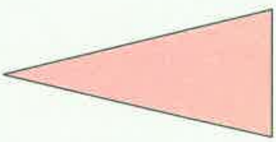
REVIEW SET 11B

1 Sketch a hexagon which is:

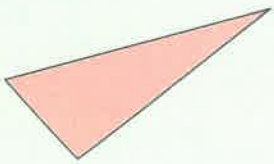
a regular **b** convex but not regular **c** non-convex.

2 Use a ruler to classify each triangle as equilateral, isosceles, or scalene:

a

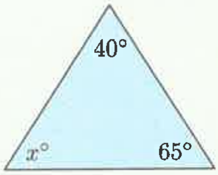


b

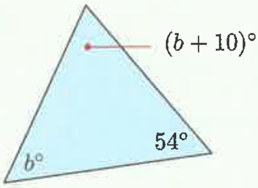


3 Find the value of the unknown:

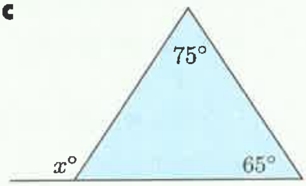
a



b

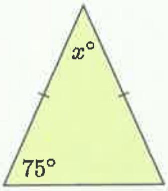


c

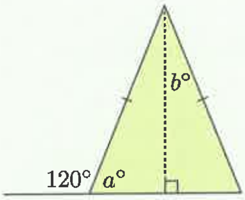


4 Find the values of the unknowns:

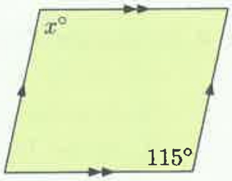
a



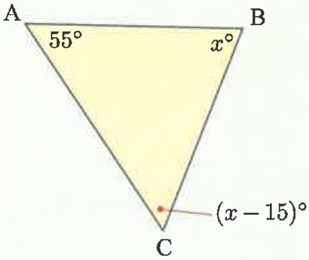
b



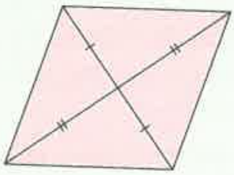
c



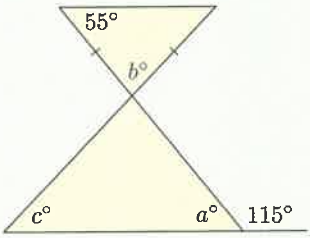
5 Find the value of x.
Hence deduce something about the triangle ABC.



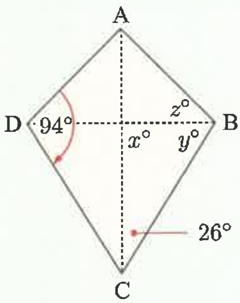
6 Name the following figure using the information given.
Give reasons for your answer.



7 Find the values of the unknowns.



8 ABCD is a kite.
Find the values of the unknowns.



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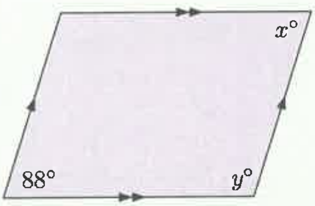


a Name this figure using the information given.

b Find the values of the unknowns.

10 Explain why it is not possible to draw a quadrilateral which has all acute angles.

11 A surveyor is measuring out the boundaries of a piece of farm land.

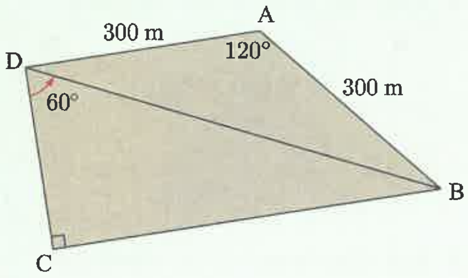
a Find the size of:

i $A\widehat{B}D$ **ii** $A\widehat{D}B$

Give reasons for your answers.

b Find $D\widehat{B}C$.

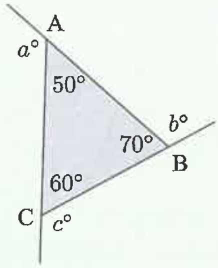
c What kind of quadrilateral is ABCD? Explain your answer.



12 **a** In triangle ABC, the exterior angles have sizes $a°$, $b°$, and $c°$.

i Find the values of a, b, and c.

ii Hence find the sum of the exterior angles.

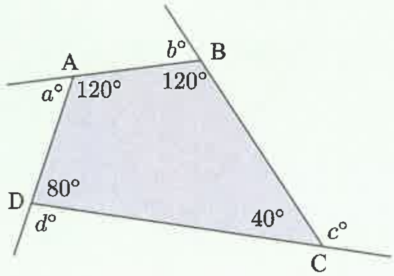


b In quadrilateral ABCD, the exterior angles have sizes $a°$, $b°$, $c°$, and $d°$.

i Find the values of a, b, c, and d.

ii Hence find the sum of the exterior angles.

c What do you suspect about the sum of the exterior angles of *any* polygon?



Chapter 12

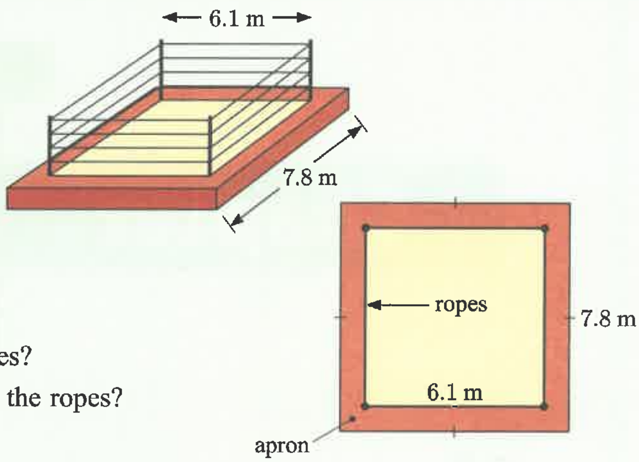
Measurement: Length and area

Contents:

OPENING PROBLEM

A boxing ring has the dimensions shown. There are 4 ropes on each side of the ring, and each rope is 6.1 m long. The ropes are connected to the corner posts.

The ring is 7.8 m long on all sides. The part of the ring outside the ropes is called the *apron*.



Things to think about:

a What is the total length of the ropes?

b What is the area of the ring inside the ropes?

c What is the area of the apron?

Measurements of length, area, volume, capacity, and mass enable us to answer questions such as:

- How far is it to work?
- How big is the swimming pool?
- How many litres of petrol did you buy?
- How heavy is the bag?

In order that we can communicate and compare measurements, we use **units**.

Early units of measurement were based on the human body or other items in the world around us. For example, in the 12th century, **King Henry I** defined a **yard** to be the distance from his nose to his outstretched fingertips.

This led to the **British Imperial System** of units, which included:

- inches, feet, yards, and miles to measure length
- pints, quarts, and gallons to measure capacity
- ounces, pounds, and tons to measure mass.

This system is still used in a few countries, but the **Metric System** developed in France in 1789 is now used more commonly throughout the world. It is an easy system to work with because it uses powers of 10 for all conversions. Common prefixes are used when naming related units.

- **Greek** prefixes are used to make the base units **larger**.

  - **kilo** means 1000
  - **mega** means 1 000 000

- **Latin** prefixes are used to make the base units **smaller**.

  - **centi** means $\frac{1}{100}$
  - **milli** means $\frac{1}{1000}$

The Metric System is more correctly called **Le Système International d'Unités** or **SI** for short.

A LENGTH

Length is a measure of distance.

The **metre** (m) is the base unit for length in the metric system.

The length of an adult's stride is about one metre.

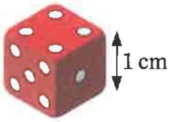
From this base unit, we define these related units for measuring smaller and larger distances:

- 1 **kilometre** (km) = 1000 metres
- 1 **centimetre** (cm) $= \frac{1}{100}$ metre
- 1 **millimetre** (mm) $= \frac{1}{1000}$ metre or $\frac{1}{10}$ centimetre

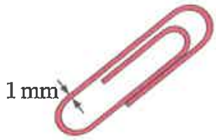
For example:

- 3 laps of a football field is about 1 km.

- The edges of a die are about 1 cm long.

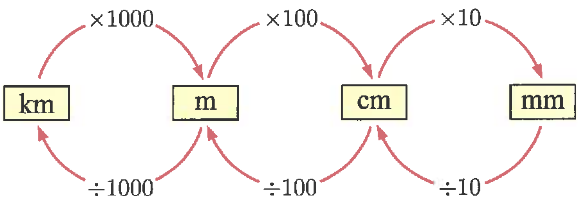


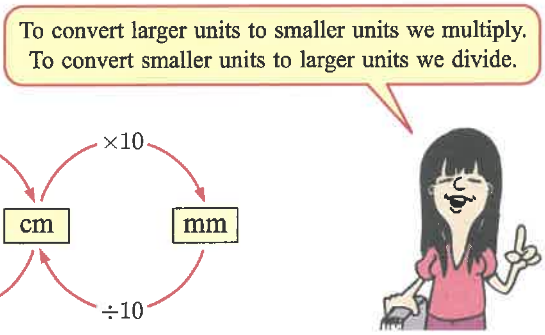
- A paper clip is about 1 mm thick.



LENGTH CONVERSIONS

1 km = 1000 m
1 m = 100 cm
1 cm = 10 mm





Example 1 Self Tutor

Convert:

a 3.2 km into m
b 65 mm into cm
c 1.2 m into mm.

a 3.2 km
= 3.2 × 1000 m
= 3200 m

b 65 mm
= 65 ÷ 10 cm
= 6.5 cm

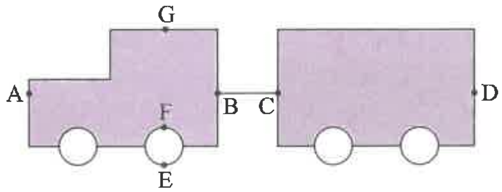
c 1.2 m
= 1.2 × 100 cm
= 120 cm
= 120 × 10 mm
= 1200 mm

EXERCISE 12A

1. State the units you would use for measuring:
   - a. the distance of a plane flight
   - b. the height of a staple
   - c. the height of a tree
   - d. the length of a pen
   - e. the length of a tennis court
   - f. the width of a blade of grass.

2. Choose the correct answer:
   - a. The length of a car is about:
     A 50 cm **B** 5 m **C** 5 mm **D** 500 mm
   - b. The length of a mosquito is about:
     A 11 cm **B** 1.1 m **C** 11 mm **D** 11 km
   - c. The distance from London to Berlin is about:
     A 9000 m **B** 9 km **C** 900 km **D** 900 m

3. The diagram shows a model train. Use your ruler to measure, in millimetres:
   - a. the length of the engine AB
   - b. the length of the carriage CD
   - c. the distance between the engine and the carriage BC
   - d. the total length AD
   - e. the height of a wheel EF
   - f. the train's height EG.



4. Use a ruler to find the length of each line. Give your answer in centimetres.
   - a. 
   - b. 

5. Convert:
   - a. 4 m into cm
   - b. 7.8 m into cm
   - c. 9 cm into mm
   - d. 13.92 cm into mm
   - e. 8 km into m
   - f. 1.09 km into m
   - g. 0.159 km into m
   - h. 5 m into mm
   - i. 3.9 m into mm.

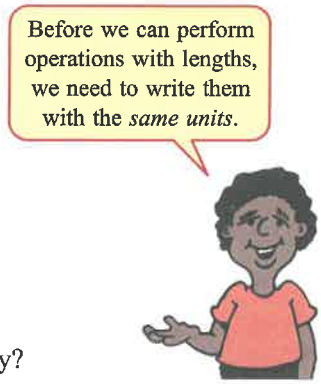
6. Convert:
   - a. 80 mm into cm
   - b. 237 mm into cm
   - c. 600 cm into m
   - d. 903 cm into m
   - e. 56.8 cm into m
   - f. 2000 m into km
   - g. 4270 m into km
   - h. 4000 mm into m
   - i. 1840 mm into m.

7. Copy and complete:
   - a. 61 cm = ... mm
   - b. 720 cm = ... m
   - c. 3020 m = ... km
   - d. 88 mm = ... cm
   - e. 11 m = ... cm
   - f. 0.4 km = ... m
   - g. 3.27 cm = ... mm
   - h. 249 cm = ... m
   - i. 3800 m = ... km
   - j. 1.7 mm = ... cm
   - k. 7.8 m = ... mm
   - l. 35 m = ... km

8 By first writing each length in centimetres, find the sum of:
- **a** 6 m, 34 cm
- **b** 3.8 m, 74 cm
- **c** 4 m, 50 cm, 95 mm
- **d** 5 m, 12 cm, 7 mm

9 By first writing each length in metres, find the sum of:
- **a** 68 cm, 1.4 m
- **b** 3 km, 430 m, 220 cm
- **c** 8 km, 920 m, 650 cm
- **d** 75 m, 32 cm, 15 mm

10 Jacquita swam 1.35 km on Monday and 840 m on Tuesday.
- **a** How far did she swim in total?
- **b** How much further did she swim on Monday than on Tuesday?



11 Tyler stacks 28 dominoes, one on top of the other. Each domino is 7 mm high. Calculate the height of the stack. Give your answer in centimetres.

12 Marie's average step length is 0.9 m. In one day she took 12 000 steps. How many kilometres did she walk?

13 Calculate the number of 8 m pipes required to lay a 32 km pipeline.

14 Find the number of 75 cm lengths of string which can be cut from a 150 m reel.

15 John planted three seedlings. After a year, he measured their heights as 76 cm, 1.08 m, and 92 cm. Find the average height of the plants.

ACTIVITY 1 **LENGTHS OF CURVES**

Finding the length of a curve accurately can be quite difficult.

In this printable Activity, we *estimate* the length of curves using string.

ACTIVITY

B PERIMETER

In English, the word *perimeter* refers to the boundary of a region.

For example, we say that:
- the boundary line of a hockey field is the playing *perimeter*
- the boundary of a prison is protected by a *perimeter* fence.

However, in mathematics the word *perimeter* refers to a *distance*.

The **perimeter** of a closed figure is the distance around its boundary.

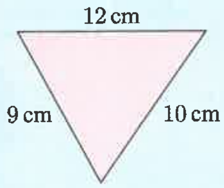
The perimeter of a polygon is found by adding the lengths of its sides.

Before you add the side lengths, make sure they are written with the *same units*.

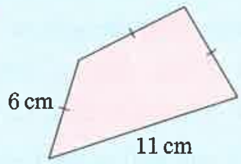
Example 2 **Self Tutor**

Find the perimeter of each polygon:

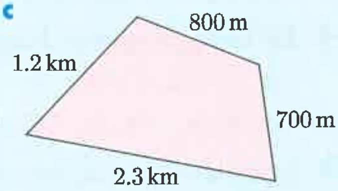
a



b

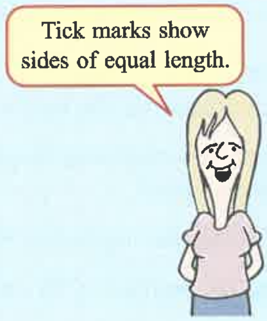


c



a Perimeter
$= (9 + 10 + 12)$ cm
$= 31$ cm

b Perimeter
$= (11 + 3 \times 6)$ cm
$= (11 + 18)$ cm
$= 29$ cm

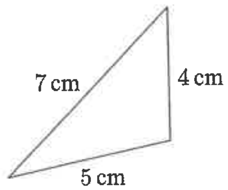


c Perimeter
$= 2.3 \text{ km} + 1.2 \text{ km} + 800 \text{ m} + 700 \text{ m}$
$= (2300 + 1200 + 800 + 700)$ m {converting to m}
$= 5000$ m

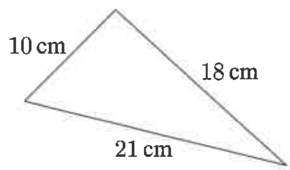
EXERCISE 12B

1 Find the perimeter of each figure:

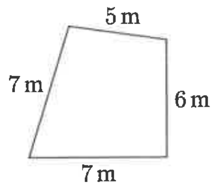
a



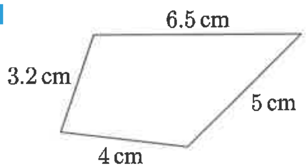
b



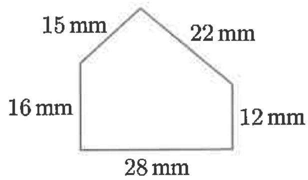
c



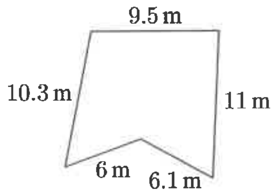
d



e

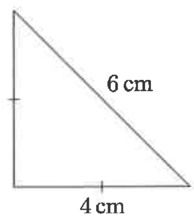


f

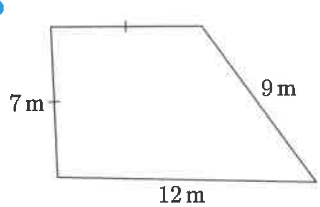


2 Find the perimeter of each figure:

a



b



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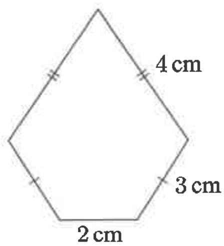

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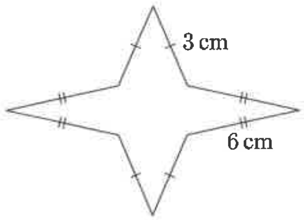

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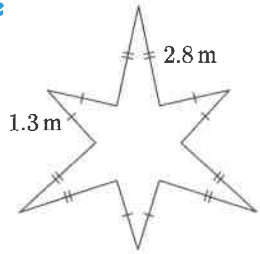
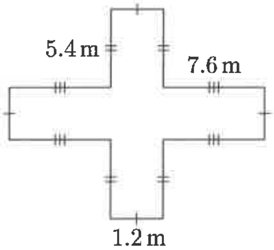
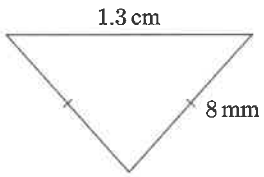


f

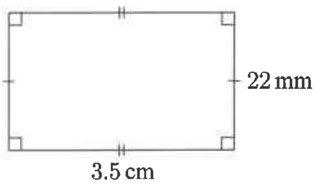


3 Find the perimeter of each figure:

a



b



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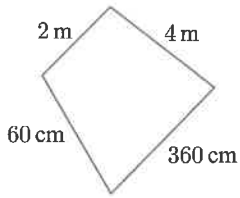

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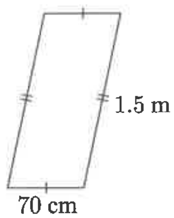

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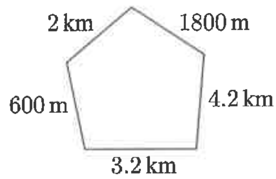

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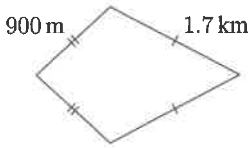
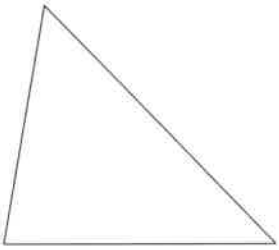


4 Use your ruler to find the perimeter of each figure:

a



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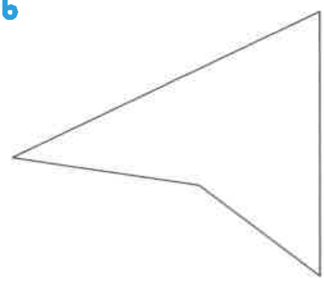

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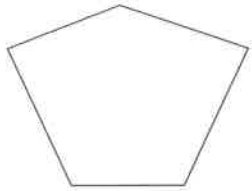
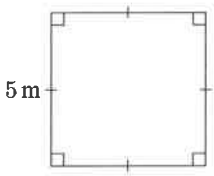


5 **a** Find the perimeter of each square:

i



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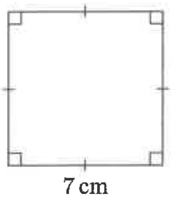

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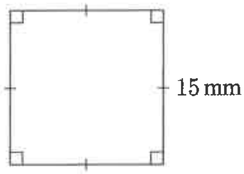
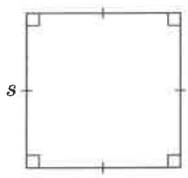
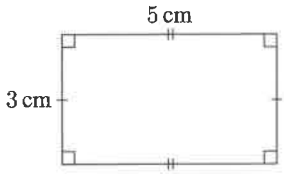
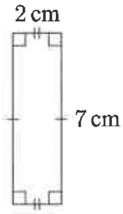


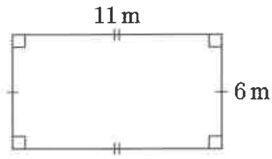
b Write down a formula for the perimeter P of a square with side length s.



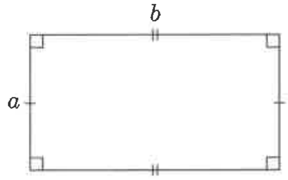
6 **a** Find the perimeter of each rectangle:

i 

ii 

iii 

b Write down a formula for the perimeter P of a rectangle with side lengths a and b.

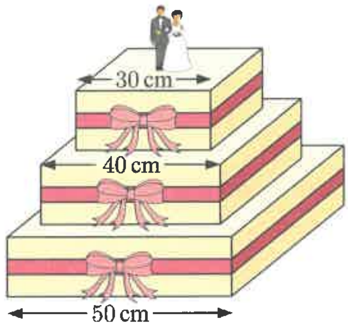


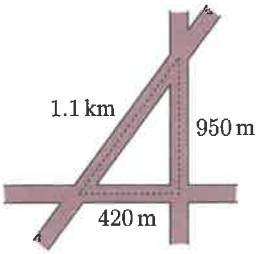
7 A piece of glass 150 cm long by 90 cm wide is placed within an 8 cm wide metal frame to make a table top. Find the perimeter of:

a the glass **b** the table top.

8 A square field has sides of length 850 metres. Find the cost of fencing the field with 3 strands of wire costing \$1.35 per metre.

9 A wedding cake has three square layers. Their side lengths are 30 cm, 40 cm, and 50 cm. A ribbon is placed around each layer, and is tied with a bow. Allowing 20 cm for each bow, how much ribbon is needed in total?



10 

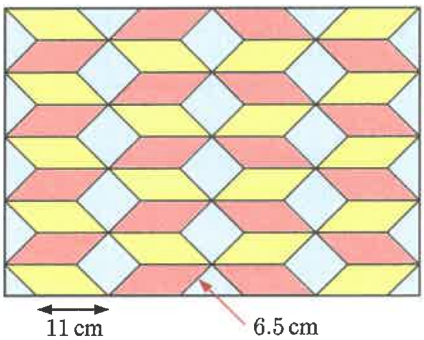
A cyclist trains by completing 14 laps of the triangular block shown. How far does she cycle? Give your answer in kilometres.

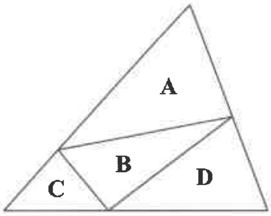
11 A stained-glass window is created from a combination of squares, parallelograms, and triangles.

a Find the perimeter of each:

i square **ii** parallelogram.

b Find the total length of glue strips required to join the pieces of glass together.



12 

A large triangle has been divided into 4 smaller triangles.
Triangle **A** has perimeter 16 cm.
Triangle **B** has perimeter 12 cm.
Triangle **C** has perimeter 5 cm.
Triangle **D** has perimeter 13 cm.
Find the perimeter of the large triangle.

ACTIVITY 2 FINDING PERIMETERS

You will need: Tape measure or metre ruler, trundle wheel.

What to do:

1 Copy the table below.

Item	*Unit of length*	*Estimate*	*Actual perimeter*
front cover of this textbook			
a bank note			
your desk			
largest window in your classroom			
your classroom			
your school gymnasium			

For each item in the table:

a Record the unit of length you will use to measure its perimeter.
b Simply by looking at the item, estimate its perimeter.
c Use an appropriate device to measure the actual perimeter.

PRINTABLE TABLE

2 Compare your measurements with your classmates, and discuss the accuracy of your estimates.

C AREA

All around your school there are flat *surfaces* such as paths, floors, desk tops, ceilings, walls, and courts for playing sport.

Area is a measurement of the size of a surface.

In the metric system, the units we use for area are related to the units we use for length.

1 **square millimetre** (mm^2) is the area enclosed by a square of side length 1 mm.

The area of a computer chip might be measured in mm^2.

1 **square centimetre** (cm^2) is the area enclosed by a square of side length 1 cm.

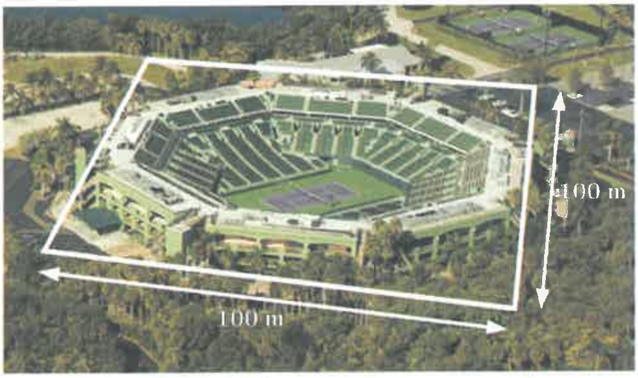
The area of a book cover might be measured in cm^2.

1 **square metre** (m^2) is the area enclosed by a square of side length 1 m.

The area of a brick paved courtyard might be measured in m^2.

1 **hectare** (ha) is the area enclosed by a square of side length 100 m.

The area of a park or a sports stadium might be measured in hectares.



Tennis stadium at Crandon Park, Miami, Florida, USA

1 **square kilometre** (km^2) is the area enclosed by a square of side length 1 km.

The area of a country or continent might be measured in km^2.

EXERCISE 12C

1 State the units that would be most appropriate for measuring the area of:

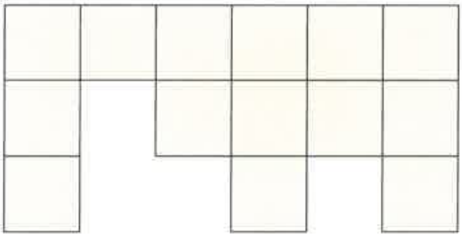
- **a** the floor of a room
- **b** a sheet of paper
- **c** Tanzania
- **d** a dot on a die
- **e** a farm.

2 Choose the correct answer:

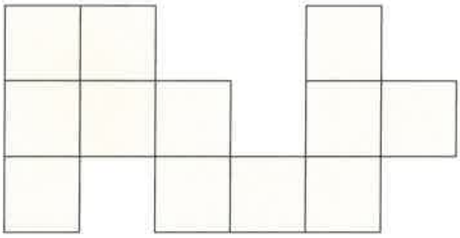
- **a** The area of a movie theatre screen is about:
  A $18\ m^2$ **B** $180\ m^2$ **C** 18 ha **D** 180 ha
- **b** The area of a doormat is about:
  A $20\ cm^2$ **B** $200\ cm^2$ **C** $2000\ cm^2$ **D** $2\ m^2$
- **c** The area of a 20 cent coin is about:
  A $65\ mm^2$ **B** $6.5\ cm^2$ **C** $65\ cm^2$ **D** $6.5\ m^2$

3 Each of the squares in these shapes has area 1 square centimetre. Count the squares to find the area of each shape.

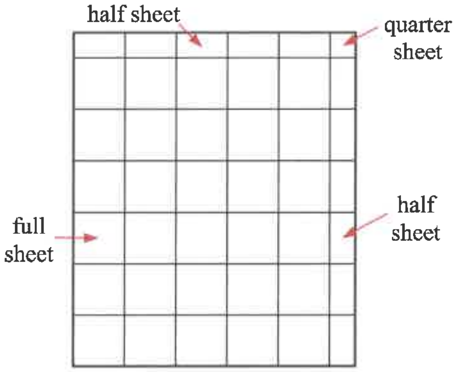
a



b



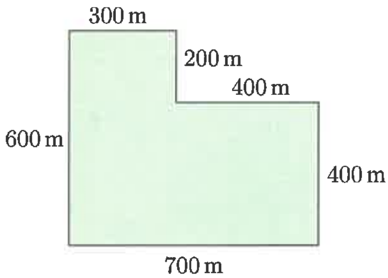
4 To find the area of their classroom, students covered the floor with 1 m by 1 m squares of paper.
Once they had used as many full sheets as possible, they used half sheets and finally one quarter sheet to finish covering the floor.
Find the area of the classroom.



5 Lucinda's farm has the dimensions shown.

a By dividing the farm into 100 m by 100 m squares, find the area of the farm in hectares.

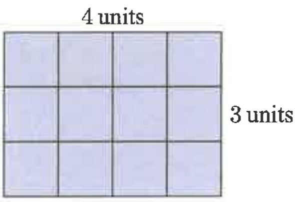
b Lucinda uses half of her farm to grow bamboo. It has an annual yield of 20 tonnes per hectare. How much bamboo does Lucinda's farm produce each year?



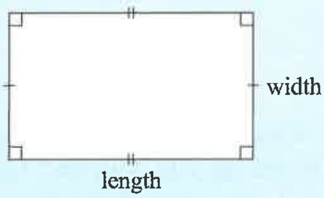
D THE AREA OF A RECTANGLE

Consider a rectangle 4 units long and 3 units wide.

The area of this rectangle is 12 units^2, and we can find this by multiplying $4 \times 3 = 12$.



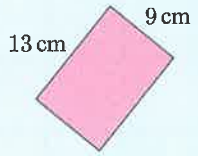
Area of rectangle = length × width



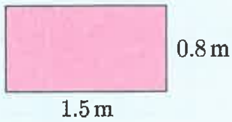
Example 3 **Self Tutor**

Find the area of each rectangle:

a



b



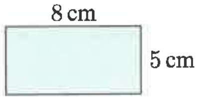
a Area
$= \text{length} \times \text{width}$
$= 13 \text{ cm} \times 9 \text{ cm}$
$= 117 \text{ cm}^2$

b Area
$= \text{length} \times \text{width}$
$= 1.5 \text{ m} \times 0.8 \text{ m}$
$= 1.2 \text{ m}^2$

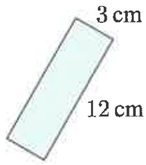
EXERCISE 12D

1 Find the area of each rectangle. Make sure you answer with the correct units.

a

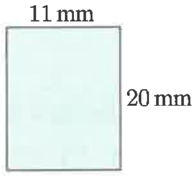


b



c

d



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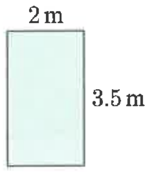

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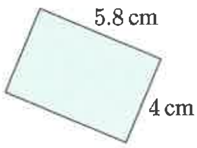
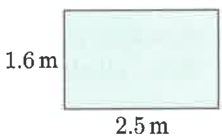
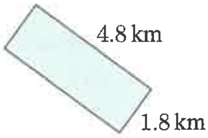


g

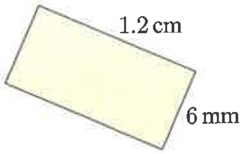


h

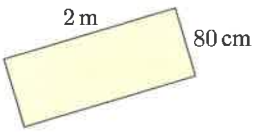


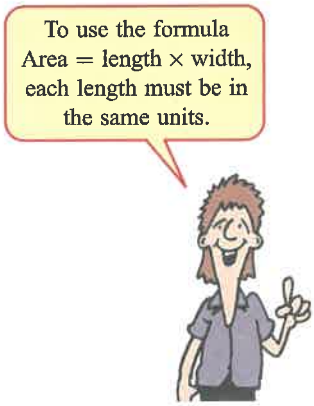
2 Find the area of each rectangle:

a

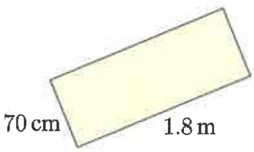


b





c



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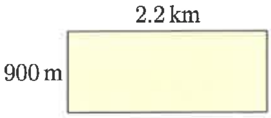


3 An office building has 10 large windows which are 2 m wide and 3 m high, and 5 small windows which are 1 m wide and 1.5 m high.
Find the total area of glass in the windows of the building.

4 A lawn bowls club has a green with dimensions 40 m by 60 m. 30 square metres can be mowed each minute.

a Find the area of the green. **b** How long will it take to mow the whole green?

5 A hallway is 9 m long and 1.8 m wide. It is to be covered in floorboards which are 1.5 m long and 15 cm wide. Each floorboard costs $21.50.
Find the:

a area of each floorboard in m^2 **b** area of the floor
c number of floorboards required **d** total cost of the floorboards.

Example 4 **Self Tutor**

A rectangle is 13 m long. Its area is 65 m^2. Find the width of the rectangle.

Let the width of the rectangle be x m.

$$\text{Area} = \text{length} \times \text{width}$$
$$\therefore\ 65 = 13 \times x$$
$$\therefore\ \frac{65}{13} = \frac{13x}{13} \quad \{\text{dividing both sides by } 13\}$$
$$\therefore\ x = 5$$

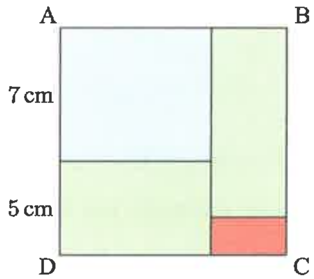
The width of the rectangle is 5 m.

6 A rectangle is 8 cm long. Its area is 48 cm^2. Find the width of the rectangle.

7 A rectangle has area 84 m^2. It is 7 m wide. Find the length of the rectangle.

8 According to the local council rules, advertising signs on a street must be no larger than 12 m^2. Neville wants his sign to be 2.5 m high. What is the maximum length his sign can be?

9 In the square ABCD, the green rectangles have the same area and the blue rectangle has area 56 cm^2. Find the area of the red rectangle.

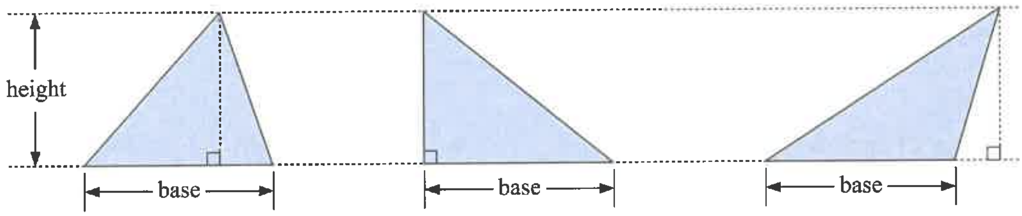


GLOBAL CONTEXT SHIKAKU PUZZLES

Global context: Scientific and technical innovation
Statement of inquiry: Solving mathematical puzzles can help us to better understand mathematical concepts.
Criterion: Investigating patterns

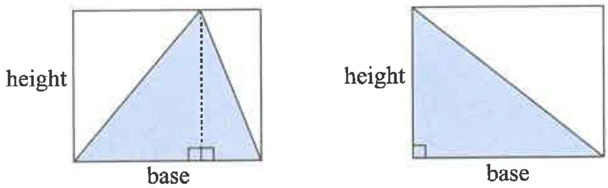
GLOBAL CONTEXT

E THE AREA OF A TRIANGLE



Area of triangle $= \frac{1}{2} \times$ **base** $\times$ **height**

The first two cases are demonstrated by drawing a rectangle with the same base and height as the triangle.

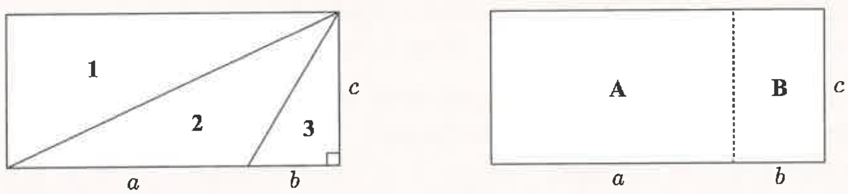


$$\text{Area} = \frac{1}{2} \text{ of area of rectangle}$$
$$= \frac{1}{2} \times \text{base} \times \text{height}$$

We prove the third case in the **Activity** below.

ACTIVITY 3 AREA OF A TRIANGLE

Below are two rectangles which each have length $(a+b)$ and height c.



The first rectangle has been divided into three triangles.

The second rectangle has been divided into two smaller rectangles **A** and **B**.

What to do:

1 In terms of a, b, and c, write down formulae for:

a area **A** **b** area **B** **c** area **3**

2 Copy and complete:
Using the first rectangle, area **1** = area **2** + area **3**
Comparing rectangles, area **1** + area **2** + area **3** = area **A** + area **B**
$\therefore$ 2 × area **2** + 2 × area **3** = area **A** + area **B**
$\therefore$ 2 × area **2** = area **A** + area **B** − 2 × area **3**

Using the formulae in **1**, $2 \times \text{area } \mathbf{2} = \ldots\ldots + \ldots\ldots - 2 \times \ldots\ldots$

$$= \ldots\ldots$$

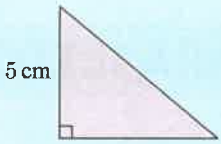
$$\therefore \quad \text{area } \mathbf{2} = \frac{1}{2} \times \ldots\ldots$$

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

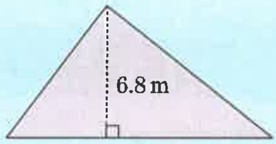
Example 5 **Self Tutor**

Find the area of each triangle:

a



b



a $\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 6 \text{ cm} \times 5 \text{ cm}$

$= 15 \text{ cm}^2$

b $\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$

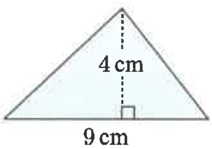
$= \frac{1}{2} \times 10 \text{ m} \times 6.8 \text{ m}$

$= 34 \text{ m}^2$

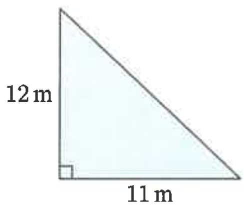
EXERCISE 12E

1 Find the area of each triangle:

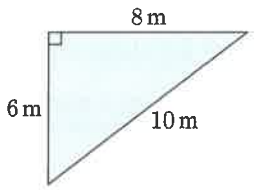
a



b



c



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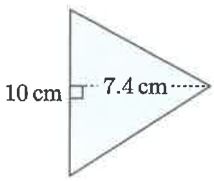

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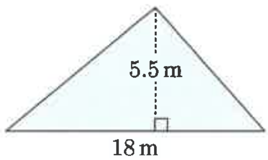

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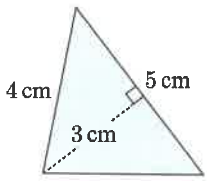

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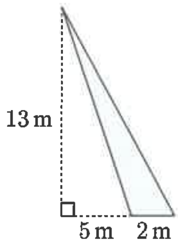

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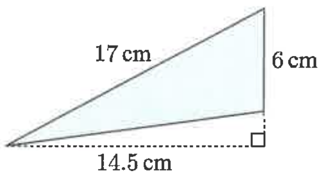
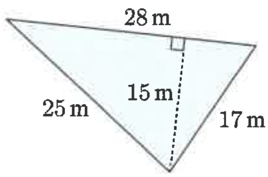
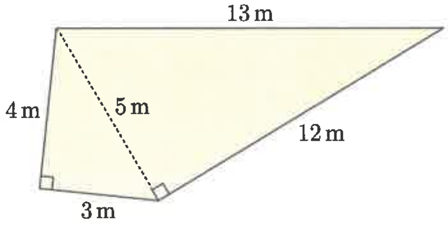


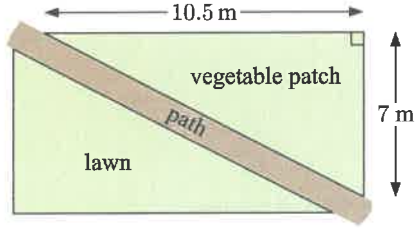
i



2 Find the area of this quadrilateral.

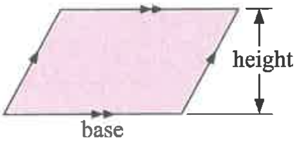


3 A path cuts across a backyard as shown. Find the area of the vegetable patch.



4 A triangle with base length 5 cm has area 15 cm^2. Find the height of the triangle.

F THE AREA OF A PARALLELOGRAM



Area of parallelogram = base × height

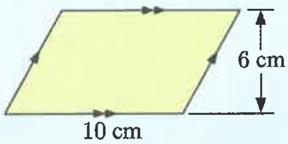
We can demonstrate this formula by cutting out a triangle from one end of the parallelogram and shifting it to the other end. The resulting shape is a rectangle with the same base and height as the parallelogram.

DEMO

Perform this demonstration for yourself using paper and scissors.

Example 6 — Self Tutor

Find the area of this parallelogram:

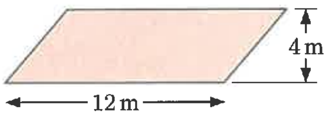


Area = base × height
$= 10 \text{ cm} \times 6 \text{ cm}$
$= 60 \text{ cm}^2$

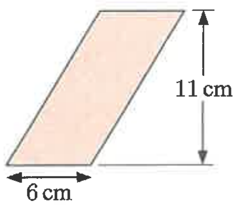
EXERCISE 12F

1 Find the area of each parallelogram:

a



b



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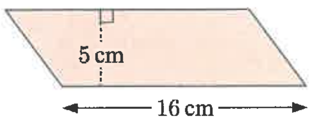

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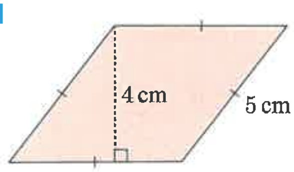

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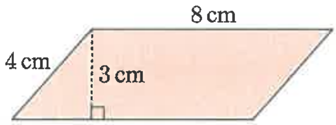

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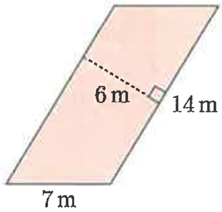

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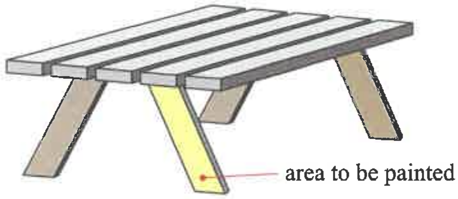
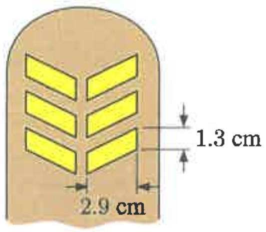
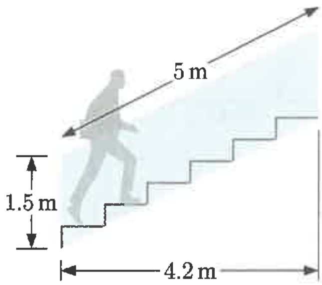


The surface of the park bench leg indicated must be repainted due to graffiti. The leg is 12 cm wide at the base, and the top of the leg is 60 cm above the ground. Find the area that needs to be repainted.

3 A soldier's uniform has these stripes on *both* shoulders.
Find the total area of the stripes on the uniform.



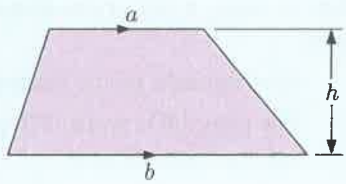
4 A perspex safety guard for one side of a staircase is to be made with the dimensions shown. Find the area of perspex required.



G THE AREA OF A TRAPEZIUM

Area of trapezium = average length of parallel sides
× distance between parallel sides

$$= \left(\frac{a+b}{2}\right) \times h$$



We can demonstrate this result using a second identical trapezium.

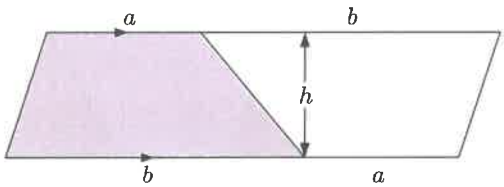
DEMO

We place the trapezia together to form a parallelogram.

$\text{Area} = \frac{1}{2}$ of area of parallelogram

$= \frac{1}{2} \times \text{base} \times \text{height}$

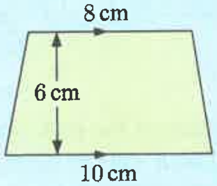
$= \frac{1}{2} \times (a+b) \times h$



Perform this demonstration for yourself using paper and scissors.

Example 7 — Self Tutor

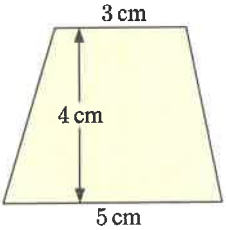
Find the area of this trapezium:

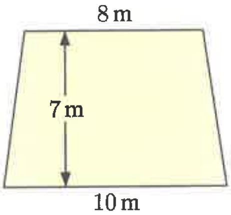


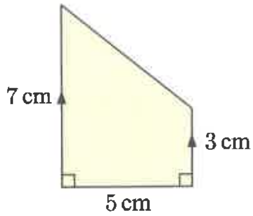
$$\text{Area} = \left(\frac{a+b}{2}\right) \times h$$
$$= \left(\frac{8+10}{2}\right) \times 6 \text{ cm}^2$$
$$= 9 \times 6 \text{ cm}^2$$
$$= 54 \text{ cm}^2$$

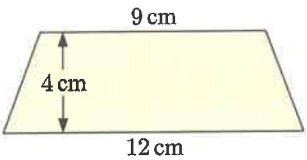
EXERCISE 12G

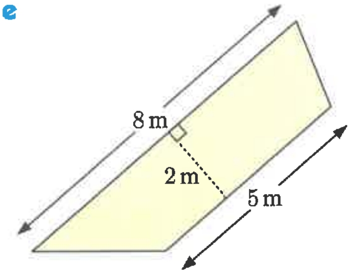
1 Find the area of each trapezium:

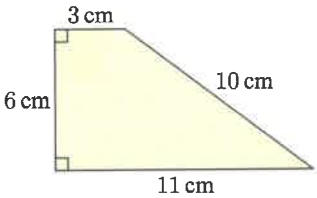
a


b


c


d


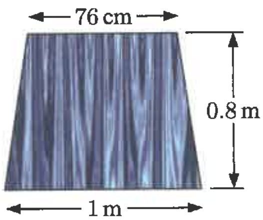
e


f


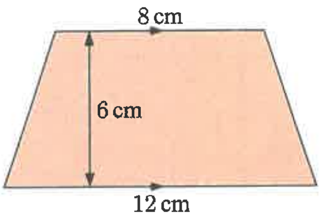
2 Joan is using a trapezium-shaped piece of material to make a skirt.

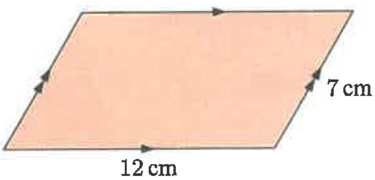
a Find the area of the material in m^2.

b The material is worth \$16 per square metre. Find the total value of the material.



3 These figures have the same area. Find the height of the parallelogram.





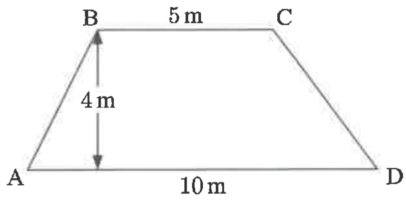
4 **a** Find the area of trapezium ABCD.

b Draw in the line segment [BD].
We divide the trapezium into triangles ABD and BCD.

i Find the area of triangle ABD.

ii Find the area of triangle BCD.

iii Verify that the sum of the areas of the triangles is equal to the area of the trapezium.



ACTIVITY 4 AREAS OF IRREGULAR SHAPES

In this Activity we estimate the areas of irregular shapes which are bounded by curves.

ACTIVITY

QUICK QUIZ

MULTIPLE CHOICE QUIZ

REVIEW SET 12A

1 Convert:

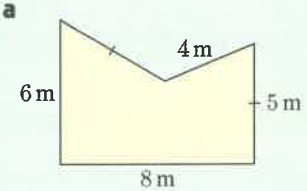
a 12.9 cm into mm **b** 3.95 km into m **c** 2.43 m into cm

d 57 mm into cm **e** 1140 cm into m **f** 640 m into km.

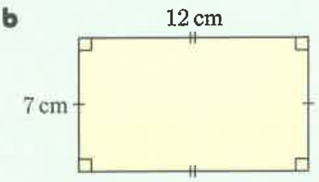
2 Find, in metres, the sum of 2 km, 510 m, and 190 cm.

3 Find the perimeter of each figure:

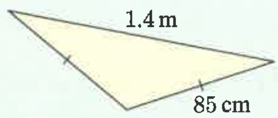
a



b



c



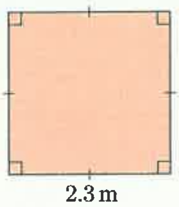
4 A 360 m length of wire is cut into equal lengths, which are then bent into equilateral triangles with side length 15 cm. How many triangles can be made?

5 State the units that would be most appropriate for measuring the area of:

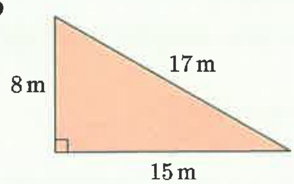
a a gymnasium floor **b** India **c** a postcard.

6 Find the area of each polygon:

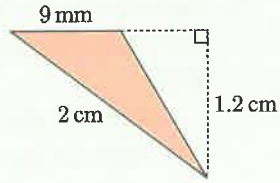
a



b



c



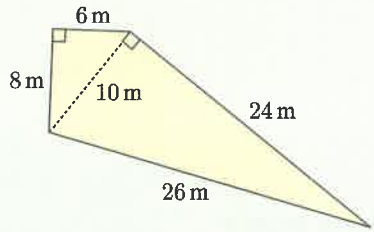
7 A driveway is 6 m long and 2 m wide. It is to be covered with concrete slabs that are 1.5 m long and 50 cm wide.

a Find the area of each slab, in square metres.

b Find the area of the driveway.

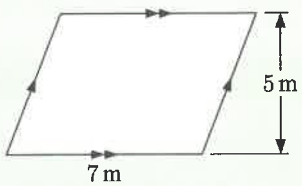
c How many slabs are required?

8 Find the area of this quadrilateral.

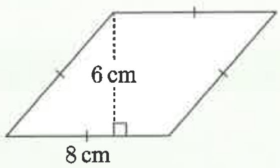


9 Find the area of each polygon:

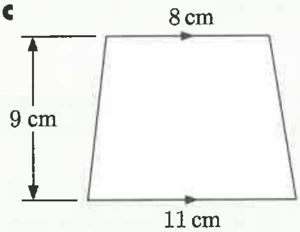
a



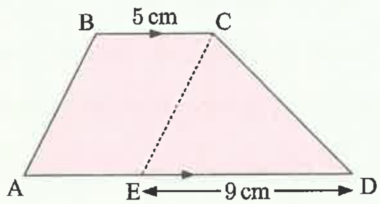
b



c



10 Parallelogram ABCE has area 30 cm^2. Find the area of trapezium ABCD.



11 **a** A brand of toilet paper is sold in rolls of 190 sheets. Each sheet of toilet paper is 11 cm long. How many *metres* of toilet paper are in:

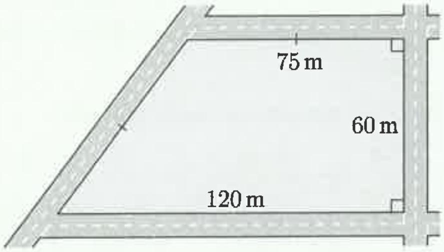
i one roll **ii** an 8-roll pack?

b The company brings out a new "long roll" of toilet paper. Each roll has 255 sheets, but only 6-roll packs are sold.

i If each sheet of toilet paper in the new roll is still 11 cm long, how many metres of toilet paper are in the 6-roll pack?

ii How many *more* metres are in this new pack than in the old 8-roll pack?

12 A city council is putting a new concrete kerb around the block shown. The kerb costs \$45 per metre.



- **a** How many metres of kerb will need to be laid?
- **b** Find the total cost of the kerb.
- **c** The council wishes to put small trees along the kerb. The trees need to be spaced 3 m apart, to allow for driveways.
  - **i** How many trees are needed to go around the whole block?
  - **ii** The trees cost \$50 each. How much will the council spend on trees?
- **d** Find the area of the block.

REVIEW SET 12B

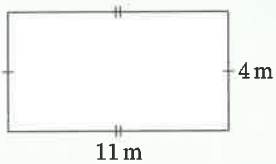
1 State the unit of length that would be most appropriate for measuring:

- **a** the width of a street
- **b** the length of an eraser.

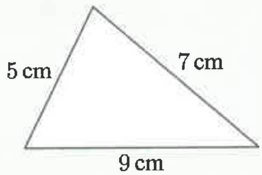
2 Grant competed in the 100 m, 200 m, 400 m, and 1500 m freestyle events during a swim meet. How many kilometres did he swim in total?

3 Find the perimeter of each figure:

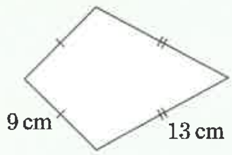
a



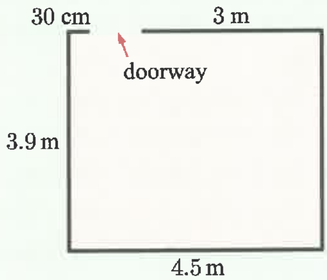
b



c



4 Adam wants to place skirting board along the bottom of the walls in his room. How many metres of board does Adam need?

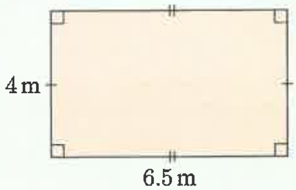


5 The area of a bathroom mirror is about:

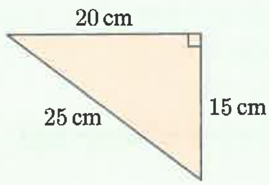
A 25 cm^2 **B** 250 cm^2 **C** 2500 cm^2 **D** 25 m^2

6 Find the area of each polygon:

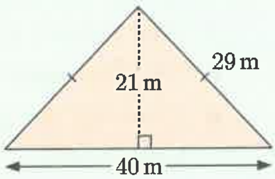
a



b



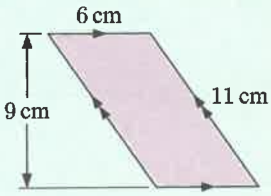
c



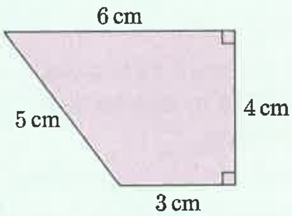
7 A soccer pitch has dimensions 100 m by 75 m. 1 kg of fertiliser covers 10 square metres of the pitch, and the fertiliser costs \$15 for a 30 kg bag. Find the cost of fertilising the pitch.

8 Find the area of each polygon:

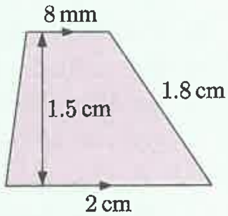
a



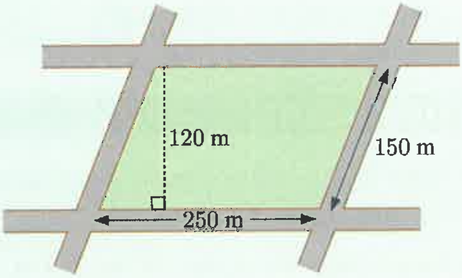
b



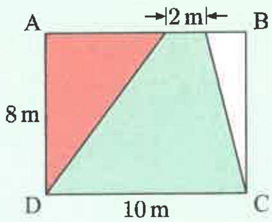
c



9 A park is surrounded by two sets of parallel roads as illustrated.
Find the area of the park.

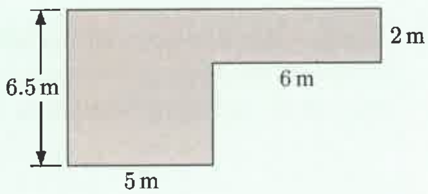


10



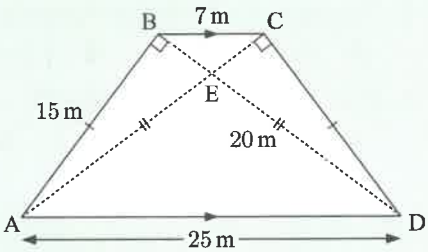
In rectangle ABCD, the green trapezium has twice the area of the red triangle.
Find the area of the white triangle.

11 Toby is landscaping his backyard. He needs to pave the area shown with 25 cm by 25 cm square pavers.



a By dividing the area into two rectangles, find the total area to be paved.
b Find the area of one paver, in square *metres*.
c How many pavers will Toby need?
d The pavers cost \$3.50 each. Find the total cost of the pavers.

12 In trapezium ABCD, ABD and ACD are right angled triangles.



a Find the area of triangle ABD.
b Hence find the height of the trapezium.
c Find the area of the trapezium.
d Find the area of triangle ABC.
e Show that triangle AED is 108 m^2 larger than triangle BEC.

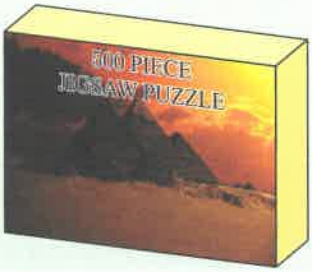
Chapter 13

Solids

Contents:

OPENING PROBLEM

Look at this diagram of a jigsaw puzzle box.



Things to think about:

a What object does the diagram represent?

b Is the *object* 2-dimensional or 3-dimensional?

c Is the *diagram* 2-dimensional or 3-dimensional?

d Can you draw a 2-dimensional shape which could be folded to create this object?

A SOLIDS

A **solid** is a three-dimensional object which occupies space.

Each solid has the three dimensions width, height, and depth.

The boundaries of a solid are called **surfaces**.

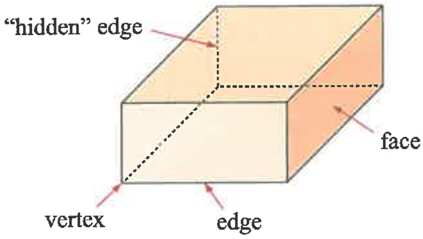
Solids may have flat surfaces, curved surfaces, or a combination of both.

Each flat surface of a solid is called a **face**.

The diagram alongside shows a solid.

Notice that:

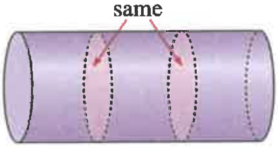
- an **edge** is where two surfaces meet
- a **vertex** is a "corner" of the solid
- we can use dashed lines to show "hidden" edges which are at the back of the solid, so we understand the solid is three-dimensional.



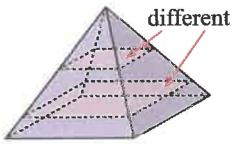
CROSS-SECTIONS OF SOLIDS

A **cross-section** of a solid is the shape of a slice through it.

For some solids, when we make a series of parallel slices along its length, the cross-section is always the same. These solids are called **solids of uniform cross-section**.



uniform cross-section

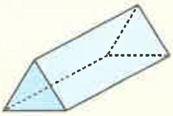
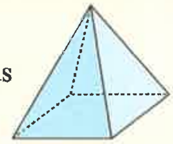
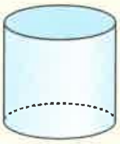
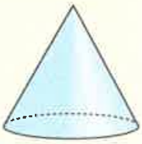


not uniform cross-section

CLASSIFYING SOLIDS

We classify solids according to their surfaces and their cross-section.

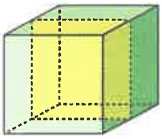
This table shows how the common families of solids are classified:

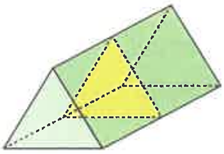
	Uniform cross-section	**Tapered solids**	**Other solids**
All flat surfaces	prisms 	pyramids 	
At least one curved surface	cylinders 	cones 	spheres 

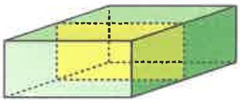
PRISMS

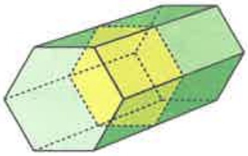
A **prism** is a solid with a uniform cross-section that is a polygon.

Each end has the shape of the cross-section. The remaining faces are all rectangles.


cube

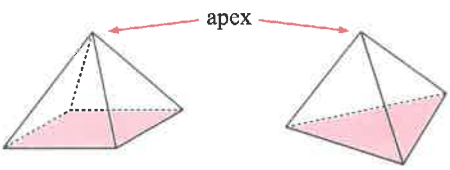

triangular prism


rectangular prism


hexagonal prism

PYRAMIDS

A **pyramid** is a solid with a polygon base, and triangular faces which come from its base to meet at a point called the **apex**.

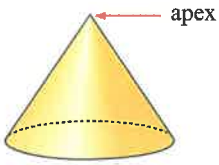


square-based pyramid

triangular-based pyramid or tetrahedron

CONES

A **cone** is a solid with a circular base and a curved surface from the base to the apex.

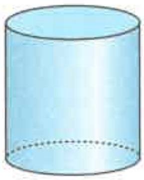


DISCUSSION

Why are pyramids and cones called *tapered* solids?

CYLINDERS

A **cylinder** is a solid with a uniform cross-section that is a circle.



SPHERES

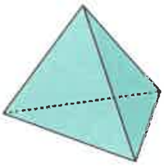
A **sphere** is a ball-shaped solid.

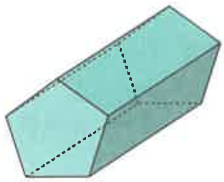
A sphere has no edges, but we often draw an "equator" around it to distinguish our drawing from a circle.

EXERCISE 13A

1 Name each solid:

a 

b 

c 

2 Draw:

a a cone **b** a rectangular-based pyramid **c** a sphere

d an octagonal prism **e** a hexagonal-based pyramid.

3 Name the solid which best resembles:

a a can of soup **b** a marble **c** a cereal box

d a witch's hat **e** a four-sided die **f** a coin.

4 What shape are the side faces of a:

a prism **b** pyramid?

5 Draw a solid which has:

a only a curved surface **b** a curved and a flat surface

c two flat surfaces and one curved surface.

DISCUSSION

How can we *define* a sphere?

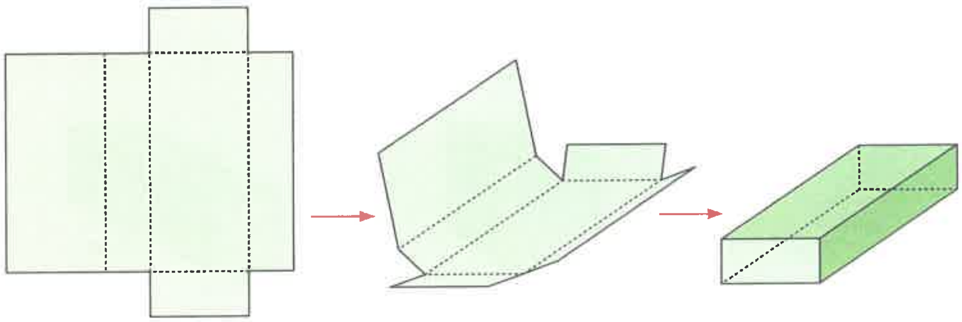
Discuss whether either of these statements is an appropriate definition:

- "a solid whose cross-section is a circle no matter which direction it is cut"
- "the set of points which are the same distance from a point called the centre of the sphere".

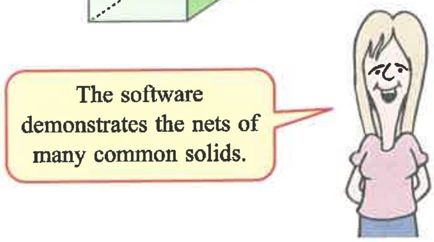
B NETS OF SOLIDS

A **net** is a two-dimensional pattern which can be folded to form a three-dimensional solid.

For example, when this net is cut out and folded along the dotted lines, we form a rectangular prism.

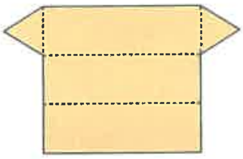


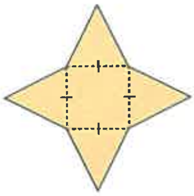
DEMO

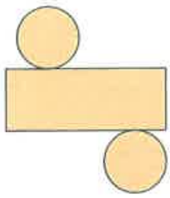


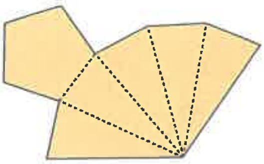
EXERCISE 13B

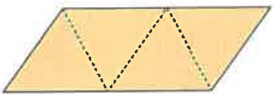
1 Draw and name the solid corresponding to each net:

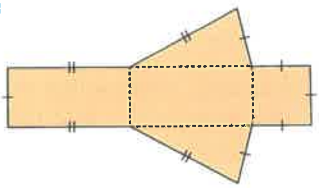
a 

b 

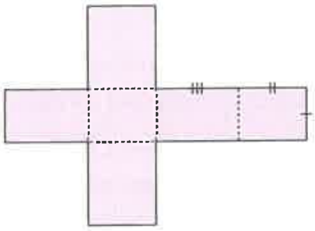
c 

d 

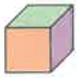
e 

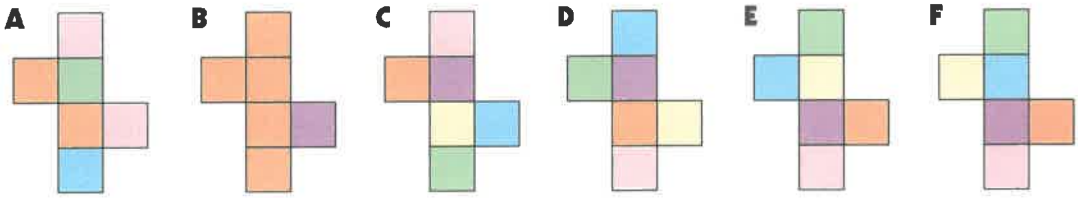
f 

2 Copy this net of a rectangular prism. Place tick marks on the remaining lines to indicate the edges of equal length.



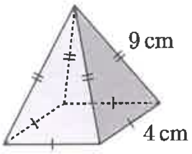
3 Which of the following nets can be used to make this cube?





4 Draw a net for each solid, clearly marking the side lengths:

a



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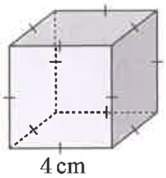

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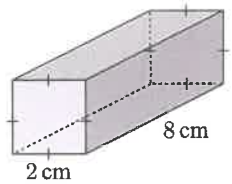
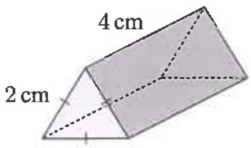
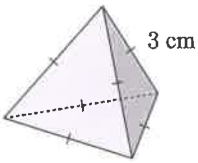


d



e



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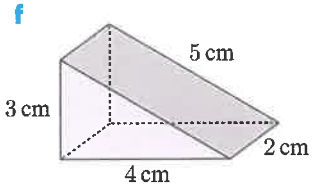
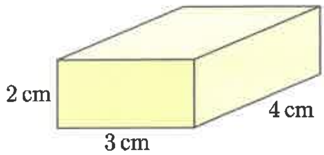


5 **a** Draw a net for this prism, clearly marking the side lengths.

b Find the total area of the net.

c What does this tell you about the total area of the surfaces of the solid?

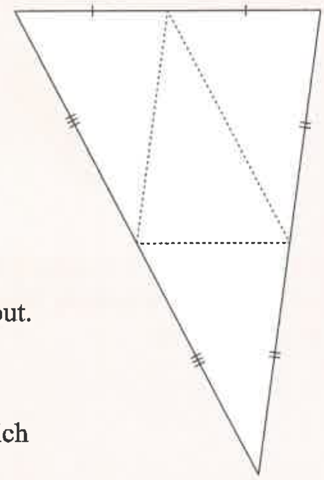


ACTIVITY 1 TETRAHEDRA

The triangle shown is acute angled and scalene.

For this triangle to be the net of a tetrahedron, the fold lines must join the midpoints of the triangle's sides.

PRINTABLE NET



What to do:

1 Print the net and fold it into a tetrahedron.

2 Draw a large acute angled triangle of your own and cut it out. Locate the midpoint of each side, then draw the fold lines. Fold the triangle into a tetrahedron.

3 Try using this method to form a tetrahedron from a net which is a right angled triangle. What goes wrong?

C OBLIQUE AND ISOMETRIC PROJECTIONS

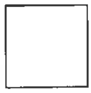
We have previously seen how to draw three-dimensional solids on paper.

In this Section we look more carefully at methods for drawing solids which can be built out of cubic blocks.

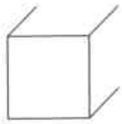
These methods are called **projections** because we *project* the image of the three-dimensional solid onto the two-dimensional paper.

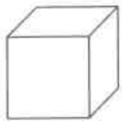
OBLIQUE PROJECTIONS

In this method we draw the front face of the object first. It is drawn in the plane of the page, with width across the page and height up the page.

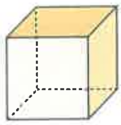


We then represent the *depth* of the object using diagonal or *oblique* lines at 45°. The oblique edges are drawn shorter than the edges on the front face.





We complete the frame of the solid, adding dashed lines to indicate hidden edges if appropriate.

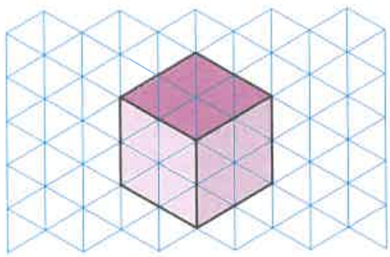


ISOMETRIC PROJECTIONS

To draw an **isometric projection** we use **isometric graph paper** made up of equilateral triangles.

We start with a vertical edge of the solid. The horizontal edges are drawn inclined at 30°.

Alongside is the isometric projection of a cube. Notice that all the edges drawn have the same length.



ISOMETRIC GRAPH PAPER

EXERCISE 13C

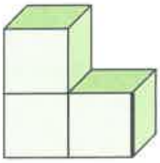
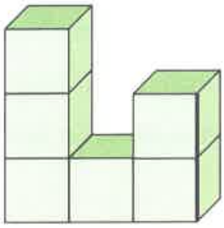
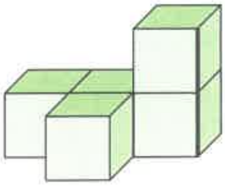
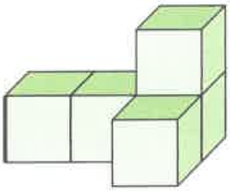
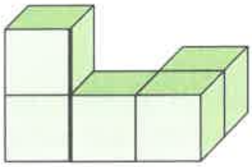
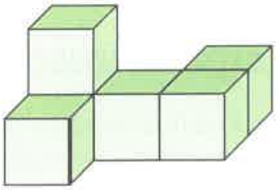
1 Draw an oblique projection of:

a a cube with side length 3 units

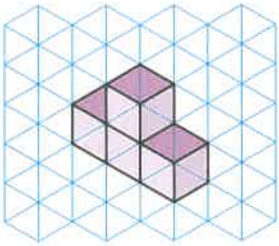
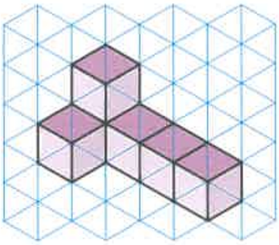
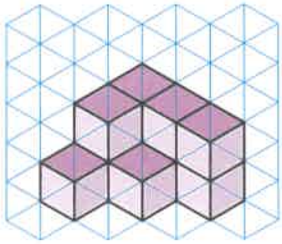
b a rectangular prism 2 units wide, 1 unit high, and 2 units deep

c a rectangular prism 3 units wide, 2 units high, and 1 unit deep.

2 Draw each solid on isometric paper. In each case, start with the thicker edge.

a 
b 
c 
d 
e 
f 

3 Draw each solid as an oblique projection:

a 
b 
c 

DISCUSSION

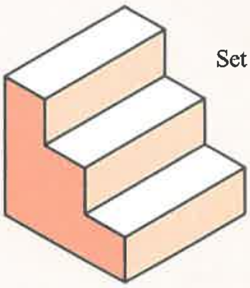
Which form of projection do you find most helpful for visualising the solid?

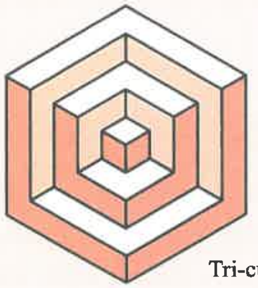
ACTIVITY 2

What to do:

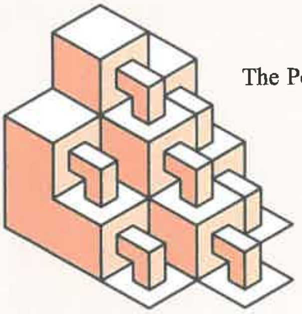
1 Use isometric paper to draw each figure:

ISOMETRIC GRAPH PAPER

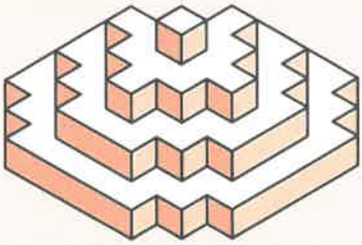
a 
Set of steps

b 
Tri-cube

c



d



2 Use an oblique projection to draw:

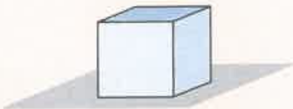
a the set of steps

b the tri-cube.

ACTIVITY 3 HUMBLE HOUSES

The Humble House Factory manufactures cubic living quarters for countries with hot climates. Heat enters every roof and exposed wall at the same rate.

For a house made of **one cube**, heat enters in equal amounts from 5 sides, but not the floor.



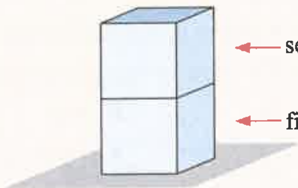
When buildings are made with more than one cube, the cubes must touch face to face, and no cube can have empty space beneath it.

What to do:

1 There are two possible building designs made from two cubes:

A

B



Count the exposed faces in each design. Hence identify which design is "best" for a hot climate.

2 Draw the *four* possible building designs made from three cubes. Identify the "best" design.

3 Investigate the possible designs for four cubes, and identify the "best" design.

4 Investigate the possible designs for five cubes, and identify the "best" design.

5 Write some general conclusions about how these buildings should be designed to minimise the amount of heat coming in.

6 Predict which design will be "best" for a building made from these numbers of cubes. Explain your choices.

a 64 **b** 62 **c** 60

ACTIVITY 4 VIEWS OF SOLIDS

When drawing a three-dimensional solid, we cannot show the details on all of the faces at the same time, because many of the faces are hidden from view.

Instead, we can make several drawings of the solid from different angles. We normally draw the solid as it appears from the **front**, **top**, **left**, **right**, and **back**.

What to do:

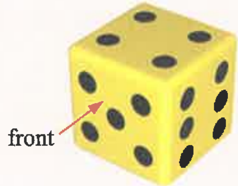
1 The numbers on a die are arranged so that the sum of each pair of opposite faces is seven.

a Which number is on the bottom of this die?

b Sketch the view of this die from the:

i front **ii** top **iii** right

iv left **v** back.

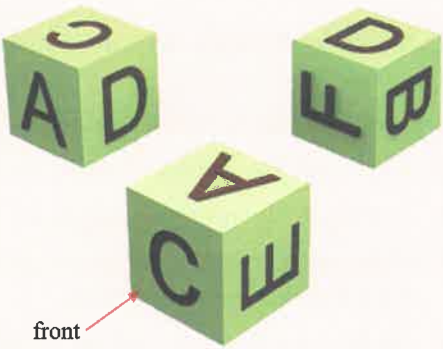


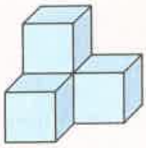
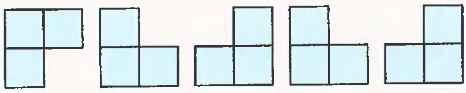
2 A cube has the letters A, B, C, D, E, and F painted on its faces. Three different views of the cube are shown alongside.

Using the letter C as the front face, draw the view of the cube from the:

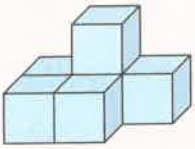
a top **b** right

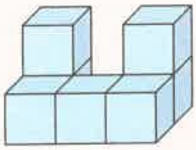
c left **d** back.

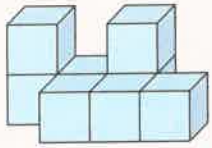


3 The solid  has views: 

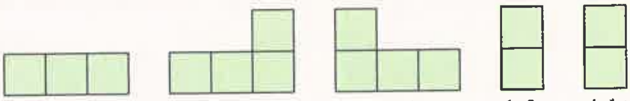
Draw the views of each solid:

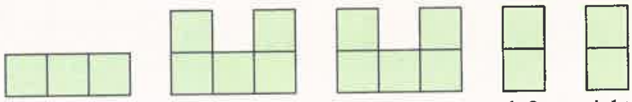
a 

b 

c 

4 Draw the solid with views:

a 

b 

5 Isla has a wooden jewellery box.

a Draw the top, front, back, left, and right views of the box.

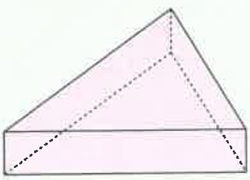
b Draw a net which could be folded to form the box. Make sure the lid can be opened!

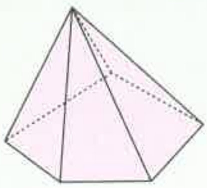
QUICK QUIZ

MULTIPLE CHOICE QUIZ

REVIEW SET 13A

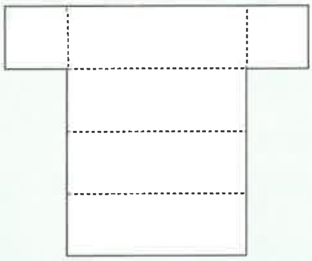
1 Name each solid:

a 

b 

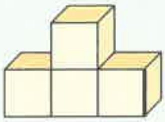
2 Draw a net for a triangular-based pyramid.

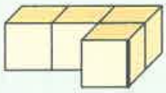
3 Name the solid corresponding to this net.

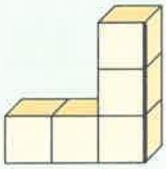


4 Draw an oblique projection of a rectangular prism which is 5 cm wide, 3 cm high, and 2 cm deep.

5 Draw each solid as an isometric projection. In each case, start with the thicker edge.

a 

b 

c 

REVIEW SET 13B

1 Draw:

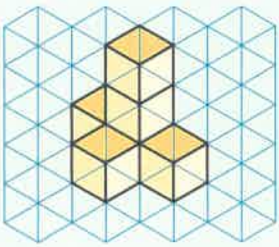
a a cylinder

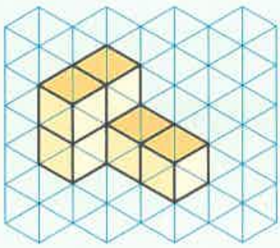
b a square-based pyramid.

2 Draw a net for a 5 cm by 3 cm by 1 cm rectangular prism.

3 Name the solid which best resembles a six-sided die.

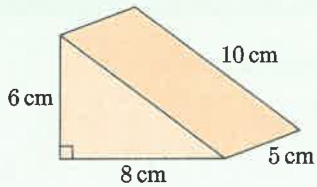
4 Draw each solid as an oblique projection.

a 

b 

5 **a** Draw the net for this triangular prism, clearly marking the side lengths.

b Find the total area of the net.



Chapter 14

Measurement: Volume, capacity, and mass

Contents:

OPENING PROBLEM

Chun's roof is leaking. 10 mL of water is dripping onto her floor every minute. She places a 30 cm by 20 cm by 10 cm container under the leak to catch the drops.

Things to think about:

a What volume of air is inside the container when it is empty?

b How much water can the container hold?

c How long will it take for the container to fill?

d What is the mass of the water in the container when it is full?

In this Chapter we complete our study of measurement by looking at **volume**, **capacity**, and **mass**.

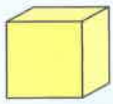
A VOLUME

The **volume** of a solid is the amount of space it occupies.

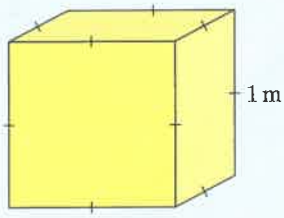
As with area, the units of volume are related to the units of length.

1 cubic millimetre (mm^3) is the volume of a cube with side length 1 mm.

1 cubic centimetre (cm^3) is the volume of a cube with side length 1 cm.



1 cubic metre (m^3) is the volume of a cube with side length 1 m.



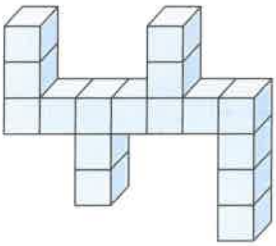
For example:

- the volume of a small marble might be measured in mm^3
- the volume of an engine cylinder might be measured in cm^3
- the volume of rock mined from a quarry might be measured in m^3.

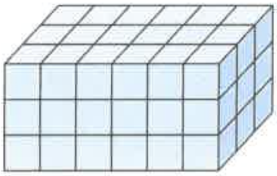
EXERCISE 14A

1 Each of the cubes in these solids has volume 1 cm^3. Find the volume of each solid.

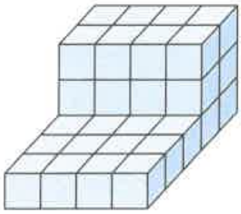
a



b



c



2 Using cubes with volume 1 cm^3, sketch a solid with volume:

a 9 cm^3 **b** 13 cm^3 **c** 24 cm^3

3 State the units you would use to measure the volume of:

a a textbook **b** a truck **c** a pea

d a mobile phone **e** an elephant **f** a plant seed.

4 The volume of an apple is about:

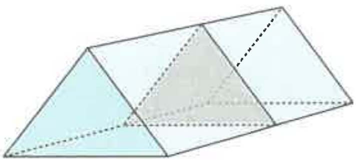
A 15 cm^3 **B** 150 mm^3 **C** 150 cm^3 **D** 1.5 m^3

5 The volume of a wardrobe is about:

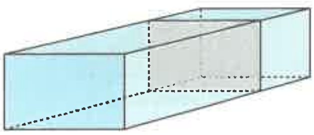
A 8 m^3 **B** 800 cm^3 **C** 80 cm^3 **D** 0.8 m^3

B THE VOLUME OF A PRISM

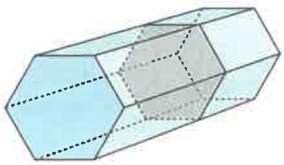
We have seen that a **prism** is a solid with a uniform cross-section that is a polygon.



triangular prism



rectangular prism



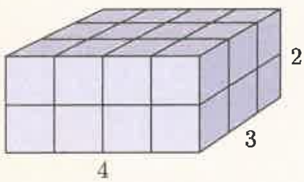
hexagonal prism

RECTANGULAR PRISMS

DISCUSSION

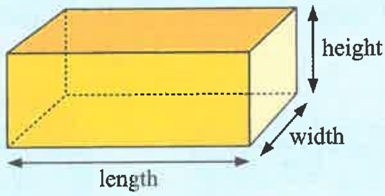
A 4 by 3 by 2 rectangular prism is shown alongside.

Discuss whether you agree with these statements:



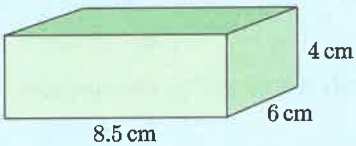
- There are $4 \times 3 = 12$ cubes in the top layer of blocks.
- There are 2 layers.
- There are $12 \times 2 = 24$ cubes in total.
- The volume of this rectangular prism is 24 units^3.
- The volume can be found by the multiplication length $\times$ width $\times$ height.

Volume of a rectangular prism
= length × width × height



Example 1 — Self Tutor

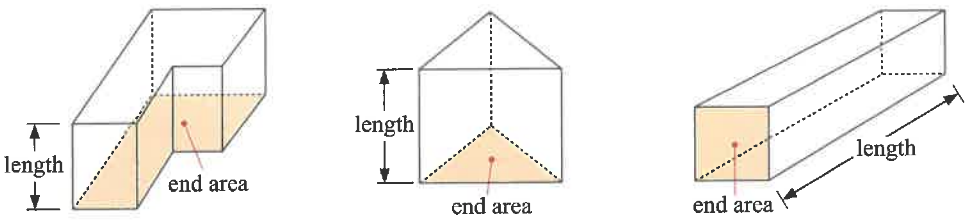
Find the volume of this rectangular prism:



$$\begin{aligned}\text{Volume} &= \text{length} \times \text{width} \times \text{height} \\ &= 8.5 \text{ cm} \times 6 \text{ cm} \times 4 \text{ cm} \\ &= 204 \text{ cm}^3\end{aligned}$$

OTHER PRISMS

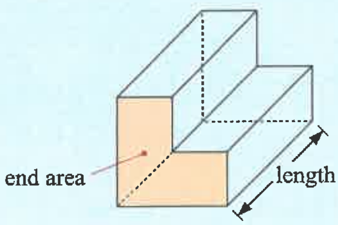
All prisms contain two identical end faces which are connected by straight edges.

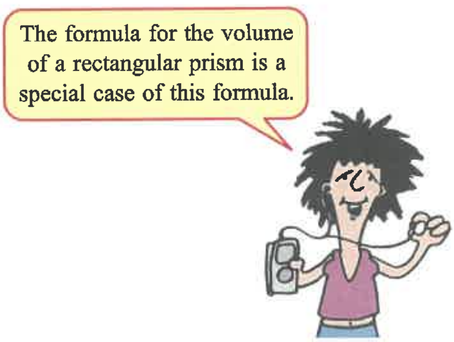


For any slice we make along the length of a prism, the *cross-section* of the prism is the same as the end.

To find the volume of a prism, we multiply the area of the end by the length of the prism.

Volume of a prism = area of end × length

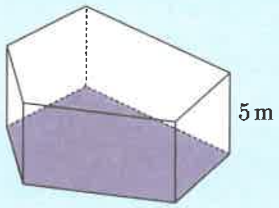




Example 2 **Self Tutor**

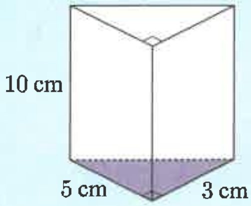
Find the volume of each prism:

a



area of end $= 20\text{ m}^2$

b



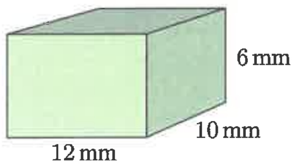
a Volume $=$ area of end $\times$ length
$= 20 \times 5\text{ m}^3$
$= 100\text{ m}^3$

b Volume
$=$ area of end $\times$ length
$= \left(\frac{1}{2} \times 3 \times 5\right) \times 10\text{ cm}^3$
{area of triangle formula}
$= 7.5 \times 10\text{ cm}^3$
$= 75\text{ cm}^3$

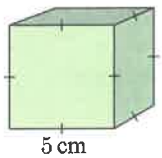
EXERCISE 14B

1 Find the volume of each rectangular prism:

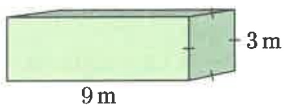
a



b

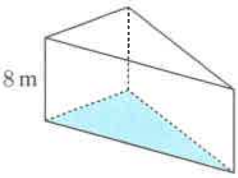


c



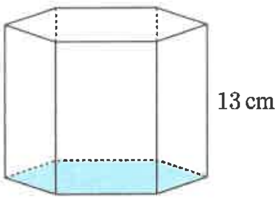
2 Find the volume of each prism:

a



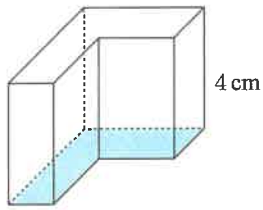
area of end $= 14\text{ m}^2$

b



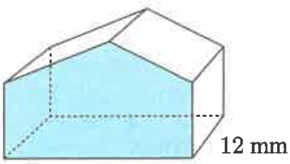
area of end $= 60\text{ cm}^2$

c



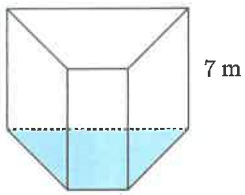
area of end $= 11\text{ cm}^2$

d



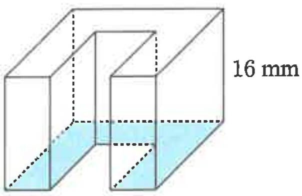
area of end $= 16\text{ mm}^2$

e



area of end $= 17\text{ m}^2$

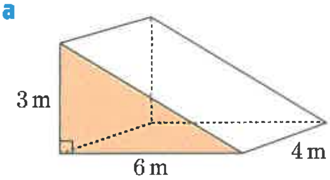
f

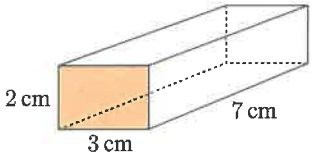


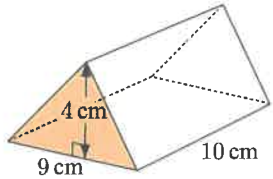
area of end $= 4\text{ cm}^2$

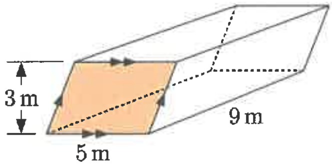
3 An empty classroom has floor area 56 m^2 and ceiling height 3 m. Find the volume of air in the classroom.

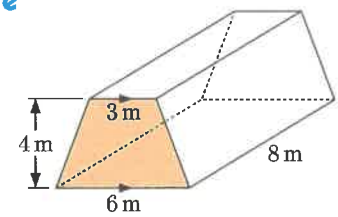
4 Find the volume of each prism:

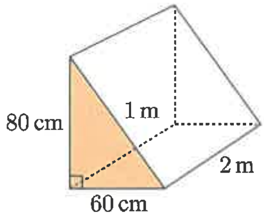
a


b


c


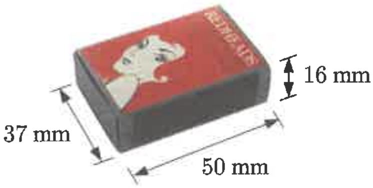
d


e


f


5 Find the volume of:

a the match box

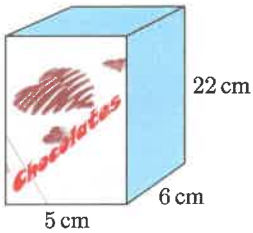


b the skip bin.

6 Which chocolate box has the larger volume?

A

B

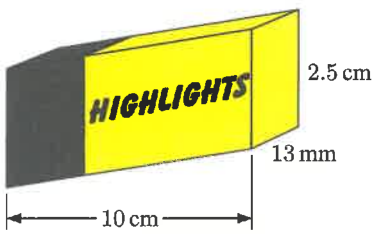


7 Find the volume of a cube with side length 2.5 cm.

8 Find the length of a prism with end area 38.5 cm^2 and volume 308 cm^3.

9 Find the volume of air in a box which is 1.5 m long, 1.2 m wide, and 80 cm high.

10 A highlighter has the dimensions shown.
Find the volume of the highlighter.



11 Mulch costs \$58 per cubic metre. How much would it cost to lay mulch in a garden 13 m long and 4.5 m wide to a depth of 8 cm?

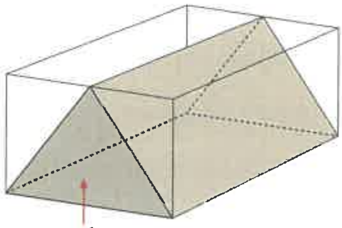
12 A rectangular container is 10 cm long and 6 cm wide. Its volume is 240 cm^3. Find the height of the container.

13 A wooden block is 60 cm long, 10 cm wide, and 8 cm high. A wedge in the shape of a triangular prism is cut from the block as shown.

a Find the volume of the original block.

b Find the volume of the wedge.

c What fraction of the block was used to make the wedge?



ACTIVITY — LUGGAGE RESTRICTIONS

Airlines frequently place restrictions on the size of luggage which passengers can carry with them onto an aircraft.

Suppose an airline has the policy that:

"*The sum of the length, width, and height of the luggage must not exceed* 90 *cm.*"

Your task is to determine the rectangular prism of largest volume which is allowed to be taken on the plane.

What to do:

1 **a** Copy and complete the following table for a rectangular prism where the sum of the length, width, and height is 90 cm. Add your own choices of dimensions for the second half of the table.

Length	*Width*	*Height*	*Volume*
10	20	60	12 000
10	30		
10	40		
20	20		
20	25		
20	30		
20	35		

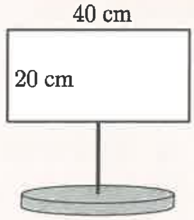
Length	*Width*	*Height*	*Volume*

b What do you suspect are the dimensions of the rectangular prism of greatest volume?

2 The airline realises that their policy is insufficient. To ensure that all luggage will fit in the overhead lockers, it introduces a further restriction that:

"*All luggage must pass through a* 40 *cm by* 20 *cm rectangle.*"

Find the rectangular prism of largest volume given this extra restriction.



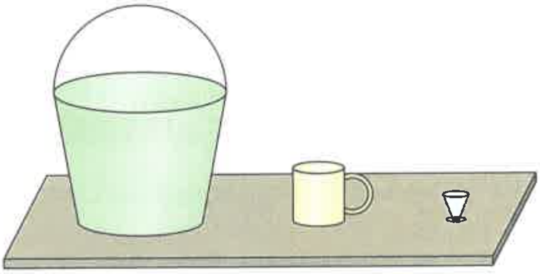
C CAPACITY

The **capacity** of a container is the amount of fluid it can contain.

The basic unit of capacity is the **litre** (L).

Other units of capacity we use are the **millilitre** (mL), **kilolitre** (kL), and **megalitre** (ML).

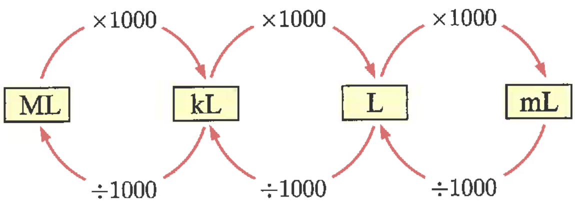
This table shows some capacities of familiar objects:



Item	*Capacity*
Medicine glass	25 mL
Bucket	8 L
Hot water system	170 L
Cup	250 mL
Petrol tank	65 L
50 m swimming pool	1500 kL
Reservoir	1000 ML

CAPACITY CONVERSIONS

1 ML = 1000 kL
1 kL = 1000 L
1 L = 1000 mL



Example 3 **Self Tutor**

Convert:

a 4500 mL into L **b** 350 kL into L.

a 4500 mL
$= 4500 \div 1000$ L
$= 4.5$ L

b 350 kL
$= 350 \times 1000$ L
$= 350\,000$ L

EXERCISE 14C

1 Match each object with its most likely capacity:

a test tube **A** 800 mL
b drink bottle **B** 28 L
c lake **C** 50 mL
d kitchen sink **D** 8.5 ML
e rainwater tank **E** 2 kL

2 Convert:

a 8000 mL into L **b** 2 ML into kL **c** 786 L into kL
d 40 mL into L **e** 3.95 kL into L **f** 1 ML into L.

3 Find the sum of 650 mL, 1.38 L, and 860 mL. Give your answer in L.

4 How many 600 mL bottles of soft drink can be filled from a 24 L container?

5 Jess takes a 10 minute shower every day. Her shower releases 8.6 litres of water per minute. How many kilolitres of water will Jess use showering in the month of January?

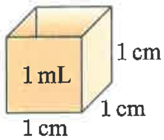
6 The dishwasher at Jock's office uses 25 L of water per cycle. The dishwasher is used for 2 cycles per day, 5 days per week, for 50 weeks of the year. If the water costs \$2.50 per kL, find the total cost of the water used by the dishwasher each year.

D CONNECTING VOLUME AND CAPACITY

The units of **capacity** and the units of **volume** are closely related.

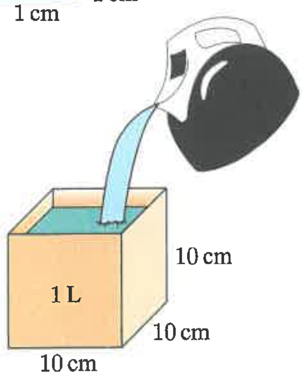
- 1 mL of fluid will fill a cube 1 cm × 1 cm × 1 cm.
  The cube has volume 1 cm^3,

  so $1 \text{ mL} \equiv 1 \text{ cm}^3$



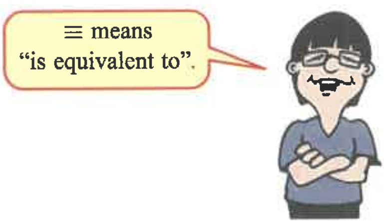
- 1 L of fluid will fill a cube 10 cm × 10 cm × 10 cm.
  The cube has volume 1000 cm^3,

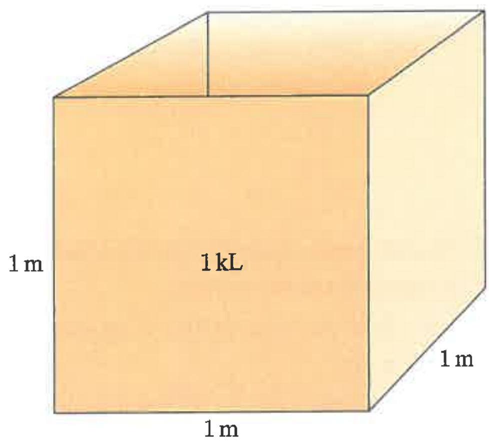
  so $1 \text{ L} \equiv 1000 \text{ cm}^3$



- 1 kL of fluid will fill a cube 1 m × 1 m × 1 m.
  The cube has volume 1 m^3,

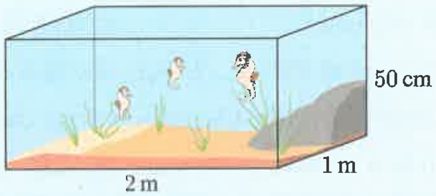
  so $1 \text{ kL} \equiv 1 \text{ m}^3$





Example 4 **Self Tutor**

Find the capacity of this fish tank.
Give your answer in litres.



Volume = length × width × height

$= 200 \text{ cm} \times 100 \text{ cm} \times 50 \text{ cm}$ {converting into centimetres}

$= 1\,000\,000 \text{ cm}^3$

Since $1 \text{ mL} \equiv 1 \text{ cm}^3$, capacity $= 1\,000\,000$ mL

$= 1\,000\,000 \div 1000$ L

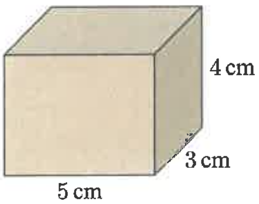
$= 1000$ L

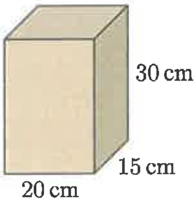
EXERCISE 14D

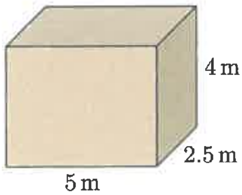
1 A container is 25 cm by 20 cm by 15 cm. Find:

a the volume of space in the container in cm^3
b the capacity of the container in mL
c the capacity of the container in litres.

2 Find the capacity of each container. Express each answer using appropriate units.

a


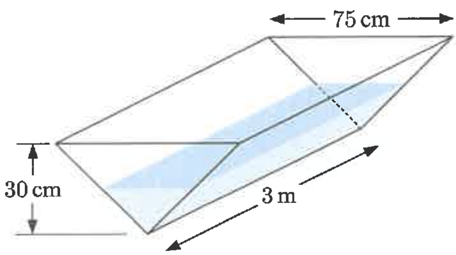
b


c


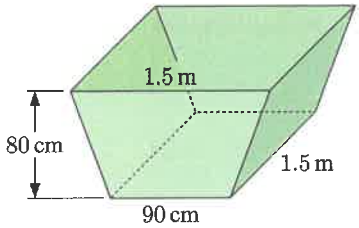
3 A rectangular ice box has dimensions 80 cm by 30 cm by 30 cm. How many litres can it hold?

4 The diagram alongside shows a water trough with a triangular cross-section.

a Find the area of the triangular cross-section in cm^2.
b Find the volume of the trough in cm^3.
c Find the capacity of the trough in:
  i litres **ii** kilolitres.

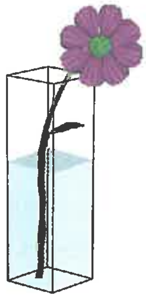


5



Beverly wants to fill 75% of this planter box with soil. How many 30 L bags of soil will she need?

6 A vase has a square base with sides 8 cm long. The vase is 30 cm high. It is filled with 1.6 litres of water. How far from the top will the water reach?



E MASS

The **mass** of an object is a measure of how heavy the object is.

The **kilogram** (kg) is the base unit of mass in the SI System. Other units of mass which are commonly used are the **milligram** (mg), **gram** (g), and **tonne** (t).

- An ant weighs approximately 5 milligrams.

- A paper clip weighs approximately 1 gram.

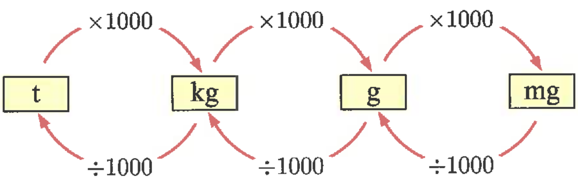
- A pineapple weighs approximately 1 kilogram.

- A small car weighs approximately 1 tonne.

MASS CONVERSIONS

The units of mass are related as follows:

1 t = 1000 kg
1 kg = 1000 g
1 g = 1000 mg



Example 5 **Self Tutor**

Write in grams:

a 3.2 kg **b** 735 mg

a 3.2 kg
$= 3.2 \times 1000$ g
$= 3200$ g

b 735 mg
$= 735 \div 1000$ g
$= 0.735$ g

EXERCISE 14E

1 State the units you would use to measure the mass of:

a a tomato **b** a bicycle **c** a pen
d a flea **e** a bus **f** a pig.

2 Write in grams:

a 3 kg **b** 6790 mg **c** 29 mg **d** 0.54 kg

3 Write in kilograms:

a 6 t **b** 4000 g **c** 350 g **d** 2.4 t

4 Convert:

a 10.37 kg into g **b** 640 kg into t **c** 5249 mg into g
d 2.63 g into mg **e** 285 g into kg **f** 0.043 g into mg.

5 Find the total mass of a box of 12 muesli bars if the box has mass 15 g and each bar has mass 42 g.

6 A car has mass 1.1 t. Four people with mass 85 kg, 80 kg, 76 kg, and 70 kg get in the car. Find the total mass of the car and the people.

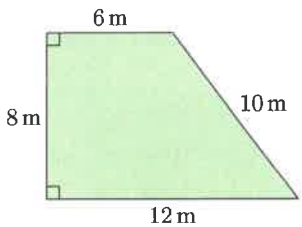
7 Find the total mass, in kilograms, of 5000 candles, each with mass 24 g.

8 A box of copy paper contains 5 reams of paper, each containing 500 sheets. Each sheet weighs 5 g, and the box itself weighs 80 g.
Find the total mass of the paper and the box in kilograms.

9 The mass of 400 suitcases is 7.6 tonnes. Find the average mass of each suitcase. Give your answer in kilograms.

10 How many bricks of mass 3.5 kg will I receive in my 14 tonne shipment of bricks?

11 Rita's lawn has the dimensions shown. She must apply 30 g of fertiliser for each square metre of lawn.
Rita only has a 1.5 kg bag of fertiliser. Once she has applied all of her fertiliser, what area of the lawn will remain unfertilised?



F THE RELATIONSHIP BETWEEN UNITS

The units for volume, capacity, and mass in the SI System are related as follows:

1000 cm^3 or 1 L of pure water at 4°C has mass 1 kg.
1 cm^3 or 1 mL of pure water at 4°C has mass 1 g.

Example 6 Self Tutor

a Find the mass of water which will fill a bucket with capacity 4 L.
b If the empty bucket has mass 250 g, what is the total mass of the bucket of water?

a 1 L of water has mass 1 kg.
$\therefore$ 4 L of water has mass 4 kg.
$\therefore$ 4 kg of water will fill the bucket.

b Mass of the water-filled bucket
$= 4\text{ kg} + 250\text{ g}$
$= 4\text{ kg} + 0.25\text{ kg}$
$= 4.25\text{ kg}$

EXERCISE 14F

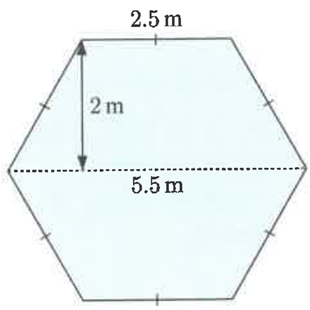
1 Find the mass of 6 mL of pure water at 4°C.

2 Find the mass of 4000 cm^3 of pure water at 4°C.

3 A watering can has mass 450 g. Find the total mass of the watering can when it is filled with 3 L of water.

4 A rectangular tray is filled with water to allow laboratory equipment to soak. The tray is 40 cm long, 20 cm wide, and 12 cm high. When empty, it has mass 1.2 kg.
   a Find the capacity of the tray.
   b What mass of water is required to completely fill the tray?
   c Suppose the tray is filled with water to a level 3 cm from the top of the tray. Find the total mass of the tray and water.

5 A fountain in a plaza has the hexagonal cross-section shown. The fountain can be filled with water to a depth of 30 cm.



   a Find the area of the cross-section.
   b Find the capacity of the fountain in litres.
   c Since the plaza is built above a car park, the total weight of the fountain and the water must not exceed 15 tonnes. The fountain itself weighs 12.2 tonnes.
      i Show that if the fountain is completely filled with water, the weight limit will be exceeded.
      ii Find the maximum depth to which the water can be filled.

MULTIPLE CHOICE QUIZ

QUICK QUIZ

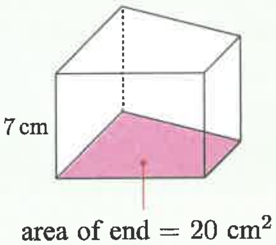
REVIEW SET 14A

1 State the units you would use to measure the volume of:

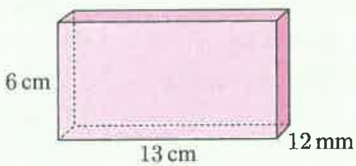
a a block of cheese **b** a shed.

2 Find the volume of each prism:

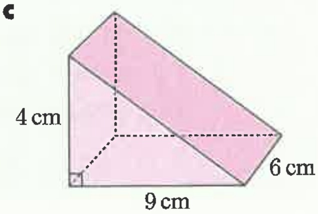
a



b



c



3 A truck driver charges $19.40 per cubic metre for delivering dirt. How much should the driver charge for filling a rectangular excavation 13 m long by 8 m wide by 2.2 m deep?

4 Convert:

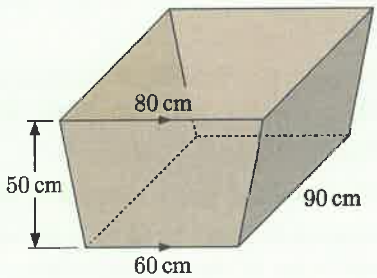
a 3.7 L into mL **b** 19 850 L into kL **c** 43 kL into ML.

5 The cross-section of this container is a trapezium.

a Find the area of the cross-section.

b Find the capacity of the container in:

i millilitres **ii** litres.

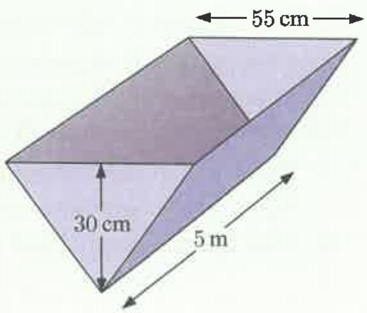


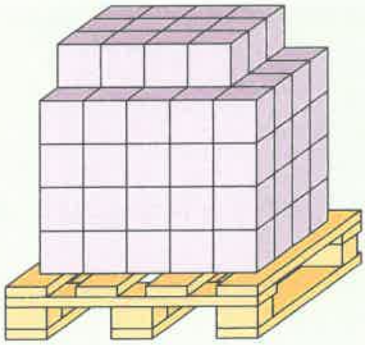
6 Write in grams:

a 5.4 kg **b** 475 mg **c** 0.032 kg

7 Find the total mass of 6000 textbooks, each with mass 750 g. Give your answer in kilograms.

8 A 5 m long water trough has a triangular cross-section.
Find the mass of water required to fill the trough.



9 

A warehouse has 20 cm by 20 cm by 20 cm boxes stacked on a pallet as shown.

- **a** How many boxes are on the pallet?
- **b** Find the volume of each box.
- **c** Find the total volume of boxes on the pallet.
- **d** Each box weighs 5 kg, and the pallet weighs 35 kg. A small forklift has a maximum lift of 500 kg. Will the forklift be able to lift the pallet?

10 Answer the **Opening Problem** on page **280**.

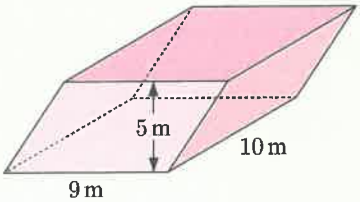
REVIEW SET 14B

1 Using cubes with volume 1 cm^3, sketch a solid with volume:

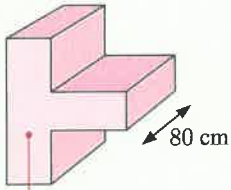
a 6 cm^3 **b** 17 cm^3

2 Find the volume of each prism:

a

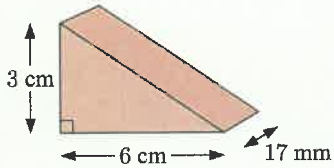


b



area of end = 1.6 m^2

3 Find the volume of wood in this door wedge.



4 There are seven 2 L bottles and six 375 mL cans of soft drink in a fridge. How many litres of soft drink are in the fridge?

5 A rectangular tank with base measuring 3.2 m by 1.5 m contains water to a height of 50 cm. If the level of the water is increased by 10 cm, how much more water has been added to the tank?

6 Convert:

a 12.13 t into kg **b** 43 g into kg.

7 Dillon weighs 60 kg, and his pet bird weighs 400 g.
How many times heavier is Dillon than his bird?

8 A large water bottle holds 7 litres of water.

- **a** Find the mass of the water.
- **b** If the bottle has mass 950 g, find the total mass of the water-filled bottle.

9 Elaine is packing books to send off to a school. Each book is 18 cm by 24 cm by 3 cm, and weighs 1.25 kg.

a Find the volume of each book.

b Books are packed in 24 cm by 36 cm by 15 cm boxes.

i Find the volume of each box.

ii How many books will fit in a box? Illustrate a method of packing the books into a box.

iii Find the total mass of books that will fit in the box.

10 A reservoir has a storage capacity of 19 000 ML.

a Find the capacity of the reservoir in:

i kL **ii** L.

b Find the volume of the reservoir in m^3.

c Estimate the mass of the water in the reservoir when it is full. Give your answer in tonnes.

Chapter 15

Coordinate geometry

Contents:

OPENING PROBLEM

Scientists and archaeologists often use grids when searching for fossils and ancient artefacts. They do this so they can accurately record the location where each object is found.

Things to think about:

a Professor Johnson has used pegs and ropes to form a grid over his excavation site. What else does he need so he can record the positions of his discoveries?

b How can Professor Johnson improve his accuracy in identifying positions? Discuss your ideas with your class.

c Suppose Professor Johnson wants to record the position of the object in his grid *and* the depth at which it is found. Discuss how he could do this.

When we want to communicate the **location** of an object or place, we often use a grid.

- To specify the location of a large object such as a building, we use grids with a letter to identify each column and a number to identify each row. We use a **map reference** or **grid reference** to tell us the square of the grid that the object lies in.

  For example, the map alongside shows part of the campus of the University of California, Los Angeles.

  We can see that Royce Hall is found in square A5.

  This grid reference does not describe the *exact* location of Royce Hall, but it tells us where to look.

- To specify the location of a small object or point exactly, we use two number lines, one horizontal and one vertical. These allow us to record a location using **coordinates**.

HISTORICAL NOTE

Frenchman **René Descartes** (1596 - 1650) first created a method for describing the position of a point in a plane. His work led to a new branch of mathematics called **coordinate geometry**.

One of Descartes' principles was "never to accept anything as true which I do not clearly and distinctly see to be so". This is a good piece of advice for your own study of mathematics.

René Descartes

A COORDINATES

In the diagram alongside, we see two number lines or **axes**.

Axes is the plural of *axis*.

The horizontal axis is called the **x-axis**.

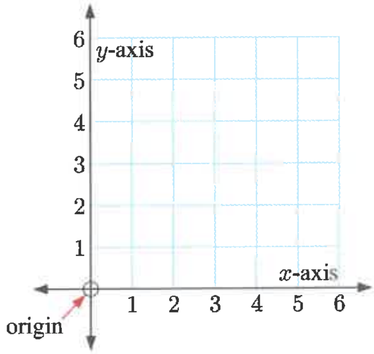
The vertical axis is called the **y-axis**.

The axes meet at a point called the **origin**, which we mark with O or with a small circle.

We draw **grid lines** from the numbers on the axes. These grid lines form a grid called a **number plane**.

We can describe any point on the number plane using a pair of numbers called **coordinates**.

We write coordinates as an **ordered pair** of numbers, in brackets, with a comma between them.

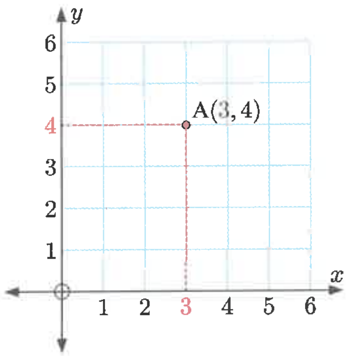


The coordinates of a point are written (x-coordinate, y-coordinate).

The x-coordinate gives the horizontal position along the x-axis and the y-coordinate gives the vertical position along the y-axis.

For example, on this number plane we see the point A has x-coordinate 3 and y-coordinate 4.

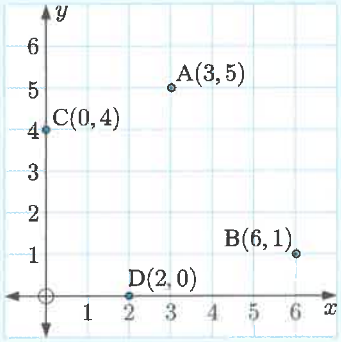
We say that A has coordinates (3, 4).



Example 1 Self Tutor

Plot these points on a set of axes:

A(3, 5), B(6, 1), C(0, 4), D(2, 0).



EXERCISE 15A

1 On the same set of axes, plot and label the points:

A(2, 2), B(4, 8), C(3, 1), D(7, 0), E(0, 5), F(5, 4),
G(9, 1), H(6, 0), I(0, 1), J(7, 7), K(0, 0), L(8, 3).

PRINTABLE GRIDS

2 Copy and complete:

a The y-coordinate of any point on the x-axis is ......

b The x-coordinate of any point on the y-axis is ......

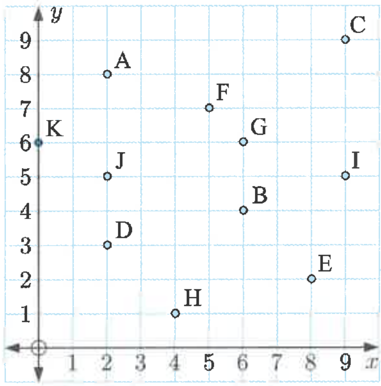
3 Look at the grid.

a Write down the x-coordinate of:
i B **ii** A **iii** C **iv** G

b Write down the y-coordinate of:
i E **ii** H **iii** J **iv** K

c Find the coordinates of each point.

d Write down the coordinates of the origin, O.

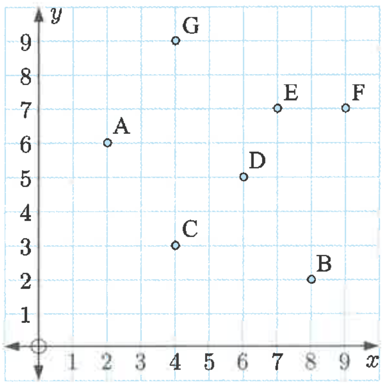


4 Look at the grid.

a Name two points with the same x-coordinate. What do you notice about these points?

b Name two points with the same y-coordinate. What do you notice about these points?

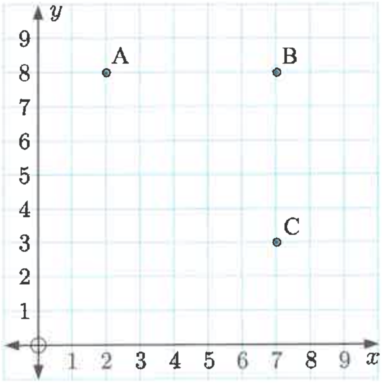
c Name the point whose x-coordinate is equal to its y-coordinate.



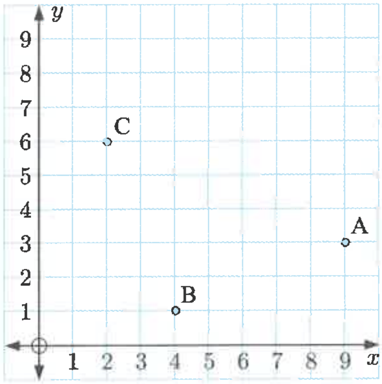
5 Explain why $(2, 5)$ and $(5, 2)$ are different points on the number plane.

6 ABCD is a square. A, B, and C are marked on the grid.
Write down the coordinates of D.

a



b



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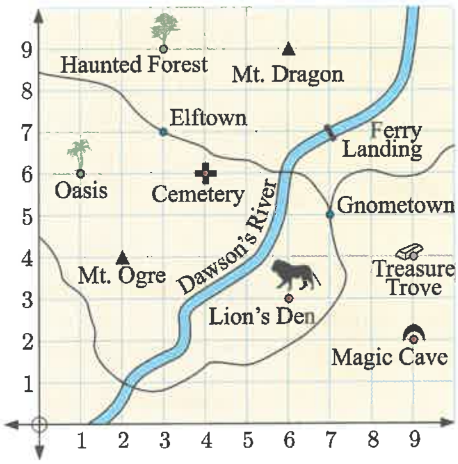


a Find the grid coordinates for:
- **i** Gnometown
- **ii** Magic Cave
- **iii** Ferry Landing
- **iv** where the roads cross Dawson's River.

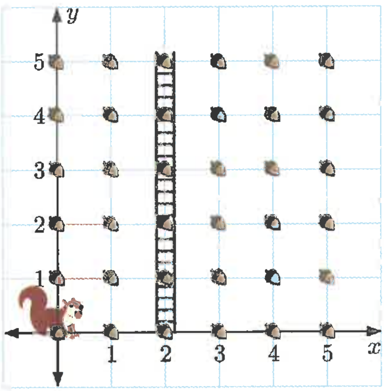
b Write down the place located at:
- **i** $(9, 4)$
- **ii** $(6, 3)$
- **iii** $(2, 4)$
- **iv** $(1, 6)$.

8 **a** On a set of axes, plot and label the points A$(3, 1)$, B$(5, 2)$, C$(7, 3)$, and D$(9, 4)$.
b If the pattern continues, what will the next point be?

9 **a** On a set of axes, plot and label the points A$(0, 10)$, B$(1, 8)$, C$(2, 6)$, and D$(3, 4)$.
b If the pattern continues, what will the next two points be?

10 Sissy the Squirrel stores her acorns in her storeroom.
She must always move her ladder horizontally first before she can climb it vertically.
If Sissy starts at the origin, describe how she needs to move to get to the point:

a $(3, 0)$ **b** $(0, 5)$
c $(2, 4)$ **d** (x, y).



ACTIVITY **HOPPING AROUND A NUMBER PLANE**

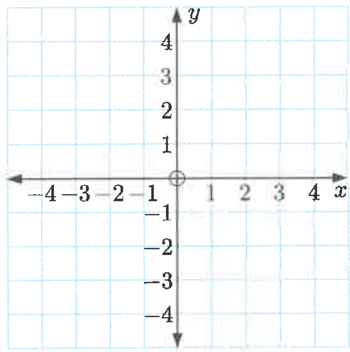
For this Activity, click on the icon to obtain instructions and a printable grid.

ACTIVITY

B POSITIVE AND NEGATIVE COORDINATES

In **Chapter 5** we saw how the number line was extended in two directions to include positive and negative numbers.

In the same way, we can extend both the x-axis and the y-axis in two directions. This allows us to consider positive and negative coordinates.



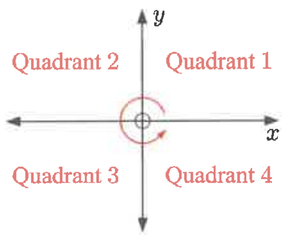
The x-axis is positive to the right of the origin O, and negative to the left of it.

The y-axis is positive above O, and negative below it.

The resulting coordinate grid is called the **Cartesian plane**, named after **René Descartes**.

The axes divide the Cartesian plane into four **quadrants**.

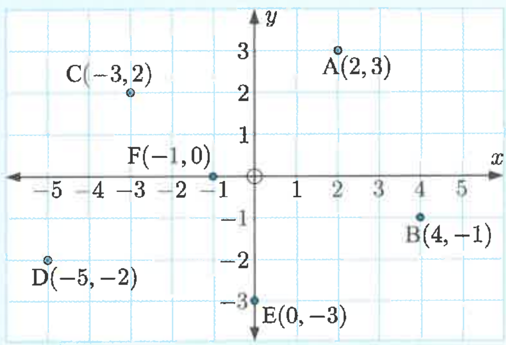
The quadrants are numbered in an anticlockwise direction, starting with the upper right hand quadrant in which x and y are both positive.



Example 2 **Self Tutor**

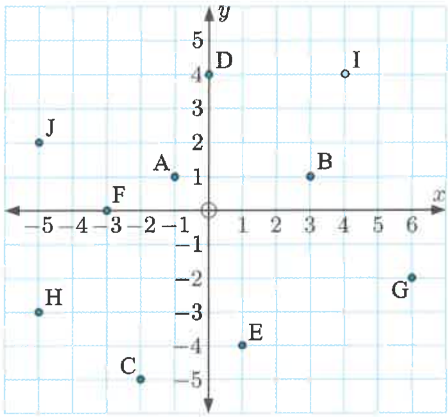
Plot these points on the Cartesian plane:

A(2, 3), B(4, −1), C(−3, 2), D(−5, −2), E(0, −3), F(−1, 0).



EXERCISE 15B

1 Look at the grid.



a Write down the x-coordinate of:

i D **ii** B **iii** J **iv** G.

b Write down the y-coordinate of:

i A **ii** C **iii** F **iv** I.

c Write down the coordinates of each point.

d Name the points which lie in the:

i first quadrant
ii second quadrant
iii third quadrant
iv fourth quadrant.

e Name the point which lies on the:

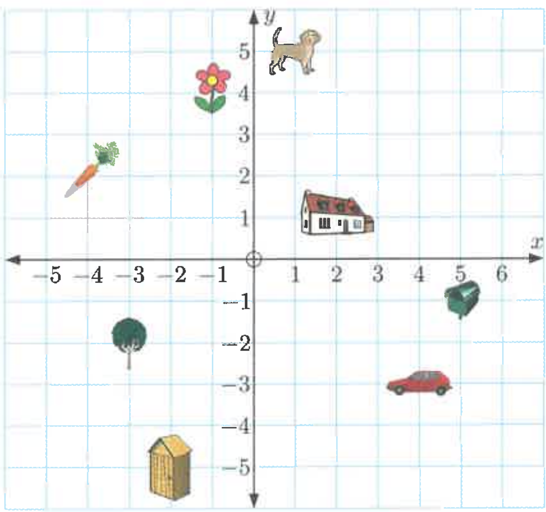
i x-axis **ii** y-axis.

2 On the same set of axes, plot and label these points:

A(3, 4), B(6, 2), C(−3, 0), D(−5, −5), E(0, −1), F(4, 0),
G(3, −4), H(0, 6), I(−5, −2), J(−4, −1), K(3, −3), L(−5, 4).

PRINTABLE GRIDS

3 Look at the map alongside.



a Write down the coordinates of the:

i house
ii tree
iii flower garden
iv car
v dog
vi carrot patch
vii letterbox
viii toolshed.

b Which of the things lie in the:

i first quadrant
ii second quadrant
iii third quadrant
iv fourth quadrant?

4 In which quadrant would you find a point where:

a both x and y are positive
b both x and y are negative
c x is negative and y is positive
d x is positive and y is negative?

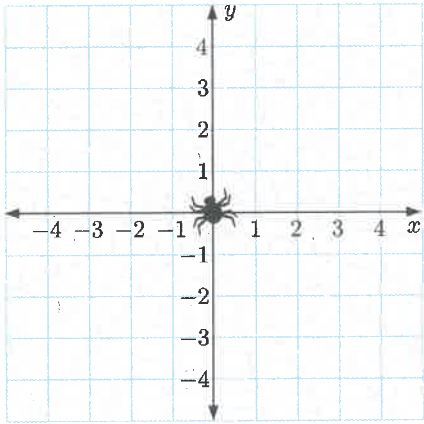
5 Determine the quadrant in which each point lies.

a A(3, 5) **b** B(2, −2) **c** C(−1, −3) **d** D(−4, 2)
e E(5, −3) **f** F(4, −4) **g** G(−2, −1) **h** H(−3, 5)

6 On a set of axes, plot these points then join them in the order given:
(−3, 4), (−1, 5), (1, 5), (3, 4), (1, 3), (−1, 3), (−3, 4), (−3, −2), (−1, −3), (1, −3), (3, −2), (3, 4), (3, 3), (4, 3), (5, 2), (5, 0), (4, −1), (3, −1), (3, 0), (4, 0), (4, 2), (3, 2).

7 Spiros the Spider always climbs horizontally and then vertically. He sits at the origin waiting for an insect to get caught in his web.
Describe how Spiros needs to move, to get to the point:

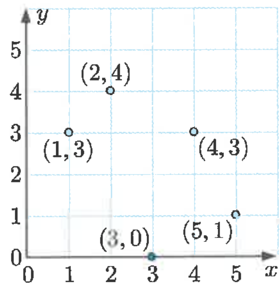
a (0, −2) **b** (−4, 0) **c** (3, −1)
d (−2, 4) **e** (−1, −4) **f** (x, y).



C PLOTTING POINTS FROM A TABLE OF VALUES

Tony plays lacrosse for his local club. The numbers of goals he has scored in the five games so far this season are shown below in this **table of values**:

Game number (x)	1	2	3	4	5
Goals scored (y)	3	4	0	3	1



For every game number x, there is a corresponding number of goals scored y. We can therefore plot a point for each game on a number plane.

EXERCISE 15C

1 Mike kept a record of the cars sold at his car yard each day last week:

Day number (x)	1	2	3	4	5	6	7
Cars sold (y)	3	1	2	0	4	7	5

Plot these points on a number plane.

2 While on ski camp, Ned recorded the minimum temperature each night. The results are given in the table:

Night number (x)	1	2	3	4	5
Temperature (y °C)	-2	0	-1	3	1

Plot these points on a number plane.

3 Plot the points in the table of values:

a

x	1	2	3	4
y	2	4	1	2

b

x	0	1	2	3	4
y	3	-2	0	5	-1

c

x	-3	-1	0	2	3
y	4	2	-1	4	-2

d

x	-2	-1	0	1	2
y	3	0	2	-4	-3

4 Bethany made a hole in the bottom of a bucket full of water. The table below shows the number of litres of water remaining in the bucket at various times after the hole was made.

Number of minutes (x)	1	2	4	6
Litres of water remaining (y)	9	8	6	4

a Plot these points on a number plane.

b Do the points lie in a straight line?

c How much water do you think was in the bucket when Bethany first made the hole?

5 The points in each of the following tables of values lie in a straight line. Plot the given points on a number plane, and use your graph to fill in the missing values.

a

x	-2	-1	0	1	2	3
y	-7	-5	-3		1	3

b

x	-2	-1	0	1	2	3
y	-3		3	6		12

c

x	-1	0	1	2	3	4
y	5				-11	

d

x	-3	-2	-1	0	1	2
y	13		9			3

e

x	-2	-1	0	1	2	3
y			7			-11

f

x	-2	0	1	2	3	4
y			-3		-13	

6 The points in this table of values lie in a straight line.

x	-3	-1	1	3	5
y	-5	-2	1	4	7

a Plot these points on a number plane.

b Identify the point on the straight line which:

i has x-coordinate 7
ii lies on the x-axis
iii lies on the y-axis
iv has x-coordinate -2
v has y-coordinate 2.

D THE EQUATION OF A LINE

A **straight line** is an infinite set of points in a particular direction.

To describe *all* of the points on a line, we need an **equation** which connects the x and y coordinates.

The equation is often given as a formula with y as the subject expressed in terms of x.

Example 3 Self Tutor

For each point on a line, the y-coordinate is 2 less than the x-coordinate. State the equation of the line.

The equation of the line is $y = x - 2$.

If we are given the equation of a line, we can use it to generate a table of values and hence draw the line.

Example 4 Self Tutor

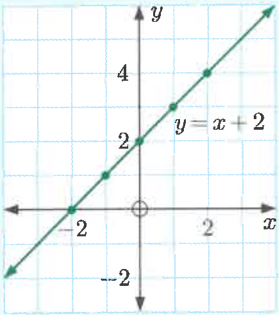
Construct a table of values with $x = -2, -1, 0, 1, 2$ for the equation $y = x + 2$.

Hence graph the line $y = x + 2$.

When $x = -2$, $y = -2 + 2 = 0$.
When $x = -1$, $y = -1 + 2 = 1$.
When $x = 0$, $y = 0 + 2 = 2$.
When $x = 1$, $y = 1 + 2 = 3$.
When $x = 2$, $y = 2 + 2 = 4$.

The table of values is:

x	-2	-1	0	1	2
y	0	1	2	3	4



EXERCISE 15D

1 State the equation of a line if, for each point, the y-coordinate is:

a the x-coordinate plus 3
b 5 more than the x-coordinate
c the x-coordinate minus 4
d 7 less than the x-coordinate
e 3 times the x-coordinate
f half the x-coordinate
g twice the x-coordinate, plus 1
h 1 minus 3 times the x-coordinate.

2 Write in words, the meaning of the equation:

a $y = x + 2$
b $y = x - 5$
c $y = 2x$
d $y = \frac{1}{4}x$
e $y = 4 - x$
f $y = 3x - 1$
g $y = 1 + \frac{1}{2}x$
h $y = 2 - 4x$

3 For each equation given, construct a table of values with $x = -2, -1, 0, 1, 2$. Hence graph the line.

a $y = x$	**b** $y = x + 4$	**c** $y = x - 2$
d $y = x + 1$	**e** $y = x - 4$	**f** $y = 2x$
g $y = 1 - x$	**h** $y = 2x + 1$	**i** $y = 2x - 3$
j $y = -3x$	**k** $y = \frac{1}{2}x$	**l** $y = 3 - 2x$

DISCUSSION

Examine the graphs you have drawn and their corresponding equations.

1 What *form* do all of these equations have?

2 What part of the equation do you think controls:

- the steepness of the line
- whether the graph slopes upwards or downwards
- where the graph cuts the y-axis?

QUICK QUIZ

MULTIPLE CHOICE QUIZ

REVIEW SET 15A

1 Look at the grid.

a Write down the x-coordinate of:

i A **ii** D.

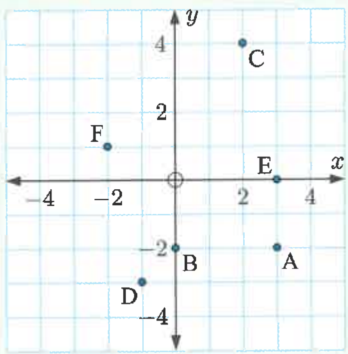
b Write down the y-coordinate of:

i B **ii** C.

c Find the coordinates of:

i A **ii** B

iii E **iv** F.



2 **a** On a set of axes, plot and label the points A(0, 1), B(1, 3), C(2, 5), and D(3, 7).

b If the pattern continues, what will the next point be?

3 On the same set of axes, plot and label the points F(4, −2), G(−5, −3), H(−1, 3), and I(0, −4).

4 Determine the quadrant in which the following points lie:

a (−2, 7) **b** (−3, −6) **c** (3, −2) **d** (5, 1)

5 The height of a plant is recorded on a weekly basis.
Plot these points on a number plane.

Week number (x)	1	2	3	4	5
Height (y cm)	6	9	11	13	14

6 Plot the points in the table of values:

a

x	1	2	3	4	5
y	4	2	5	0	2

b

x	-2	-1	0	1	2
y	-7	-4	-1	2	5

7 The points in each of the following tables of values lie in a straight line. Plot the given points on a number plane, and use your graph to fill in the missing values.

a

x	-2	-1	0	1	2	3
y	-5	-2		4		10

b

x	-3	-2	-1	0	1	2
y	6				-2	

8 State the equation of a line if, for each point, the y-coordinate is:

a 3 less than the x-coordinate **b** twice the x-coordinate.

9 For the equation $y = 2x - 1$, construct a table of values with $x = -2, -1, 0, 1, 2$. Hence graph the line.

10 A new movie has been released in the cinema. In week 1, the movie has 12 session times per day. In week 2, it has 10 session times per day, and in week 5, it has 4 session times per day.

a Complete this table.

b Plot the points on a number plane.

c Do the points lie in a straight line?

Week number (x)	1	2	5
Sessions per day (y)			

d Use your graph to predict the number of session times per day in:

i week 3 **ii** week 4.

e Explain why it is unreasonable to use your graph to predict the number of session times per day in week 8.

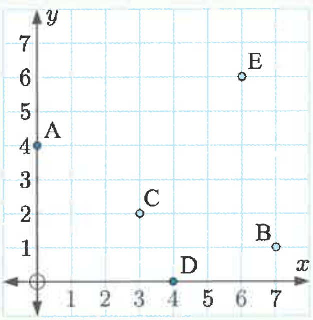
REVIEW SET 15B

1 **a** Write down the coordinates of:

i A **ii** B **iii** C

iv D **v** E

b Which point has an x-coordinate equal to its y-coordinate?



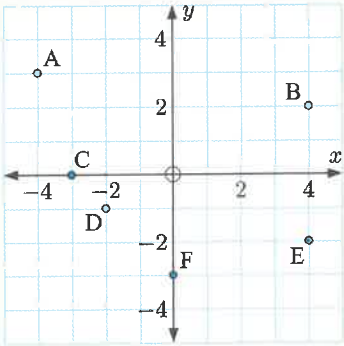
2 In which quadrant would you find a point with negative x and y-coordinates?

3 Match each ordered pair with the correct point on the number plane:

a $(4, -2)$ **b** $(0, -3)$

c $(-4, 3)$ **d** $(4, 2)$

e $(-3, 0)$ **f** $(-2, -1)$



4 **a** Plot and label the points $A(-2, 5)$, $B(3, 5)$, and $C(3, 2)$.

b Find the coordinates of D, the fourth vertex of rectangle ABCD.

5 Are $(3, 4)$ and $(4, 3)$ the same point on the number plane? Explain your answer.

6 Tina measured the rainfall at her house each day for 5 days. Her results are shown below:

Day number (x)	1	2	3	4	5
Rainfall (y mm)	5	2	9	0	4

Plot these points on a set of axes.

7 On a set of axes, plot these points, then join them in the order given:
$(1, 4)$, $(2, 1)$, $(5, 1)$, $(3, -1)$, $(4, -4)$, $(1, -2)$, $(-2, -4)$, $(-1, -1)$, $(-3, 1)$, $(0, 1)$, $(1, 4)$.

8 The points in this table of values lie in a straight line.

x	-2	-1	0	1	2	3
y	7	5	3	1	-1	-3

a Plot these points on a number plane.

b Identify the point on the straight line which has y-coordinate -5.

9 The y-coordinate of each point on a line is two less than its x-coordinate. Write down the equation of this line.

10 For each equation given, construct a table of values with $x = -2, -1, 0, 1, 2$. Hence graph the line.

a $y = -\frac{1}{2}x$ **b** $y = 2x - 4$

11 For each point on a line, the y-coordinate is 4 less than the x-coordinate.

a State the equation of the line.

b Construct a table of values with $x = -2, -1, 0, 1, 2$ for the line.

c Draw the graph of the line.

d On the same set of axes, draw the graph of $y = 2 - x$.

e State the coordinates of the point at which the two lines meet.

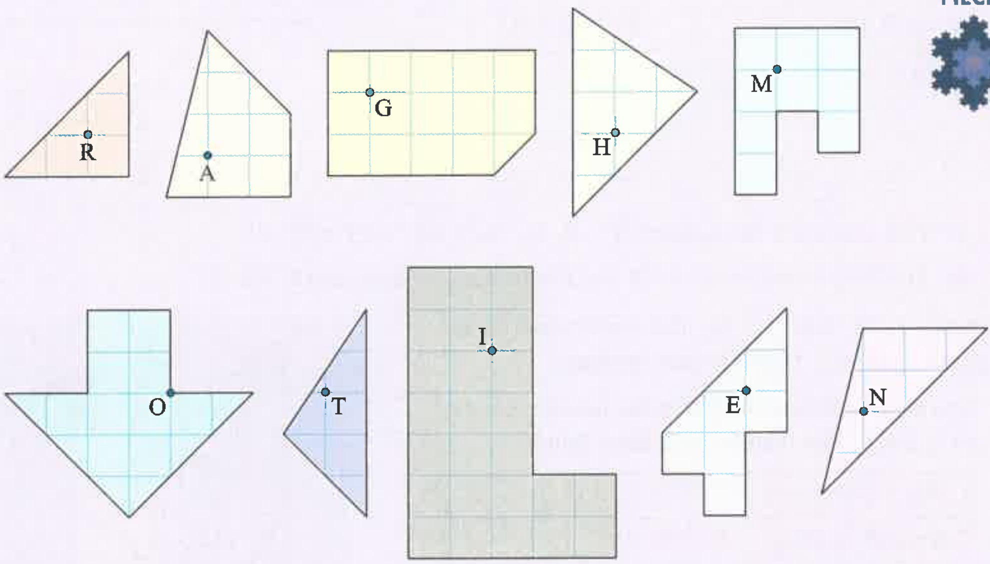
f Name the quadrants that *both* lines pass through.

PUZZLE

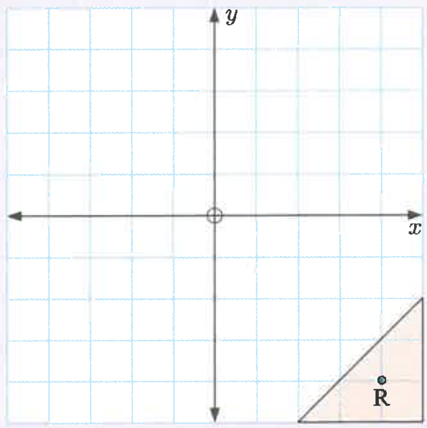
What to do:

1 Print these jigsaw pieces, and cut them out.

PRINTABLE PIECES



2 Arrange the pieces onto this coordinate grid. You may rotate the pieces, but you must not turn them over.
One piece has been placed for you.



3 Solve the riddle below by writing the letter on your completed grid corresponding to each of the coordinates given.

What did the x-axis say to the y-axis?

___ ___ ___ ___   ___ ___   ___ ___
(3, 1) (3, −3) (3, −3) (−4, 0)   (3, 1) (3, −3)   (−3, 4) (−4, 0)

___ ___ ___   ___ ___ ___ ___ ___ ___
(−4, 0) (−2, −1) (3, −3)   (0, 0) (4, −4) (−3, −3) (4, 3) (−3, −3) (−2, 3)

Chapter 16

Ratio and rates

Contents:

OPENING PROBLEM

To make a chocolate milkshake, Joel usually combines 20 mL of chocolate topping with 200 mL of milk. However, when he looks in the fridge he finds there is only 100 mL of milk left.

Things to think about:

a If Joel still adds 20 mL of chocolate topping to the milk, will it taste the same as usual?

b How much chocolate topping should Joel add so that his milkshake will taste the way he likes it?

In this Chapter we look at ways to **compare** quantities. We will consider **ratios** which compare quantities of the same kind, and **rates** which compare quantities of different kinds.

A RATIO

We often hear statements about:
- a team's win-loss ratio
- the teacher-student ratio in a school
- mixing ingredients in a particular ratio.

A **ratio** is an ordered comparison of quantities of the **same kind**.

Carol bought some industrial strength disinfectant for use in her hospital ward. It is important to mix the disinfectant and water in the correct ratio so that the disinfectant will be effective but the chemicals will not be wasted.

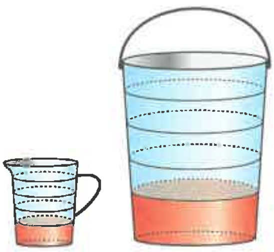
The bottle instructs her to "mix one part disinfectant to four parts water".

Disinfectant and water are both liquids, so this statement can be written as a **ratio**.

The required ratio of disinfectant to water is $1:4$ or "1 is to 4".

Notice that the ratio is written *without units* such as mL or L. It does not matter what the units of volume are, as long as the same units are used to measure both.

Carol may mix a jug or a bucket of disinfectant. As long as she mixes it in the correct ratio, it will be effective.



Example 1 **Self Tutor**

Write as a ratio:

a 3 km is to 5 km **b** 7 minutes is to 2 hours.

a 3 km is to 5 km $= 3 : 5$

b 7 minutes is to 2 hours $=$ 7 minutes is to 120 minutes
$= 7 : 120$

EXERCISE 16A

1 Write as a ratio:

a \$4 is to \$5 **b** 15 mL is to 8 mL **c** 1 tonne is to 4 tonnes

d 8 m is to 7 m **e** 9 kg is to 5 kg **f** 2 mm is to 11 mm

2 Find the ratio of:

a red balloons to blue balloons

b teachers to students

c cats to mice

d basketballs to tennis balls

3 Write as a ratio:

a 17 cents is to \$1 **b** 50 seconds is to 1 minute

c 1 kg is to 150 g **d** 9 months is to 2 years

e 12 minutes is to 3 hours **f** 400 kg is to 1 tonne

g 2 weeks is to 9 days **h** 780 m is to 3.1 km

Example 2 **Self Tutor**

Write as a ratio:

Keith spends two hours watching TV and three hours doing homework.

TV : homework $= 2 : 3$

4 Write as a ratio:

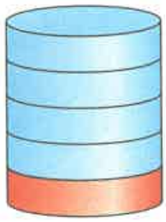
a Jess is 152 cm tall and Carly is 164 cm tall.
b At the cricket there are 2 female spectators for every 5 male spectators.
c A farmer has 3 dogs for every 500 sheep.
d There are 20 people skiing for every 12 people snow boarding.
e I spend €8 for every €5 I save.
f Mix 200 mL of cordial concentrate with 800 mL of water.
g For every 2 km I walk, I run 700 m.
h A restaurant makes 850 g of chips for every 1 kg of meat served.

B RATIO AND FRACTIONS

The ratio of quantities in a mixture is related to the fraction of each quantity in the whole.

We consider the *total number of parts* in the ratio, then divide these parts according to the ratio.

For example, Carol's mixture of hospital disinfectant combines disinfectant and water in the ratio $1:4$.



The ratio contains $1+4=5$ parts in total.

Of the 5 parts, 1 part is disinfectant and 4 parts are water.

So, $\frac{1}{5}$ of the mixture is disinfectant, and $\frac{4}{5}$ of the mixture is water.

Example 3 **Self Tutor**

The ratio of girls to boys in a class is $3:4$.
What fraction of the class are:

a girls
b boys?

The ratio contains $3+4=7$ parts in total.

Of the 7 parts, 3 parts are girls and 4 parts are boys.

a $\frac{3}{7}$ of the class are girls.
b $\frac{4}{7}$ of the class are boys.

EXERCISE 16B

1 The ratio of adults to children visiting a theme park is $3:5$.
What fraction of the visitors are:

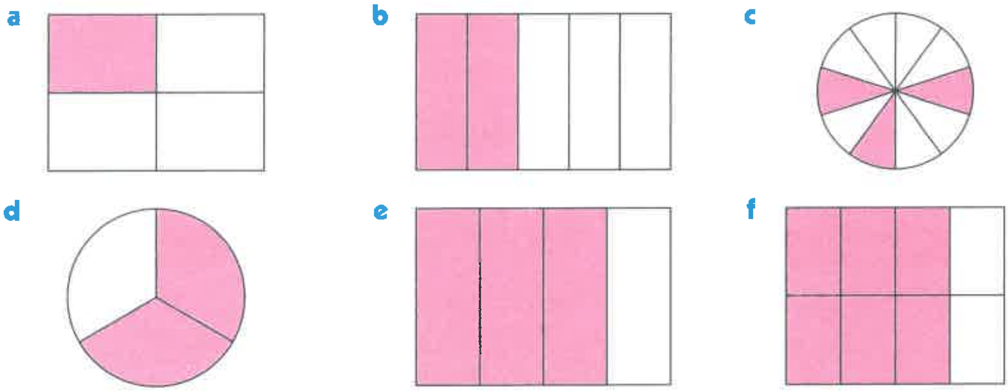
a adults
b children?

2 A fruit punch is made by mixing pineapple juice and orange juice in the ratio $2:3$.
What fraction of the fruit punch is:

a pineapple juice
b orange juice?

3 For each figure, find:

i the ratio of the shaded area to the unshaded area
ii the fraction of the figure which is shaded
iii the percentage of the figure which is shaded.



4 At a tennis tournament, Naomi won three times as many matches as she lost.

a Write the ratio of matches won to matches lost.
b What fraction of her matches did Naomi lose?

5 Belinda has 16 pages in her passport. 7 of the pages are filled with stamps. The remaining pages are blank. Find:

a the fraction of pages filled with stamps **b** the ratio of filled pages to blank pages.

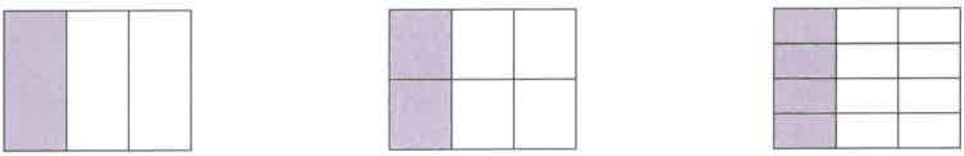
6 $\frac{1}{40}$ of a fuel mixture is oil. The remainder is petrol. Find the ratio of oil to petrol in the fuel mixture.

7 $\frac{8}{15}$ of Adrian's fruit trees are apple trees, $\frac{4}{15}$ are orange trees, and the rest are peach trees.
Find the ratio of:

a apple trees to orange trees **b** apple trees to peach trees
c orange trees to peach trees **d** apple trees to orange trees to peach trees.

C EQUAL RATIOS

Under each diagram below is written the ratio shaded area : unshaded area.



1 : 2 2 : 4 4 : 8

Notice that the fraction of the total area which is shaded is the same in each case. The three ratios are therefore **equal**.

We can write that $1:2 = 2:4 = 4:8$.

Notice that $\underset{\times 2}{\overset{\times 2}{2:4 = 4:8}}$ and $\underset{\div 2}{\overset{\div 2}{2:4 = 1:2}}$.

Comparing with the equal fractions $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$, we conclude that we can find equal ratios in the same way as we find equal fractions.

> If we multiply or divide both parts of a ratio by the same non-zero number, we obtain an **equal ratio**.

Example 4 — Self Tutor

Write a ratio equal to $6:9$ by:

a multiplying both parts by 4 **b** dividing both parts by 3.

a $6:9 = 6 \times 4 : 9 \times 4$
$= 24:36$

b $6:9 = 6 \div 3 : 9 \div 3$
$= 2:3$

EXERCISE 16C

1 Write a ratio equal to $4:10$ by:
a multiplying both parts by 3 **b** dividing both parts by 2.

2 Write a ratio equal to $12:30$ by:
a multiplying both parts by 2 **b** dividing both parts by 3.

3 Write a ratio equal to $16:400$ by:
a dividing both parts by 4 **b** multiplying both parts by $\frac{5}{2}$.

4 Which *two* of these ratios are equal to $18:24$?
A $6:9$ **B** $3:4$ **C** $54:96$ **D** $6:12$ **E** $36:48$

5 Decide whether each pair of ratios are equal:
a $3:2$ and $9:6$ **b** $4:8$ and $12:24$
c $4:7$ and $16:21$ **d** $20:50$ and $4:10$
e $30:35$ and $5:7$ **f** $42:24$ and $7:4$

To obtain an equal ratio, both parts must be multiplied or divided by the *same number*.

6 Are the ratios $5:3$ and $3:5$ equal?
Explain your answer.

7 Decide whether each pair of ratios are equal:
a $0.8:1.4$ and $4:7$
b $2.4:1.8$ and $4.2:3$
c $\frac{3}{5}:\frac{1}{2}$ and $\frac{9}{10}:\frac{3}{4}$

D LOWEST TERMS

A ratio is in **lowest terms** or **simplest form** when it is written in terms of whole numbers with no common factors.

To write a ratio in lowest terms, we divide both parts of the ratio by their **highest common factor**.

Example 5 Self Tutor

Write in lowest terms:

a $8:16$ b $35:20$

a $8:16$
$= 8 \div 8 : 16 \div 8$ {HCF of 8 and 16 is 8}
$= 1:2$

b $35:20$
$= 35 \div 5 : 20 \div 5$ {HCF of 35 and 20 is 5}
$= 7:4$

Two ratios are equal if they can be written in the same lowest terms.

EXERCISE 16D

1 Write in lowest terms:

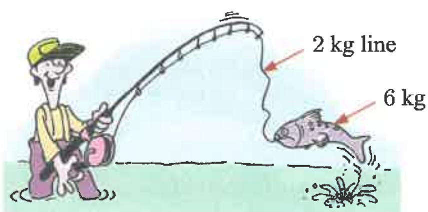
a $3:6$ b $9:3$ c $2:10$ d $12:4$
e $40:50$ f $14:8$ g $10:25$ h $36:24$
i $12:18$ j $16:56$ k $72:48$ l $28:84$

2 Decide whether each ratio is in lowest terms:

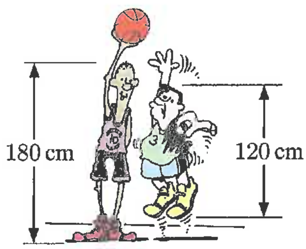
a $10:16$ b $7:4$ c $7:21$ d $22:12$
e $20:6$ f $5:9$ g $15:28$ h $45:27$

3 Write as a ratio in lowest terms:

a the weight of the fish to the breaking strain of the fishing line



b the height of the tall player to the height of the short player



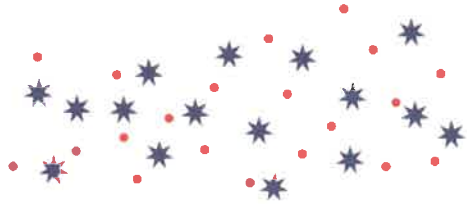
4 Write as a ratio in lowest terms:

a 25 kg to 250 kg b 14 cm to 28 cm c 21 min to 14 min

5 Write as a ratio in lowest terms:

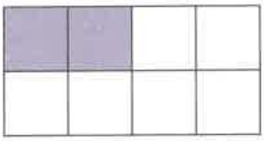
a the number of giraffes to the number of zebra

b the number of dots to the number of stars



6 Write the ratio shaded area : unshaded area in lowest terms.

a



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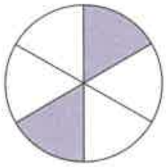

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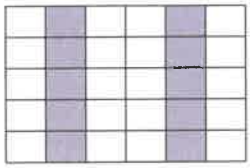
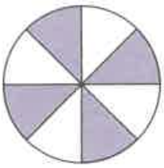


d



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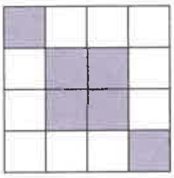

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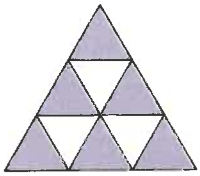
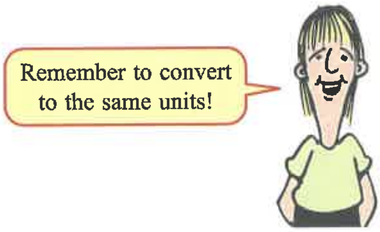


7 Write as a ratio in lowest terms:

a 30 cents to \$1
b 400 mL to 2 L
c 3 months to 1 year
d 200 m to 2 km
e 350 g to 7 kg
f 2 hours to 30 min
g 5 days to 1 week
h 4 m to 80 cm
i 600 g to 1 kg
j 24 min to 1 hour
k 4 seconds to 1 min
l 280 mL to 0.32 L



8 By writing each ratio in lowest terms, determine whether each pair of ratios are equal:

a $2:4$ and $6:12$
b $9:3$ and $16:4$
c $6:9$ and $3:2$
d $6:15$ and $5:2$
e $49:35$ and $21:15$
f $32:24$ and $45:30$
g $12:15$ and $20:25$
h $9:16$ and $16:25$

E PROPORTIONS

A **proportion** is a statement that two ratios are equal.

For example, the statement $2:5=4:10$ is a proportion.

We can use our knowledge of equal ratios to find unknown values in proportions.

Example 6 — Self Tutor

Find the unknown value $\square$ in $2:5=6:\square$

$$2:5=6:\square$$

$\times 3$

$$\therefore\ 2:5=6:\square$$

$\times 3$

$$\therefore\ \square = 5\times 3 = 15$$

EXERCISE 16E

1 Find the unknown value $\square$ in each proportion:

- **a** $2:3=4:\square$
- **b** $3:1=9:\square$
- **c** $2:11=6:\square$
- **d** $3:8=\square:40$
- **e** $2:3=\square:27$
- **f** $9:2=36:\square$
- **g** $15:20=3:\square$
- **h** $8:12=\square:3$
- **i** $32:8=4:\square$
- **j** $20:\square=2:1$
- **k** $\square:21=4:3$
- **l** $8:\square=2:5$

Example 7 — Self Tutor

The student to leader ratio at a youth camp must be $9:2$. If there are 63 students enrolled, how many leaders are needed?

students : leaders $=9:2$

$\times 7$

$$\therefore\ 9:2=63:\square$$

$\times 7$

$$\therefore\ \square = 2\times 7 = 14$$

So, 14 leaders are needed.

2 A hospital employs nurses and doctors in the ratio $10:3$.

- **a** If there are 120 nurses, find the number of doctors.
- **b** If there are 45 doctors, find the number of nurses.

3 The ratio of teachers to students in a school is $1:18$. If there are 360 students, find the number of teachers.

4 A store sells 8 bracelets for every 3 necklaces they sell. If the store sold 56 bracelets yesterday, how many necklaces did it sell?

5 A car manufacturer produces station wagons and sedans in the ratio $2:5$. Last month they made 140 sedans. How many station wagons did they make?

6 Consider the **Opening Problem** on page **310**.

- **a** Find the ratio of chocolate topping to milk when Joel makes a chocolate milkshake.
- **b** If Joel only has 100 mL of milk, how much chocolate topping should he use?

7 After school, Sasha likes to make a snack by mixing raisins and nuts in the ratio $3:5$. When she checked the cupboard today, there were only 60 g of raisins and 75 g of nuts left. What is the largest snack Sasha can make using the correct ratio of ingredients?

F USING RATIOS TO DIVIDE QUANTITIES

If we are given a quantity to be divided in a certain ratio, we can use **fractions** to determine the size of each portion.

Example 8 Self Tutor

I wish to divide \$100 in the ratio $2:3$ to give to my children Petra and Sam. How much does each child receive?

The ratio contains $2+3=5$ parts in total.

Petra receives $\frac{2}{5}$ of the money, and Sam receives $\frac{3}{5}$ of the money.

$\therefore$ Petra receives $\frac{2}{5}$ of \$100 and Sam receives $\frac{3}{5}$ of \$100

$= \frac{2}{5} \times \$100$ $\qquad$ $= \frac{3}{5} \times \$100$

$= \$40$ $\qquad$ $= \$60$

Check: $\$40 + \$60 = \$100$ ✓

EXERCISE 16F

1 A bag of 18 chocolates is divided between Nick and Petrov in the ratio $2:1$.

a What fraction of the chocolates is received by: **i** Nick **ii** Petrov?

b How many chocolates are received by: **i** Nick **ii** Petrov?

2 Christina makes beetroot dip by combining beetroot and yoghurt in the ratio $5:3$. How much of each ingredient will she need to make:

a 200 g of dip **b** 600 g of dip?

3 In a recipe for scones, the ratio of butter to flour is $2:3$. How much of each ingredient is needed to make:

a an 800 g batch of scones **b** a 2 kg batch of scones?

4 Divide:

a £30 in the ratio $1:5$ **b** €28 in the ratio $5:2$ **c** \$600 in the ratio $4:1$.

5 ¥160 000 is divided in the ratio $3:7$. What is the smaller share?

6 Rob filled a 250 mL beaker by combining chemicals X and Y in the ratio $2:3$.
Celeste filled a 300 mL beaker by combining chemicals X and Y in the ratio $1:5$.
The contents of the beakers are combined. Find, in lowest terms, the ratio of chemical X to chemical Y in the resulting mixture.

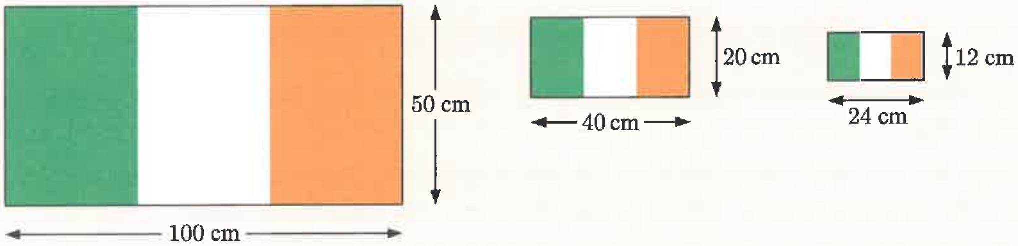
ACTIVITY 1 FLAG RATIOS

Every country in the world has a **national flag**. National flags are often seen in schools, on government buildings, at international conferences, and at sporting events.

Most countries specify the dimensions of their flag as **ratios** rather than lengths. This means that the flag can be made in many different sizes, but all copies will be in **proportion**.

What to do:

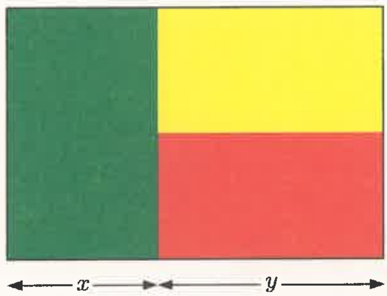
1 The flag of Ireland has three equal vertical stripes coloured green, white, and orange. The **height to length ratio** of the Irish flag is $1 : 2$. This means that the flag must be twice as long as it is high. So, acceptable dimensions of the Irish flag include:



Suppose an Irish flag is 90 cm high. Find the:

a length of the flag **b** width of each vertical stripe.

2 The flag of Benin is shown alongside. The flag consists of two equal horizontal stripes coloured yellow and red, and a vertical green stripe.
The height to length ratio of the flag is $2 : 3$.
The width of the green stripe is such that $x : y = 2 : 3$.
Suppose a Beninese flag is 40 cm high.



a Find the length of the flag.

b Find the dimensions of the:
i green stripe **ii** yellow and red stripes.

c Find the area of the:
i green stripe **ii** yellow stripe **iii** red stripe.

d What percentage of the flag is:
i green **ii** yellow **iii** red?

e Are the percentages you found in **d** true for all Beninese flags, or only this one?

f Find the dimensions, in centimetres, of the smallest Beninese flag such that each stripe has whole number dimensions.

3 Research the dimensions of the Australian flag. You should include:

- the dimensions of the Union Jack
- the shape of the Southern Cross.

GLOBAL CONTEXT NUTRITION

Global context:	Identities and relationships
Statement of inquiry:	Understanding the ratios in which we should eat certain food groups can improve our health and well-being.
Criterion:	Applying mathematics in real-life contexts

G RATES

We have seen that a **ratio** is an ordered comparison of quantities of the **same** kind.

For example, we can have a ratio of lengths or a ratio of times.

A **rate** is an ordered comparison of quantities of **different** kinds.

One of the most common rates we use is **speed**, which is a comparison between a *distance travelled* and the *time taken* to travel it.

Because we are comparing quantities of different kinds, the units cannot be omitted as they are with ratios. We use the word *per* which means "for every", to separate the units.

In the case of speed, the units we use are either kilometres per hour, or metres per second.

Other common examples of rates are:

Rate	*Example units*
Rate of pay	dollars per hour
Petrol consumption	litres per 100 km
Fuel efficiency	km per litre
Annual rainfall	mm per year
Unit cost	dollars per kg
Population density	people per square kilometre

Example 9 Self Tutor

A tap fills a 9 litre bucket in 3 minutes. Express this as a rate.

$$\text{rate} = \frac{9\text{ L}}{3\text{ minutes}}$$

$$\therefore \text{ rate} = \frac{9}{3}\text{ L per minute}$$

$$\therefore \text{ rate} = 3\text{ L per minute}$$

EXERCISE 16G

1 Describe the meaning of each rate:

- **a** 5 km per h
- **b** 15 dollars per h
- **c** 7 L per s
- **d** 99 cents per L
- **e** 30 kg per h
- **f** 14 g per min
- **g** 96 dollars per day
- **h** 66 m per s

2 Copy and complete:

- **a** A car uses 10 L of petrol every 160 km. The rate of fuel efficiency is ...... km per litre.
- **b** A train travels 416 km over 8 hours. This is a rate of ...... km per hour.
- **c** 28 L of water drains from a tank in 8 seconds. This is a rate of ...... L per s.
- **d** A driver works for 3 hours and receives £51. His rate of pay is £...... per hour.
- **e** A bottle of milk costs $4.98 for 2 L. This is a rate of $...... per L.
- **f** 30 bananas bought for €12 is at a rate of ...... cents per banana.

3 Annie travels 25 km by train to school, a journey which takes 45 minutes.
Victoria travels 20 km by car, a journey which takes 40 minutes.

- **a** Find the rate of travel for each girl in km per min.
- **b** Which mode of transport has the greater speed?

4 Xinsong works 8 hours as a waiter, earning $168.
Jay works 6 hours in the kitchen, earning $132.

- **a** Find the rate of pay for each person in dollars per hour.
- **b** Who is paid at a higher rate?

5 A greyhound runs 520 m in 30 seconds.
A horse gallops 2200 m in 2 minutes 5 seconds.
Which animal moves faster?

DISCUSSION

1 Why do we need to include units for rates but not for ratios?

2 How can we *convert* between units of rates?

3 Discuss how to convert:

- **a** dollars per kilogram into cents per kilogram
- **b** dollars per kilogram into cents per gram
- **c** kilometres per hour into metres per hour
- **d** kilometres per hour into metres per second.

H UNIT COST

When shopping, it is important to get good value for money. However, it is not always obvious which item represents the best value for money, because the same product can come in several different sized packages.

To properly compare prices, we can convert the cost of each sized package into a rate called the **unit cost**. It could be the cost per item, the cost per 100 grams, the cost per kilogram, or the cost per litre. We can then compare the unit costs to decide which sized package gives the best value.

ACTIVITY 2 — UNIT COST

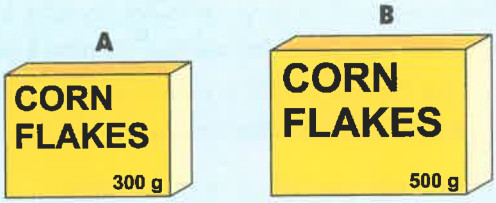
Most supermarkets include unit costs on the price tags of their items.

Next time you are at a supermarket, find out how the unit costs for the following items are measured:

- milk
- batteries
- steak
- flour
- dishwashing liquid
- paper towels

Example 10 — Self Tutor

By comparing the cost per 100 g, decide which box of cereal is better value for money.



$2.19 $3.29

A 300 g = 3 lots of 100 g

$\therefore$ the cost per 100 g

$= \dfrac{\$2.19}{3}$

$= \$0.73$ per 100 g

B 500 g = 5 lots of 100 g

$\therefore$ the cost per 100 g

$= \dfrac{\$3.29}{5}$

$= \$0.658$ per 100 g

So, box **B** is better value for money.

EXERCISE 16H

1 Use your calculator to find the unit cost of each item. Write your answer in the units in brackets.

a a twin pack of chocolate bars costs \$2.40 (\$ per bar)

b a packet of 3 tennis balls costs RM 11.40 (RM per ball)

c 5 kg of potatoes costs \$8.45 (\$ per kg)

d a 250 g packet of chips costs \$2.40 (cents per g)

e a 1.25 L bottle of soft drink costs \$0.99 (cents per L)

f 4.2 m of ribbon costs €8.40 (€ per m)

g 8 nights in a hotel costs £1323.20 (£ per night)

h 35 L of petrol costs \$43.40 (cents per L)

i 124 m^2 of concrete costs \$13 208.48 (\$ per m^2)

2 Decide which item is better value for money, by comparing:

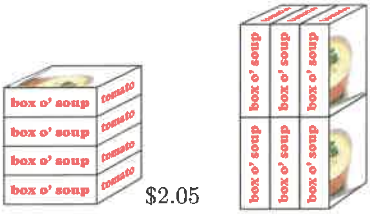
a the cost per 100 g

$5.10 $7.85

b the cost per 100 mL

£1.79 £2.99

c the cost per box



$2.05 $3.19

d the cost per tablet

$8.90 $6.45

e the cost per 10 m

50 m
€3.65

20 m
€1.55

f the cost per 10 g

420 g
$3.85

250 g
$2.15

3 A supermarket sells 110 g tubes of toothpaste for $3.19, and 160 g tubes of toothpaste for $3.99.

a Calculate the price per 10 g for each size tube.

b Which size tube is better value for money?

c The supermarket offers a "3 for 2" deal where if you buy two 110 g tubes of toothpaste, you receive a third one free. Does this represent better value for money than buying 160 g tubes?

DISCUSSION

Is it always best to buy the package size with the lowest unit cost?
What other things might you consider?

GLOBAL CONTEXT — GREAT EMPIRES

Global context: Orientation in space and time

Statement of inquiry: Mathematics can help give us an understanding of people, history, and culture.

Criterion: Applying mathematics in real-life contexts

GLOBAL CONTEXT

QUICK QUIZ

MULTIPLE CHOICE QUIZ

REVIEW SET 16A

1 Write as a ratio:

a \$9 is to \$4 **b** 11 g is to 5 g **c** 7 mm is to 3 cm

2 At a school, the ratio of right handed students to left handed students is $13 : 2$.
What fraction of the students are:

a right handed **b** left handed?

3 $\frac{9}{11}$ of a particular concrete mixture is cement. The remainder is water. Find the ratio of cement to water in the mixture.

4 Decide whether each pair of ratios are equal:

a $2 : 5$ and $6 : 15$ **b** $5 : 8$ and $20 : 36$ **c** $40 : 15$ and $8 : 3$

5 Write in lowest terms:

a $2 : 8$ **b** $24 : 15$ **c** $16 : 44$

6 Find the unknown value $\square$ in each proportion:

a $7 : 2 = 21 : \square$ **b** $18 : 10 = \square : 5$ **c** $\square : 35 = 9 : 7$

7 A commercial vehicle yard has vans and trucks in the ratio $5 : 3$. If there are 35 vans in the yard, how many trucks are there?

8 When Craig exercises, he does push-ups and sit-ups in the ratio $5 : 4$.
In one session Craig completed 60 push-ups. How many sit-ups did he complete?

9 €300 is divided between Courtney and Wendy in the ratio $3 : 7$.

a What fraction of the money does Courtney receive?

b How much money does Courtney receive?

10 A petrol pump delivers 42 litres of petrol into a car in 3 minutes. Write this rate in litres per minute.

11 **a** Which of these packets of cereal is better value for money?

b The 750 g packet is put on special, and now costs only \$3.49. Which packet is better value for money now?

\$5.50 \$3.99

12 An LED lightbulb consumed 125 kilojoules of energy over 5 hours.
A CFL lightbulb consumed 120 kilojoules of energy over 4 hours.

a Find the rate of energy consumption for each lightbulb in kilojoules per hour.

b Which lightbulb is more energy efficient?

13 At school, Grace has Maths and Science classes in the ratio 3 : 5.

a If Grace has 16 of these lessons each week at school, how many are:

i Maths **ii** Science?

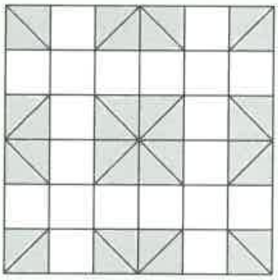
b Grace allocates study time at home in the same ratio.

i On Monday night she spent 24 minutes on Maths. How much time did she spend on Science?

ii On Tuesday night she spent 30 minutes on Science. How much time did she spend on Maths?

iii On Wednesday night she spent a total of 2 hours on these subjects. How much time did she spend on Maths?

14 Leo will use the tiling pattern alongside to tile his house. The pattern is made from a combination of square tiles and triangular tiles.



a Find, in lowest terms, the ratio of:

i the number of square tiles to the number of triangular tiles used

ii the area of square tiles to the area of triangular tiles used.

b Leo will use 30 square tiles in his kitchen. How many triangular tiles will he use?

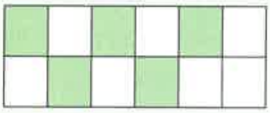
c Leo will use 40 triangular tiles in his bathroom. How many tiles will he use in the bathroom in total?

d Leo's dining room has area 36 m^2. How much area will the triangular tiles cover?

REVIEW SET 16B

1 Write as a ratio: "I saw seven white cars for every two red cars."

2 Find:



a the ratio of the shaded area to the unshaded area

b the fraction of the figure which is shaded.

3 Write a ratio equal to 32 : 12 by:

a multiplying both parts by 2 **b** dividing both parts by 4.

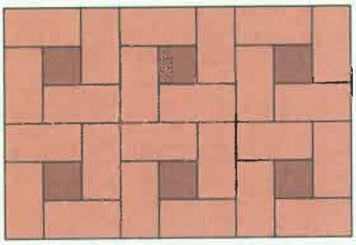
4 Write as a ratio in lowest terms:

a 53 minutes to 2 hours **b** 3 cm to 9 mm **c** 600 mL to 4 L

5 The width to height ratio of a television screen is 16 : 9. If the screen is 48 cm wide, find the height of the screen.

6 During the netball season, a club's win-loss ratio was $4 : 3$. If the team lost 12 matches, how many did they win?

7 This tile pattern is made from a combination of square tiles and rectangular tiles.
Find, in lowest terms, the ratio of:



a the number of square tiles to the number of rectangular tiles used

b the area of square tiles to the area of rectangular tiles used.

8 Water and vinegar are mixed in the ratio $4 : 5$ to make cleaning liquid for a coffee pot. How much vinegar is needed to make 720 mL of cleaning liquid?

9 A fruit grower plants apple trees and pear trees in the ratio $6 : 5$. If he plants a total of 1320 trees, how many of each type does he plant?

10 Margot receives \$220 for an eight hour nursing shift. Find her hourly rate of pay.

11 Kym cycled 75 km in 3 hours. Steve cycled 140 km in 5 hours. Who cycled faster?

12 Which chocolate bar is better value for money?

13 A butcher sells chicken pieces in 1 kg bags for \$7.50, and 400 g trays for \$3.75.

a Find the price per 100 g of chicken pieces bought in:

i a 400 g tray **ii** a 1 kg bag.

b Which size pack is better value for money?

c The 400 g trays are put on special, so that if you buy three trays you get a fourth tray free.

i Calculate the price per 100 g for chicken pieces bought with this special.

ii Does the special provide better value than the 1 kg bags?

14 Seymour looks after the drinks for his soccer team. He mixes cordial made from cordial concentrate and water in the ratio $1 : 6$.

a For each full 2 L bottle of cordial concentrate, how much water will Seymour need?

b Suppose Seymour filled a 56 L container with cordial. How much cordial concentrate and how much water did he need?

c When the container in **b** was half full, Seymour filled it up to the top again with water.

i How much cordial concentrate was present before Seymour added extra water?

ii How much water is present in the final mixture?

iii Write the new cordial concentrate : water ratio in lowest terms.

Chapter 17

Probability

Contents:

OPENING PROBLEM

Maurice wants to know the probability that a person who enters his store will buy something. One morning he counts 25 people entering his store. He notices that 9 of the people buy something.

Things to think about:

a Can you use this information to estimate the probability that a person entering Maurice's store will buy something?

b How accurate do you think the estimate is?

c How can the accuracy of the estimate be improved?

The **probability** of an event is the chance or likelihood of it occurring.

We can determine probabilities based on:

- what we theoretically expect to happen (theoretical probability)
- observing the results of an experiment (experimental probability).

In this Chapter we consider both theoretical and experimental probability.

A DESCRIBING PROBABILITY

We often use words to describe the probability of something happening in the future.

The probability for any event must lie between two extremes:

- If an event will *definitely not* happen, we say it is **impossible**.
- If an event will *definitely* happen, we say it is **certain**.

DISCUSSION

Consider these statements involving probability:

"We will probably buy a new television soon."

"It is likely that the storms will damage the crops."

"I am almost certain that I passed the exam."

"It is unlikely that our team will win today."

1 In each statement, which word or phrase describes the probability of the event occurring?

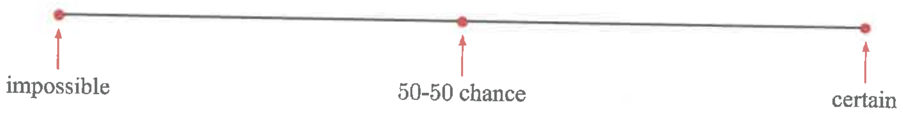
2 Discuss how each of these words is associated with probability:

a maybe **b** often **c** doubtful **d** rare

EXERCISE 17A

1 Copy the line below, then add the following words using arrows in appropriate positions:

- likely
- rarely
- almost impossible
- almost certain
- slightly less than 50-50 chance



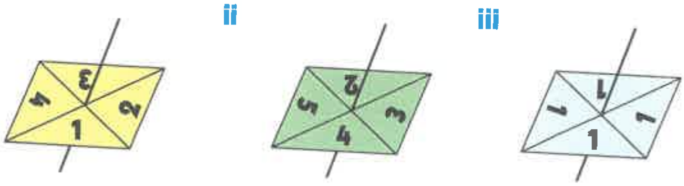
2 Describe, using a word or phrase, the probability that:

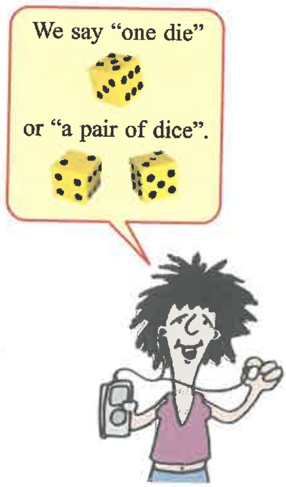
a Ken, who is now 13, will live to the age of 100 years
b you will win the major prize in Lotto in your lifetime
c there will be water at the beach
d you will be struck by lightning next year
e when tossed once, a coin will land on heads
f you will get homework tonight
g in your next car trip through the city, you will need to stop at a red light.

3 Describe as either *certain*, *possible*, or *impossible*:

a When rolling a die, a 6 results.
b When rolling a pair of dice, a sum of 7 results.
c When rolling a pair of dice, a sum of 14 results.
d When tossing a coin, a head results.
e When tossing a coin twelve times, it lands tails every time.
f When twirling each of the given square spinners, a 1 results:

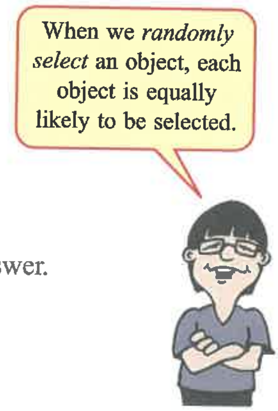
i **ii** **iii**





4 A bag contains 1 pink ticket and 99 purple tickets. A ticket is randomly selected from the bag.

a Is it certain that the ticket will be purple?
b Describe how likely it is that the ticket is purple.



5 A box contains 4 blue marbles and 7 red marbles. A marble is randomly selected from the box.

a Is the marble more likely to be blue or red? Explain your answer.
b Describe the chance that the marble will be:

i blue **ii** yellow **iii** red.

6 A black cat has 7 kittens. 2 kittens are white and 5 are black. 3 kittens are female and 4 are male. One of the eight cats is selected at random.

a Describe the chance that it is:

i black **ii** female.

b Is it reasonable to describe the chance that it is black *and* female? Explain your answer.

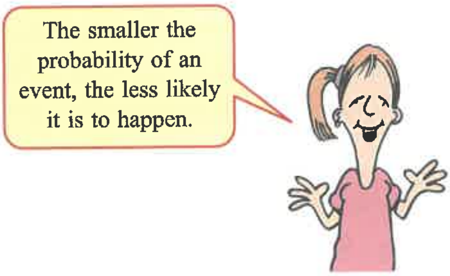
B USING NUMBERS TO DESCRIBE PROBABILITIES

To more accurately describe the probability of an event occurring, we use a number from 0 to 1.

An **impossible** event has probability 0 or 0%.

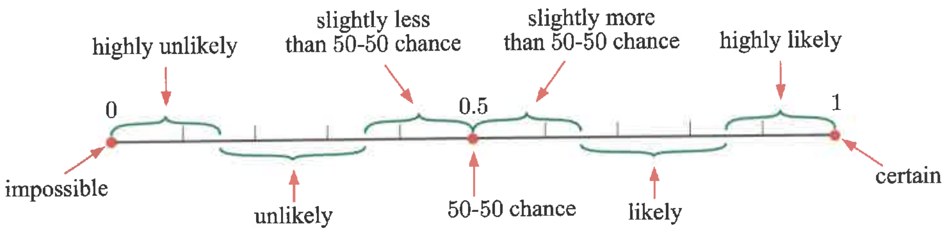
A **certain** event has probability 1 or 100%.

All other events have probability between 0 and 1.



An event that is equally likely to occur as not occur has probability 0.5, $\frac{1}{2}$, or 50%.

We can therefore place probabilities on a number line:



PROBABILITY NOTATION

In probability, we often use capital letters to represent events.

For example, we could let E represent the event that you will be on time for school tomorrow.

The probability of event E occurring is written $\mathrm{P}(E)$.

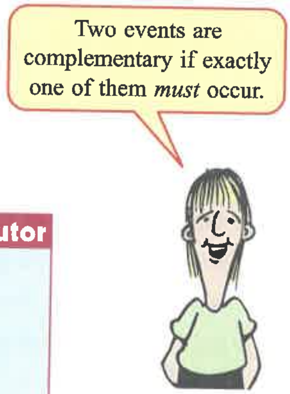
COMPLEMENTARY EVENTS

The **complement** of an event E is the event that E does *not* occur.

The complement of E is written E'.

For example, if A is the event that Adam wins the race, then A' is the event that Adam does *not* win the race.

For any event E, either E or E' *must* occur. Their probabilities must therefore sum to 1.



For any event E with **complementary** event E',
$\mathrm{P}(E) + \mathrm{P}(E') = 1$ or $\mathrm{P}(E') = 1 - \mathrm{P}(E)$.

Example 1 **Self Tutor**

Jill has a 70% chance of winning her squash game tonight.

a Describe this probability with a word.
b Write this using probability notation.
c Describe the complementary event and find its probability.

a It is likely that Jill will win her squash game tonight.
b Let J be the event that Jill will win her squash game tonight.
$\therefore \mathrm{P}(J) = 70\% = 0.7$
c J' is the event that Jill will lose her squash game tonight.

$$\begin{aligned}\mathrm{P}(J') &= 1 - \mathrm{P}(J)\\ &= 1 - 0.7\\ &= 0.3\end{aligned}$$

EXERCISE 17B

1 Write a word or phrase to describe the probability value:
a 0.97 **b** 0.3 **c** 0.5 **d** 1 **e** 0.56

2 People entering a charity raffle are told they have a $\frac{1}{5}$ chance of winning a prize.
a Write this probability as a decimal.
b Write a word or phrase to describe the probability that a particular person will win a prize.

3 A weather forecast reported a 45% chance of snow on Saturday, and an 80% chance of snow on Sunday.
a Write a word or phrase to describe the probability of snow on:
i Saturday **ii** Sunday.
b Is it more likely to snow on Saturday or on Sunday?

4 Write down the event which is complementary to:
a Terry will go to school tomorrow. **b** Jennifer has at least 3 pets.
c When selecting a ball from a bag, the result is either red or blue.

5 Suppose E is the event that Evelyn will remember to get her diary signed tonight. $\mathrm{P}(E) = 0.63$.
a State E', the complementary event of E. **b** Find $\mathrm{P}(E')$.
c Which is more likely to occur, E or E'?

6 The probability that Cameron's laptop will work today is 0.87. Find the probability that Cameron's laptop will *not* work today.

7 Each morning, Naomi catches the train to school. She catches the 8:00 am train $\frac{1}{10}$ of the time, the 8:10 am train $\frac{1}{2}$ of the time, and the 8:20 am train $\frac{2}{5}$ of the time.

a Write each of these probabilities as a decimal.

b Find the sum of the probabilities. Explain your answer.

c On any particular morning, which train is Naomi most likely to catch?

d Find the probability that, on any particular morning, Naomi catches *either* the 8:00 am *or* the 8:10 am train.

8 A container holds the same number of black discs as white discs.
Let B be the event that a randomly selected disc is black.

a Describe $\text{P}(B)$ using:

i a word or phrase **ii** a number.

b Describe the complementary event B' and find its probability.

9 A bag contains blue and green tokens only.
Let R be the event that a randomly selected token is red.

a Find $\text{P}(R)$. Explain your answer. **b** Find $\text{P}(R')$. Explain your answer.

10 Annabeth, Burt, and Callum are taking a History test today. Let A, B, and C be the events that Annabeth, Burt, and Callum pass the test respectively.
$\text{P}(A) = 0.92$, $\text{P}(B) = 0.66$, and $\text{P}(C) = 0.34$.

a Use a word or phrase to describe the probability that Burt will pass the test.

b Who is most likely to pass the test?

c **i** State A', the complementary event of A. **ii** Find $\text{P}(A')$.

d **i** Find $\text{P}(B) + \text{P}(C)$.

ii Are B and C complementary events? Explain your answer.

11 Lily and Ralph enjoy playing a sideshow game where they must try to knock over a set of 3 pins with a ball.
The table alongside shows the probability of each player knocking over 0, 1, 2, or 3 pins with a particular throw.

	Lily	Ralph
0 pins	0.2	0.05
1 pin	0.45	0.35
2 pins	0.3	0.5
3 pins	0.05	0.1

a For a particular throw, find the probability that:

i Ralph will knock over all 3 pins

ii Lily will not knock over any pins.

b Who is more likely to knock over:

i exactly 1 pin **ii** exactly 2 pins?

c A prize is won if at least 2 pins are knocked over.
Let L be the event that Lily wins a prize.
Find $\text{P}(L)$ and $\text{P}(L')$.

d Who would you say is better at the game?
Explain your answer.

C SAMPLE SPACE

When we consider the probability of a particular event occurring, it is useful to know what the possible **outcomes** are.

The set of possible outcomes is called the **sample space**.

For example, when we twirl this spinner, the sample space is $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

There are 8 possible outcomes.

EXERCISE 17C

1 List the sample space, and state the number of possible outcomes for:

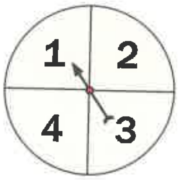
a flipping a disc with a smile on one side and a frown on the other

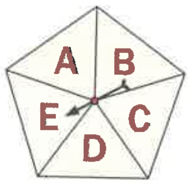
b choosing a day of the week

c selecting an integer from 10 to 19

d choosing a month of the year

e twirling the spinner:

i


ii


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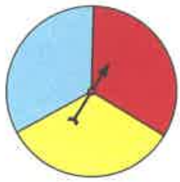


2 A two digit prime number is selected at random.

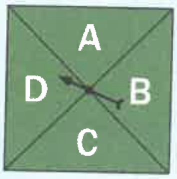
a List the sample space.

b How many possible outcomes are there?

c How many of the possible outcomes are greater than 50?

Example 2 Self Tutor

List the sample space for *two* spins of this spinner.
State the number of possible outcomes.



Let AB represent a result of A with the first spin and B with the second spin.

The sample space is:
{AA, AB, AC, AD, BA, BB, BC, BD, CA, CB, CC, CD, DA, DB, DC, DD}.

There are 16 possible outcomes.

3 List the sample space, and state the number of possible outcomes for:

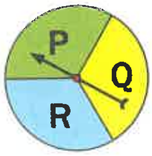
a tossing a 5 cent and a 10 cent coin

b the different orders in which 3 students Anna, Barry, and Catherine may line up

c tossing 3 different coins simultaneously

d the different orders for the genders of 3 kittens in a litter

e two spins of this spinner



f rolling two dice simultaneously

g the different orders in which 4 alphabet blocks W, X, Y, and Z may be placed in a line.

4 Tickets numbered 1 to 5 are placed in a bag. One ticket is selected from the bag and put to one side. A second ticket is then selected from the bag.

a List the sample space for the two selections.

b How many possible outcomes are there?

c How many of the outcomes contain two odd numbers?

DISCUSSION

A ball is in the middle of a flat, square table. It could roll off any edge of the tabletop with equal chance. How could you write down the sample space of possible outcomes?

D THEORETICAL PROBABILITY

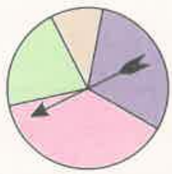
In *some* situations involving probability, the possible outcomes are all **equally likely**.

DISCUSSION — EQUALLY LIKELY OUTCOMES

For each of these situations, discuss whether the possible outcomes are equally likely:

- tossing a coin
- the winner of a tennis match
- the colour of the next car driving through the school gates
- spinning this spinner:

- spinning this spinner:



- choosing a student by placing each student's name in a hat and drawing one name out
- choosing a student by giving each student a tennis ball and seeing who can throw it furthest.

If the possible outcomes of an experiment are equally likely, the **theoretical probability** of a particular event is given by

$$\textbf{probability} = \frac{\textbf{number of outcomes in the event}}{\textbf{total number of possible outcomes}}.$$

Example 3 **Self Tutor**

Tickets numbered 1 to 9 are placed in a hat, and one is drawn at random.

Find the probability of obtaining:

a a 3 or a 7 **b** a number greater than 4.

There are 9 possible outcomes which are equally likely: 1, 2, 3, 4, 5, 6, 7, 8, and 9.

a Two of the outcomes are 3 or 7.

So, P(a 3 or a 7) $= \frac{2}{9}$.

b The five outcomes 5, 6, 7, 8, and 9 are greater than 4.

So, P(a number greater than 4) $= \frac{5}{9}$.

EXERCISE 17D

1 A six-sided die is rolled once.

a List the sample space of possible outcomes.

b How many possible outcomes are there?

c Determine the probability that the result is:

i a 4 **ii** greater than 3 **iii** not a 6.

2 A letter of the alphabet is chosen at random.

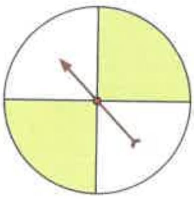
a How many possible outcomes are there?

b Find the probability of choosing:

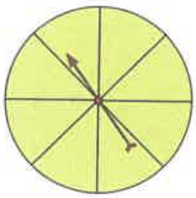
i T **ii** A, B, C, or D **iii** a letter from the word CHOCOLATE.

3 Determine the probability that the spinning needle will finish on green:

a



b



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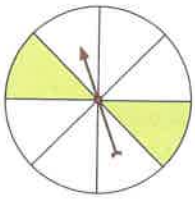

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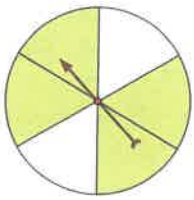


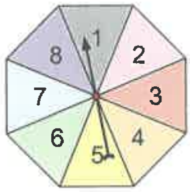
4 A month of the year is chosen at random. Find the probability that the chosen month:

a is July **b** starts with an "A" **c** contains the letter "r"

d has 30 days **e** is later in the year than May.

5 A class contains 29 students, 5 of whom suffer from asthma. Find the probability that a randomly chosen student does *not* suffer from asthma.

6 This spinner is a regular octagon.
If the spinner is spun once, find the probability of getting:

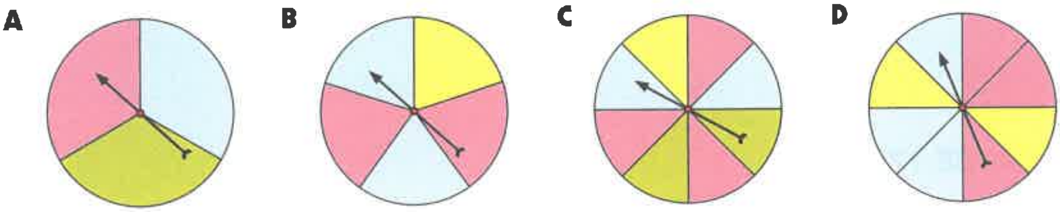


- **a** a 7
- **b** a 2 or a 4
- **c** an even number
- **d** a number less than 1
- **e** a number greater than 3.

7 A bag contains 1 blue, 2 yellow, and 5 green discs. One disc is randomly selected. Find the probability that the disc is:

- **a** blue
- **b** yellow
- **c** green
- **d** red
- **e** not blue
- **f** not yellow
- **g** neither yellow nor blue
- **h** blue, yellow, or green
- **i** neither blue, nor yellow, nor green.

8 If each of the following spinners is spun, which is most likely to end on pink?

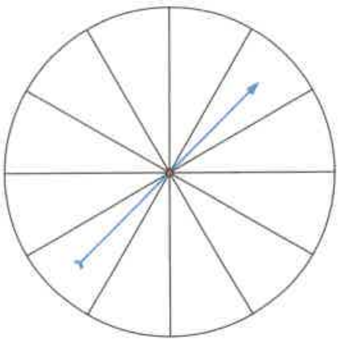


9 Frank selects one of these cards at random.

7	3	2	1	5
4	9	6	8	4
5	1	7	6	3

- **a** Find the probability that the card Frank selects is:
  - **i** a 3
  - **ii** the highest number in its row
  - **iii** not a 1, but is in the same row or column as a 1.
- **b** Let E be the event that the selected card is horizontally or vertically adjacent to exactly three other cards.
  - **i** Find $P(E)$.
  - **ii** State the complementary event E'.
  - **iii** Find $P(E')$.

10 For this question you should draw spinners with 12 sectors like the one alongside.
Draw a coloured spinner for which the probability of spinning:



- **a** red is 25% and blue is 50%
- **b** green is $\frac{1}{3}$ and blue is $\frac{1}{4}$
- **c** red is $\frac{2}{3}$, black is $\frac{1}{4}$, and yellow is $\frac{1}{12}$.

PRINTABLE SPINNERS

11 A number from 1 to 10 is selected at random. Let A be the event that a factor of 10 is selected, and B be the event that a number greater than 4 is selected.

a Find $P(A)$. **b** Find $P(B)$.

c Show that $P(A) + P(B) = 1$.

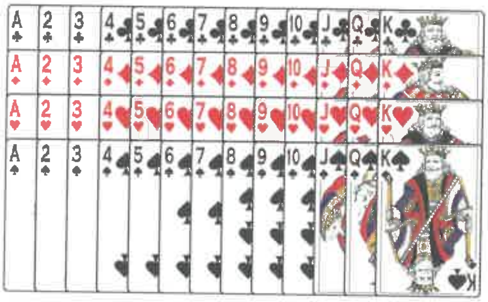
d Does this mean that A and B are complementary events? Explain your answer.

12 There are 52 cards in a pack of playing cards. They are divided into four suits:

- the two red suits Hearts and Diamonds
- the two black suits Spades and Clubs.

In each suit there is an Ace, the numbers 2 to 10, and three picture cards Jack, Queen, and King.

One card is randomly selected from a pack of cards. Determine the chance that it is:



a the Queen of ♥ **b** a club ♣ **c** an 8

d not an 8 **e** a picture card **f** a red card

g a black 2 **h** a red picture card **i** a 10 or an Ace.

Example 4 Self Tutor

3 coins are tossed simultaneously. Find the probability of getting:

a exactly one head **b** at least two heads.

The possible outcomes are {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

There are 8 possible outcomes.

a The three outcomes HTT, THT, and TTH have exactly one head.

So, $P(\text{exactly one head}) = \frac{3}{8}$.

b The four outcomes HHT, HTH, THH, and HHH have at least two heads.

So, $P(\text{at least two heads}) = \frac{4}{8} = \frac{1}{2}$.

13 2 coins are tossed simultaneously.

a List the sample space of possible outcomes.

b How many possible outcomes are there?

c Find the probability of getting:

i two heads **ii** two tails **iii** exactly one head **iv** at least one head.

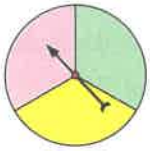
d Which two of the events in **c** are complementary?

14 Three seats are placed in a row. Three children A, B, and C enter the room and sit down randomly, one on each chair. Determine the probability that:

a A sits on the leftmost chair **b** they sit in the order BCA from left to right

c C sits in the middle **d** B does not sit in the middle.

15 This spinner is spun twice. Find the probability of getting:



- **a** two pinks
- **b** no greens
- **c** a yellow and a pink
- **d** at least one yellow.

16 One ball is randomly selected from this bag, and is put to one side. A second ball is then selected from the bag.
Find the probability of selecting:

- **a** a B and a D
- **b** first an E, then a blue ball
- **c** two consonants
- **d** two balls of the same colour
- **e** two balls of different colour.

DISCUSSION

When you roll an ordinary die, each outcome 1, 2, 3, 4, 5, or 6 is equally likely.

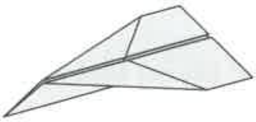
Suppose you rolled two dice and *added* the numbers.

- What are the possible outcomes for the sum?
- Are the different possible outcomes equally likely?
- What are the probabilities for each possible outcome?

E EXPERIMENTAL PROBABILITY

Isaac has made a paper plane. He throws it up into the air, and observes how it lands. There are two possible outcomes for how the plane lands:

- the *right way up*



- *upside down.*

We cannot argue that the probability of each outcome occurring is $\frac{1}{2}$, because there is no reason to think that these outcomes are equally likely.

In a situation like this, we can *estimate* the probability of each outcome occurring by conducting an **experiment**.

Suppose Isaac throws his plane into the air 50 times. It lands the right way up 31 times, and it lands upside down 19 times.

We say that:

- the *frequency* of landing the right way up is 31
- the *relative frequency* or fraction of times for which the plane landed the right way up is $\frac{31}{50}$.

Based on the experiment, we *estimate* the probability of the plane landing the right way up as the relative frequency.

So, we write P(right way up) $\approx \frac{31}{50} \approx 0.62$.

In a probability experiment:

- the **frequency** of a particular event is the number of times that this event is observed
- the **relative frequency** of an event is the frequency of that event divided by the total number of trials
- the relative frequency is the **experimental probability** of the event.

Experimental probabilities are not *exact*, but they are the best estimates we can make using the information available.

Example 5 **Self Tutor**

A ball is thrown to Joel 40 times. He catches the ball 29 times.
Estimate the probability that Joel will:

a catch the next throw **b** drop the next throw.

Joel caught the ball 29 times, and dropped it 11 times.

a P(catch)

$\approx \dfrac{\text{frequency of a catch}}{\text{number of trials}}$

$\approx \dfrac{29}{40}$

≈ 0.725

b P(drop)

$\approx \dfrac{\text{frequency of a drop}}{\text{number of trials}}$

$\approx \dfrac{11}{40}$

≈ 0.275

EXERCISE 17E

1 Adam is practising hitting baseballs in a cage. Out of 52 attempts, he hits the ball 38 times. Estimate the probability that Adam will hit the next baseball pitched at him.

2 Read the **Opening Problem** on page **328**. Estimate the probability that a person who enters Maurice's store will buy something.

3 Paula catches a bus to her daily yoga class. During a period of 66 days, she arrives at class on time on 47 occasions. Estimate the probability that tomorrow Paula will arrive:

a on time **b** late.

4 When Aina is taking her washing off the line, she throws her pegs at the peg basket nearby. She successfully throws 34 pegs into the basket, and misses with 17 pegs.
Estimate the probability that with her next throw, Aina will:

a throw the peg into the basket **b** miss the basket.

5 Jeffrey asked 80 people how many cousins they had. 6 people did not have any cousins, and 10 people only had one cousin. Estimate the probability that a randomly selected person has:

a one cousin **b** more than one cousin.

DISCUSSION

How can we improve the accuracy of a probability we calculate by experimentation?

How might the following affect the accuracy of an experimental probability:

- whether the experiment is performed with a consistent procedure
- the number of times the experiment is performed?

F THE ACCURACY OF EXPERIMENTAL PROBABILITIES

INVESTIGATION 1 THE NUMBER OF TRIALS IN AN EXPERIMENT

SIMULATION

Click on the icon to run a simulation for tossing a coin.

Each toss is a *trial* of the experiment.

For each toss, the probability that the coin lands on *heads* is $\frac{1}{2}$ or 0.5.

What to do:

1 Suppose our experiment involves 10 tosses of the coin.

a Use the simulation to toss a coin 10 times.
Copy the table and record your results as Experiment 1. Write the relative frequency as a decimal.

b Repeat the experiment 9 more times, recording your results in the table.

c In general, how close are the relative frequencies for your experiments to the theoretical probability?

Experiment	*Number of heads*	*Relative frequency of heads*
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

2 Repeat the procedure from **1** to perform 10 experiments with 100 tosses each.
Record your results in a table.
In general, are the relative frequencies closer to the theoretical probability than your results were with 10 tosses?

3 Repeat the procedure for 10 experiments with 1000 tosses each, and then 10 experiments with 10 000 tosses each.
Which number of trials gives the most accurate experimental probabilities?

From the **Investigation** you should have seen that as the number of trials in the experiment increased, the experimental probability of obtaining a head gets closer to the theoretical probability of 0.5 .

The more times an experiment is performed, the more accurate the experimental probability will be.

EXERCISE 17F

1 Don threw a tin can into the air 180 times. It landed on its side 137 times.
Later that afternoon, Don threw the same can into the air 75 more times. It landed on its side 46 times.


side end

a Find the experimental probability of the can landing on its side for:
 i the first experiment **ii** the second experiment.

b Which probability is likely to be more accurate? Explain your answer.

2 **a** In the first round of a competition, Sasha recorded 77 hits out of 80 shots at her target. Use this result to estimate her probability of hitting the target on a given shot.

b In the next round, Sasha scored 185 hits out of 200 shots. Use this result to estimate her probability of hitting the target on a given shot.

c Which of your estimates is more likely to reflect Sasha's actual probability of hitting the target?

3 Kelly rolled a die 200 times. Lucy rolled a die 50 times. Their experimental probabilities for obtaining each number are given in the tables below.

A

1	2	3	4	5	6
0.16	0.3	0.1	0.24	0.08	0.12

B

1	2	3	4	5	6
0.2	0.14	0.18	0.16	0.18	0.14

Suggest which set of results is Kelly's, and which is Lucy's. Explain your answer.

4 Two coins were tossed simultaneously 120 times.
The number of heads was recorded each time, with the results given in the table.
For each possible outcome, calculate the:

Number of heads	0	1	2
Frequency	32	57	31

a experimental probability **b** theoretical probability.

5 Keith and Megan want to estimate the probability that a randomly chosen person is vegetarian. Keith surveyed 150 people, and found that 36 of them were vegetarian. Megan surveyed 60 people, and found that 18 of them were vegetarian.

a Estimate the probability that a randomly chosen person is vegetarian, using:
 i Keith's results **ii** Megan's results.

b Which estimate is likely to be more accurate? Explain your answer.

c Combine Keith's and Megan's results to obtain an even more accurate estimate.

INVESTIGATION 2 THE CEREAL BOX PROBLEM

There is a plastic toy animal inside each box of Weet-Plus cereal. There are six different animals to collect: an elephant, a bear, a buffalo, a lion, a tiger, and a giraffe. The animals are equally likely to be found in any given box.

Gareth's mother buys 8 boxes of Weet-Plus cereal. What is the probability that Gareth will have a complete set of animals?

Since the animals are equally likely to occur in any given box, we can model the result from each cereal box using a roll of a die, with each number corresponding to a different animal:

1 = elephant   2 = bear   3 = buffalo   4 = lion   5 = tiger   6 = giraffe

What to do:

1 **a** Roll a die 8 times. Record your results in a table like the one alongside. Determine whether each number has appeared at least once, indicating that Gareth has collected the complete set.

Trial	*Rolls*	*Complete set?*
1	1, 4, 4, 6, 5, 3, 5, 1	No
2	2, 5, 1, 1, 3, 6, 6, 4	Yes
3	5, 2, 2, 6, 1, 5, 3, 2	No
⋮	⋮	⋮

b Repeat the experiment 50 times in total, recording your results.

c Use your results to estimate the probability that Gareth will collect a complete set of animals from 8 cereal boxes.

d Compare your results with those of your classmates.

2 To obtain a more accurate estimate of the probability, we need to run the experiment many more times. We will therefore use technology to help us. Click on the icon to open a spreadsheet which is set up to run 500 trials of the experiment.

SPREADSHEET

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Trial		Box 1	Box 2	Box 3	Box 4	Box 5	Box 6	Box 7	Box 8		Complete Set?		Probability
2														
3	1		1	5	1	5	5	1	6	4		No		0.106
4	2		4	1	3	6	5	4	2	6		Yes		
5	3		4	6	2	3	5	6	5	5		No		

The results in column L indicate whether each trial generates a complete set of animals. The value in cell N3 gives the experimental probability of obtaining the complete set based on these trials.

Pressing **F9** will generate a new set of random numbers, and a new experimental probability. Compare the results you obtain with your results in **1**. Which results do you expect are more accurate?

QUICK QUIZ

MULTIPLE CHOICE QUIZ

REVIEW SET 17A

1 Describe, using a word or phrase, the probability that:

a the next person to cross the street will be older than 70 years of age

b at least one goal will be scored in the next hockey match.

2 Choose the word or phrase from the list alongside which best describes each probability value:

probable
certain
impossible
extremely likely
"50-50" chance
unlikely

a $\frac{1}{5}$ **b** 0.97 **c** 0

d 0.5 **e** 1 **f** 75%

3 Suppose $P(A) = 0.94$.

a Find $P(A')$.

b Describe, using a word or phrase, the probability of:

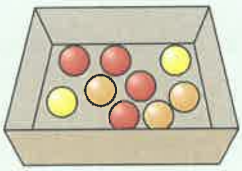
i A occurring **ii** A' occurring.

4 List the sample space for choosing:

a a prime number between 20 and 40

b a single digit odd number then a single digit even number.

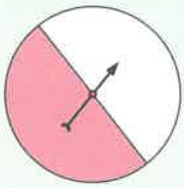
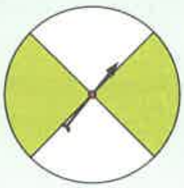
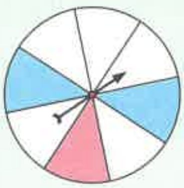
5 A box contains 4 red, 3 orange, and 2 yellow marbles. One marble is randomly selected from the box. Find the probability that the marble is:



a red **b** orange **c** not yellow

d yellow or red **e** neither yellow nor red.

6 Find the probability that the spinning needle will land on white:

a  **b**  **c**  **d** 

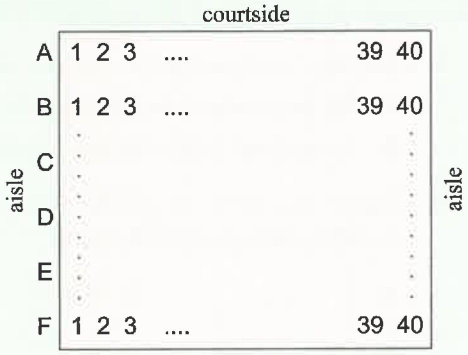
7 Raffle tickets numbered 1 to 100 are placed in a big bag. A ticket is selected at random. Find the probability that the ticket number is:

a 40 **b** an odd number **c** closer to 90 than to 20.

8 Four seats are placed in a row. Four children A, B, C, and D enter the room and sit down randomly, one on each chair. Find the probability that:

- **a** they sit in the order CADB from left to right
- **b** A and B sit on the ends
- **c** C and D sit next to each other
- **d** there is exactly one seat between B and C.

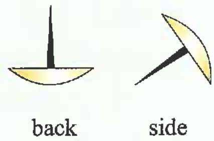
9 Carolyn has bought a ticket to a basketball match. Her seat will be in the Southern stand shown, but the seat is chosen for her at random.



- **a** How many seats are in this stand?
- **b** Find the probability that Carolyn will be sitting:
  - **i** in row A
  - **ii** in seat D25
  - **iii** in an aisle seat
  - **iv** in an odd-numbered seat.

10 A survey of forty five 18 year olds was conducted. It was found that 19 enjoyed camping. Estimate the probability that a randomly selected 18 year old likes camping.

11 Tom threw 47 drawing pins in the air, and 25 of them landed on their back.
Emily threw 85 drawing pins in the air, and 49 of them landed on their back.



- **a** Estimate the probability of a drawing pin landing on its back, using:
  - **i** Tom's results
  - **ii** Emily's results.
- **b** Which estimate is likely to be more accurate? Explain your answer.

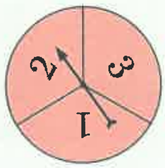
12 A company which makes plates was concerned that some plates were being chipped during the manufacturing process.
Two workers took samples of the plates, and counted how many were chipped.

Worker	*Sample plates*	*Chipped plates*
Donald	50	6
Nyree	300	24

- **a** Estimate the probability that a randomly chosen plate will be chipped using:
  - **i** Donald's sample
  - **ii** Nyree's sample.
- **b** Which of these estimates is likely to be more accurate?
- **c** Combine these samples to obtain a more accurate estimate.

REVIEW SET 17B

1 The spinner alongside is spun once. Describe the following events as either *certain*, *possible*, or *impossible*:



a an even number **b** a number less than 4

c a composite number.

2 Use a word or phrase to describe the probability of these events:

a There is a 95% chance that I will have to stand on the bus on my way to school.

b There is a 1% chance that Steve will quit his job.

c There is a 52% chance that Melina's first child will be a girl.

3 Jacqueline and Richard will compete in a tennis match tomorrow.
E is the event that Jacqueline will win, and $P(E) = 0.6$.

a State E', the complementary event of E. **b** Find $P(E')$.

4 List the sample space, and state the number of possible outcomes for:

a selecting a planet in our solar system

b choosing one letter and one number from this number plate.

5 The numbers 1 to 40 are marked on separate cards and placed in a hat. Determine the probability that a randomly chosen card is a multiple of 8.

6 A flower bulb packet contains 6 daffodil bulbs, 9 tulip bulbs, 4 iris bulbs, and 1 amaryllis bulb. Clive picks a bulb at random and plants it. Determine the probability that the resulting flower is not a daffodil.

7 Is it possible to make a spinner with the probabilities $P(\text{red}) = \frac{3}{8}$, $P(\text{blue}) = \frac{1}{4}$, $P(\text{green}) = \frac{1}{2}$? Explain your answer.

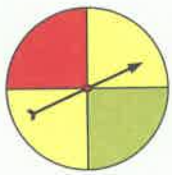
8 A tetrahedral die with sides numbered 1, 2, 3, and 4 is rolled twice.
Find the probability that:

a the first roll is 3 and the second roll is 1

b both rolls are the same number

c at least one 4 is rolled.

9 Jiao spun this spinner 40 times. Michelle spun it 150 times. Their experimental probabilities for obtaining each colour are given in the tables below. Suggest which set of results is Jiao's, and which is Michelle's. Explain your answer.



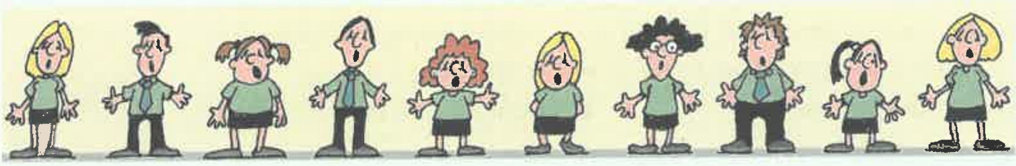
A

Red	Yellow	Green
0.15	0.45	0.4

B

Red	Yellow	Green
0.22	0.48	0.3

10 The members of a school choir are shown below:



Teegan	Daniel	Casey	Ben	Trish	Deb	Skye	Tim	Wendy	Donna
Age 12	Age 11	Age 11	Age 12	Age 10	Age 10	Age 11	Age 12	Age 10	Age 12

A member of the choir will be selected at random to perform a solo.

a List the sample space for this selection.

b Determine the probability that the selected member:

i has a name that starts with "T" **ii** is 12 years old.

c A 10 year old is randomly selected from the choir to introduce the conductor at a concert. Find the probability that the chosen person has red hair.

11 In the previous week, Aaron received 12 emails from his friend Patrick, 2 from his mother, 4 from his father, and 11 spam emails. Estimate the probability that the next email Aaron receives will be from:

a Patrick **b** one of his parents.

12 Hannah has a bag of tiles, each with a letter on it.

Letter	*Number of tiles*
E	12
A	9
D	4
F	2
P	2
K	1
X	1
Z	1
S	4

a How many tiles are in the bag?

b Hannah selects a tile from the bag.

i Which letter has a 25% chance of being selected?

ii Which letter has the greatest chance of being selected? Use a word or phrase to describe the probability of selecting this letter.

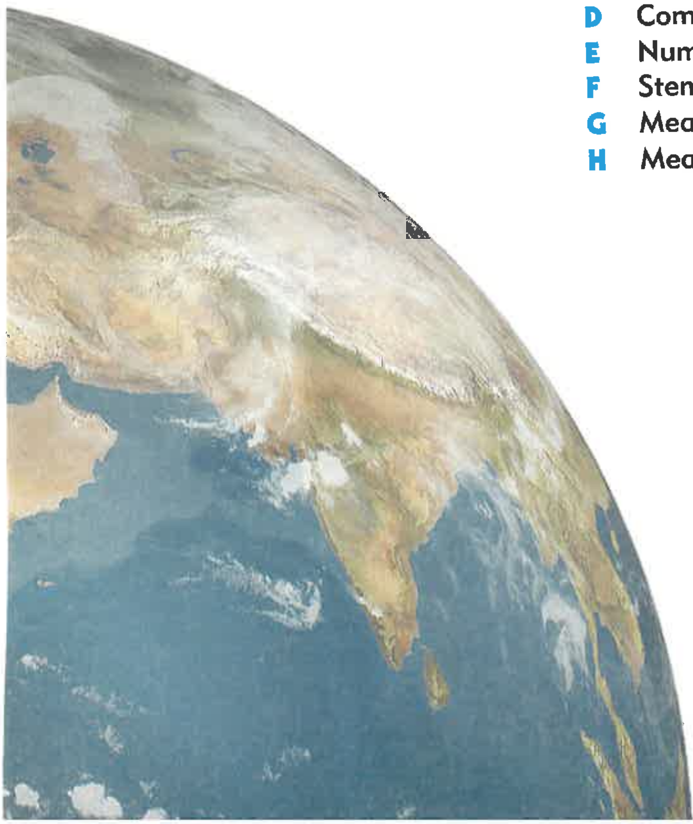
iii What is the probability of selecting a tile with a vowel on it?

c The tile Hannah selected is returned to the bag, and four more "P" tiles are added to the bag.
Write, as a decimal, the probability of selecting a "P" tile from the bag.

Chapter 18

Statistics

Contents:



OPENING PROBLEM

Zach and Ed both enjoy fishing with their father. They record how many fish they catch each time they go fishing over the holidays:

Zach:	5	8	4	6	9	7	9	6	9	9
Ed:	8	7	10	5	4	8	4	6		

Things to think about:

a By just looking at these values, is it easy to tell who catches more fish?

b Would it be fair to compare the boys by finding the total number of fish they caught?

c How can we determine which boy generally catches more fish?

When we collect facts or information about something, we call this information **data**.

For example, in the **Opening Problem**, the data are the numbers of fish caught by each boy.

Statistics is the study of collecting, organising, and analysing data.

Governments, businesses, sports organisations, manufacturers, and scientific researchers all use statistics to examine things.

HISTORICAL NOTE

The collection and analysis of data has been important to people for thousands of years.

- Before 3000 BC, the **Babylonians** recorded yields for their crops on small clay tablets.
- Pharaohs in **Ancient Egypt** recorded their wealth on walls of stone.
- Censuses were conducted by the **Ancient Greeks** so that taxes could be collected.
- After **William the Conqueror** invaded and conquered England in 1066, his followers took possession of estates previously occupied by Saxons. Confusion reigned over who owned what.
  In 1086 William ordered that a census be conducted to record population and wealth. A person's wealth was recorded in terms of land, animals, farm implements, and the number of peasants on the estate. All this information was collated in the **Domesday Book**. Regarded as the greatest public record of Medieval Europe, the Domesday Book is displayed in the National Archives in Kew.

William the Conqueror

A DATA COLLECTION

When we conduct a statistical investigation, there is always a target **population** which we want information from. The population might be:

- the entire population of a country
- an entire animal species
- the items produced by a machine.
- all the students at a school
- all the animals in a colony

One of the first decisions we need to make is who or what we will collect data from.

- A **census** involves collecting data about *every* individual in the whole population.
  A census is detailed and accurate but is expensive, time consuming, and often impractical.
- A **sample** involves collecting data about a *part* of the population only.
  A sample is cheaper and quicker than a census but is not as detailed or as accurate. Conclusions drawn from samples usually involve some error.

Example 1 — Self Tutor

Discuss whether a census or a sample would be used to investigate:

a the length of time an electric light globe will last

b the causes of serious car accidents in a particular state over one weekend.

a We would use a sample.
It is impractical to test every light globe produced as there would be none left for sale!

b We would use a census.
An accurate analysis of all accidents is important, and we hope there is only a small number to investigate.

EXERCISE 18A

1 State whether a census or a sample would be used to investigate:

a the number of goals scored each week by a waterpolo team

b the sizes of paintings in an art gallery

c the most popular radio station in Singapore

d the number of new cars that fail crash tests

e the number of litres of milk bought each week by a family

f the pets owned by students in a given Grade 7 class

g the number of camping holidays Canadians have taken.

2 Troy wants to know his classmates' favourite subject. He asks the entire class which subject they preferred. Is this a census or a sample?

DISCUSSION

1 For each of the following situations, discuss:
- *how* the information could be collected
- *why* the information would be useful.

a A hat manufacturer would like to know the head measurements of people in different age groups.

b The manager of your school canteen is interested in the types and quantities of food you like to eat.

c A company keeps records of what it buys throughout the year.

d Meteorologists measure temperature, rainfall, and atmospheric pressure throughout the world.

2 Suppose your school is next to a busy road. You wish to convince your local council that installing traffic lights outside the gates will make it safer for students to cross the road. Discuss what data you would like to collect and how you could collect it.

DISCUSSION

What information about yourself do you regard as *personal* information?

Research on the internet what is legally considered to be "personal information".

Discuss the rules and ethics of collecting, storing, and sharing personal information.

Why is it important to protect your personal information?

B CATEGORICAL DATA

Categorical data is data which can be placed in categories.

For example, the *breeds of dog* are categories. If we record the breeds of dog at a dog show, the result is categorical data.

We can organise categorical data using a **tally and frequency table**.

- The **tally** is used to count the data in each category.
  We use | to represent 1 and $\cancel{||||}$ to represent 5.
- The **frequency** gives the total number in each category.

A tally and frequency table allows us to identify features of the data such as the *mode*.

The **mode** is the most frequently occurring category.

Example 2 **Self Tutor**

Alan recorded the favourite subject of each student in his class. He used a letter to represent each subject: Mathematics (M), Art (A), Science (S), History (H), and English (E).

His data was:

A H M S H E S M S A S H H S M S S A H M S H S

a Draw a tally and frequency table for the data.

b Which subject was chosen least?

c Find the mode of the data. Explain what it means.

a

Favourite subject	*Tally*	*Frequency*
Mathematics	IIII	4
Art	III	3
Science	~~IIII~~ IIII	9
History	~~IIII~~ I	6
English	I	1
	Total	23

b English was chosen the least. It was the favourite subject of only one student.

c Science has the highest frequency, so it is the mode. The most common subject chosen was Science.

EXERCISE 18B

1 Students in a science class obtained the following grades:

D C C A A C C D C B C C C D
B C C C C E B A C C B C B C

a Complete a tally and frequency table for this data.

b How many students obtained a C?

c What fraction of students obtained a B?

d Find the mode of the data. Explain what it means.

2 A group of children at a summer camp were asked which sport they wanted to play. The choices were tennis (T), swimming (S), cricket (C), basketball (B), and athletics (A).

The data was: A A C T C C S A S T T T B A S A A C S A T A T B C

a Draw a tally and frequency table for the data.

b Find the mode of the data.

c What fraction of the children chose cricket? Write your answer in lowest terms.

3 People visiting the local fair were asked whether they preferred the side shows (S), the animals (A), the events in the main ring (E), the rides (R), or the food (F).

The results were: S E F S S F A R R S E S A F S E S E F S S S E E A

a Draw a tally and frequency table for the data.

b What was the most popular attraction?

c Find the percentage of people surveyed who preferred the main ring events.

4 The children exiting an Engineering exhibition were asked which exhibit they found most interesting. The exhibits included aerodynamics (A), nanocircuits (N), robotics (R), space travel (S), and telecommunications (T).

The results were: N S A T A S A R N N R S S R A A A S R N
N A T R A N A S A S S R R N S A

a Draw a tally and frequency table for the data.

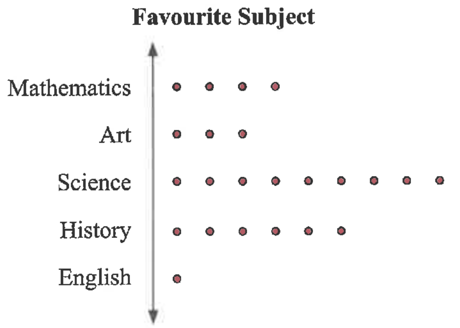
b Find the fraction of children who were most interested in space travel.

c Find the percentage of children who were most interested in aerodynamics.

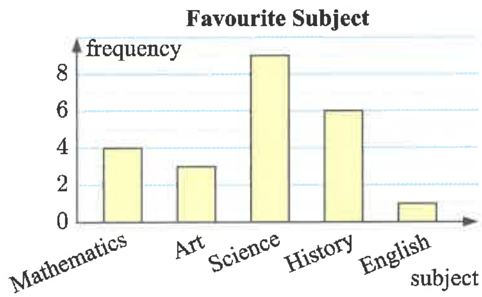
C DISPLAYING CATEGORICAL DATA

There are several types of graphs which we use to display categorical data:

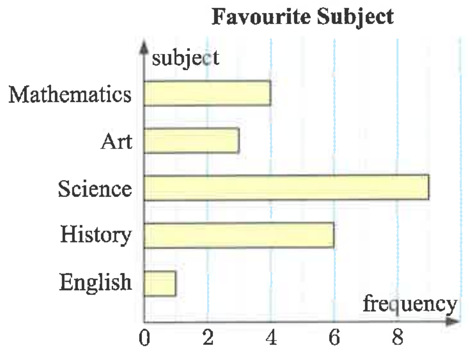
- A **dot plot** uses a dot to represent each data value.



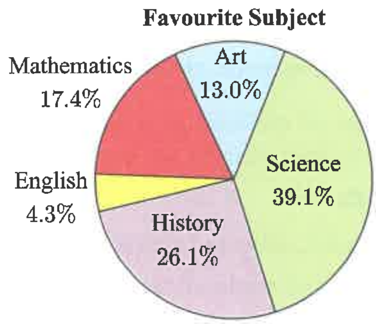
- A **vertical column graph** shows the frequency of each category by the height of its column.



- A **horizontal bar chart** shows the frequency of each category by the length of its bar.



- A **pie chart** shows the relative frequency of each category by the angle of its sector.



DISCUSSION

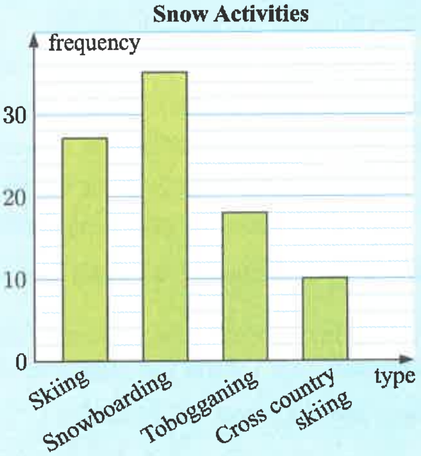
Each of the graphs above displays the data from **Example 2.**

- Which graph do you think is most effective in illustrating the data?
- Add up the percentages on the pie chart. Why do they not add up to exactly 100%?

Example 3 **Self Tutor**

The column graph shows the activities a group of children chose at a ski resort.

a What was the least popular activity?
b What is the mode of the data?
c How many children chose skiing?
d What percentage of children went tobogganing?



a The least popular activity was cross country skiing.
b The mode is snowboarding, since it has the highest frequency.
c 27 children chose skiing.
d The total number of children in the group $= 27 + 35 + 18 + 10 = 90$

So, the percentage of children who went tobogganing is $\frac{18}{90} \times 100\% = 20\%$.

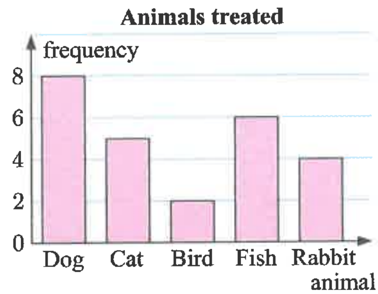
You should be able to generate each of these types of graph by hand and also using technology.

STATISTICS PACKAGE

EXERCISE 18C

1 A vet clinic kept a record of the animals they treated last week. The results are displayed in the column graph.

a How many cats were treated?
b How many animals were treated?
c Find the mode of the data.
d What percentage of the animals treated were rabbits?



2 The 20 players in a football team voted to decide who should be their captain. The results are given in the table alongside.

Candidate	*Votes*
Cameron	3
Greg	7
Luke	4
Steve	6

a Draw a horizontal dot plot to display the data.
b Which candidate received the:
 i most votes ii least votes?
c What percentage of the team voted for:
 i Luke ii Greg or Steve?

3 A group of people were asked whether they paid for streamed television, and if so, which provider they used. The results are shown in the table.

Television service	*Frequency*
N	63
F	38
S	19
D	56
other	15
no service	59

a How many people were surveyed?

b Which service was the most popular?

c What percentage of people surveyed:

i used the most popular service

ii did not use a streaming service?

d Draw a horizontal bar chart to display the data.

4 At a school camp, the students chose an ice cream flavour out of chocolate (C), strawberry (S), vanilla (V), and lime (L).

Their choices were: C V C S S V L S C V C V S L V S C C V V C S L C V
V C L S C C C V L S S L V C V C L C S C L C V L C

a Organise this data into a tally and frequency table.

b How many students chose vanilla?

c What percentage of the students chose lime?

d Find the mode of the data.

e Draw a column graph to display the data.

Example 4

Self Tutor

All of the Year 7 students at a school were asked to choose different countries to write about in a project.

The table alongside shows the continents of the countries they chose.

Continent	*Frequency*
Europe	15
Africa	18
Asia	16
North America	2
South America	9

a Draw a pie chart to display the data.

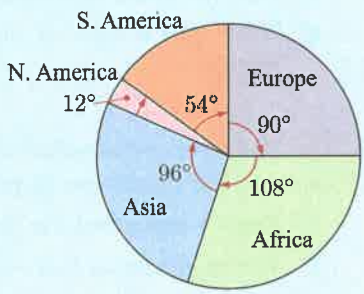
b What fraction of the students chose a country in Europe?

a In total there are $15 + 18 + 16 + 2 + 9 = 60$ students.

$\frac{1}{60}$ of $360° = \frac{360°}{60} = 6°$

so each country corresponds to an angle of $6°$ at the centre of the circle.

The Europe sector has angle $15 \times 6° = 90°$.
The Africa sector has angle $18 \times 6° = 108°$.
The Asia sector has angle $16 \times 6° = 96°$.
The North America sector has angle $2 \times 6° = 12°$.
The South America sector has angle $9 \times 6° = 54°$.



b The fraction of students who chose a country in Europe $= \frac{15}{60} = \frac{1}{4}$.

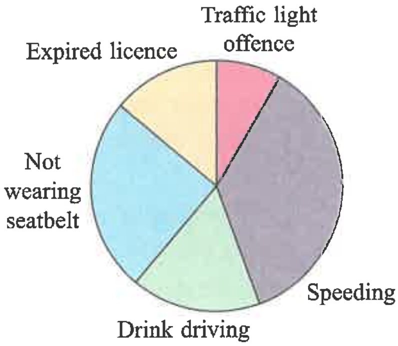
5 The pie chart illustrates the traffic fines handed out by a police officer over one month.
Determine whether each statement is true or false:

a The most common fine was for drink driving.

b About one quarter of the fines were for not wearing a seatbelt.

c More than half of the fines were for either speeding or drink driving.

d There were more traffic light offence fines than expired licence fines.



6 A group of 400 teenagers were surveyed about the type of films they would be most likely to see in a cinema. The results are shown in the pie chart.

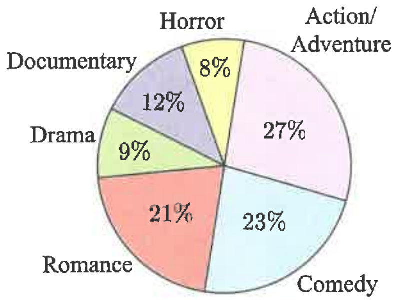
a State the mode of the data.

b How many of the teenagers would be most likely to see:

i a romance film

ii a documentary or action/adventure film?

c Find the angle at the centre of the circle of the sector for comedy films.



7 A survey of eye colour in a group of 30 teenagers gave these results:

Eye colour	Blue	Brown	Green	Grey
Number of students	9	12	2	7

a Illustrate these results on a pie chart.

b What percentage of the group have: **i** green eyes **ii** blue or grey eyes?

c What fraction of the group does *not* have blue eyes?

GLOBAL CONTEXT **CHARLES BOOTH'S POVERTY MAPS**

Global context:	Fairness and development
Statement of inquiry:	Collecting data and presenting it in a visually appealing way can enhance society's awareness and understanding of important issues.
Criterion:	Applying mathematics in real-life contexts

GLOBAL CONTEXT

D COMPARING CATEGORICAL DATA

To *compare* two data sets, we can draw a column graph for each data set on the same axes. We call this a **side-by-side column graph**.

We use a different colour for each data set so we can clearly see which is which.

Example 5 Self Tutor

Alan and Bill both recorded the favourite subject of each student in their respective classes. Their results are summarised in the table.

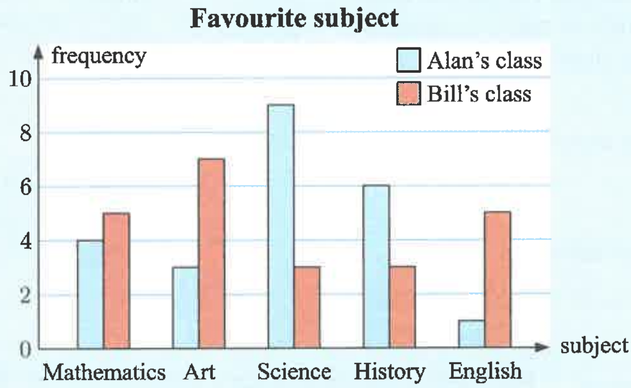
a Draw a side-by-side column graph to display the two sets of data.

b Compare the modes of the data sets.

c Compare the number of students who liked English in the two classes.

Subject	*Alan's class*	*Bill's class*
Mathematics	4	5
Art	3	7
Science	9	3
History	6	3
English	1	5
Total	23	23

a



b The mode for Alan's class is Science, whereas the mode for Bill's class is Art.

c There were more students who liked English in Bill's class than in Alan's class.

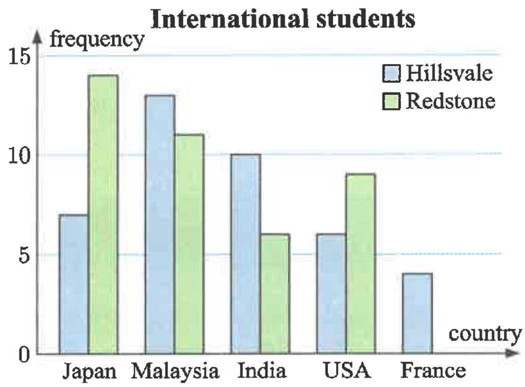
DISCUSSION

1 Why would it not be sensible to do a comparison like this if the number of students in Bill's class were different from the number of students in Alan's class?

2 If the numbers of students were different, what could we do with the data so that a valid comparison could be made?

EXERCISE 18D

1 Hillsvale School and Redstone School each have 40 international students. This side-by-side column graph shows the countries that these international students come from.



a How many of Hillsvale's international students come from India?

b Which school does not have any students from France?

c Which school has more students from:
  i Japan **ii** Malaysia?

d Compare the modes of the data sets.

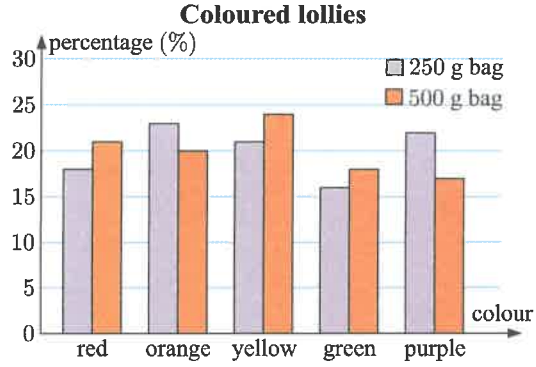
2 This side-by-side column graph shows the percentage of different coloured lollies in two different size bags.

a Find the mode for each data set.

b Which bag had a greater percentage of orange lollies?

c Which bag had *more* orange lollies?

d Explain why the graph uses the *percentages* of each colour rather than the *frequency* of each colour.



3 30 children and 30 adults were asked which section of the newspaper they enjoyed the most.

a Draw a side-by-side column graph to display the data.

b Find the mode of each data set.

c Which sections have the most difference in popularity between children and adults? Discuss your answer.

Section	*Children*	*Adults*
News	5	10
Sport	7	9
Comics	10	4
Puzzles	8	7

4 On a particular day, a fire truck and an ambulance each received 20 call-outs. The data below shows the location of each call-out, using the categories house (H), apartment (A), office (O), and factory (F).

Fire truck

F O A H F O H F O F
H F O F H F A H O H

Ambulance

H F O H A F A H F H
A H F H H O H F H A

a Draw a tally and frequency table for each set of data.

b Draw a side-by-side column graph to display the data.

c Find the mode of each data set.

d Which vehicle was called out to more offices?

E NUMERICAL DATA

Numerical data is data which is in number form.

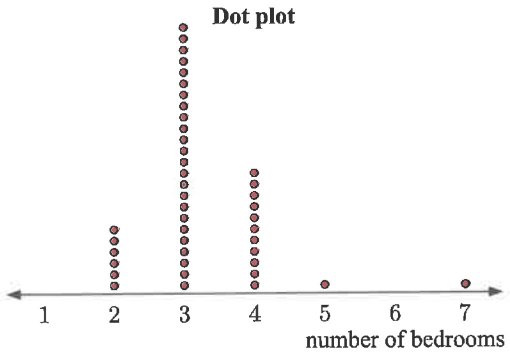
For example, the number of bedrooms in the houses sold by a real estate agent is shown below. The data collected is numerical data.

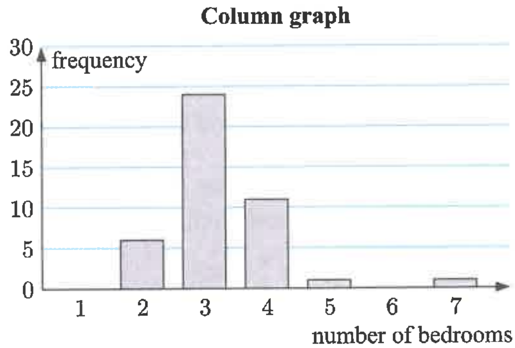
3 3 4 2 2 4 3 5 3 2 3 3 3 3 7
3 3 4 2 3 3 4 4 3 4 2 2 3 3 3
3 4 4 3 3 3 3 4 3 3 4 3 4

As with categorical data, numerical data can be organised using a tally and frequency table.

Number of bedrooms	*Tally*	*Frequency*
1		0
2	𝍸 𝍷	6
3	𝍸 𝍸 𝍸 𝍸 𝍷𝍷𝍷𝍷	24
4	𝍸 𝍸 𝍷	11
5	𝍷	1
6		0
7	𝍷	1
	Total	43

Numerical data can be displayed using a **dot plot**, **column graph**, or **stem-and-leaf plot**.





We will learn about stem-and-leaf plots in the next Section.

OUTLIERS

You should have noticed in the graphs above that the data value "7" is *separate* from the rest of the data. We call data values like this **outliers**.

Outliers are data values that are either much larger or much smaller than the general body of data.

Example 6 — Self Tutor

A tennis player has won the following numbers of matches in tournaments during the last two years:

1 2 0 1 3 1 4 2 1 2 3 4 0 0 1 2 2 3 2 1 6 3 2 1
1 1 1 2 2 0 3 4 1 1 2 3 0 2 3 1 4 1 2 0 3 1 2 1

a Organise the data using a tally and frequency table.

b Draw a column graph of the data.

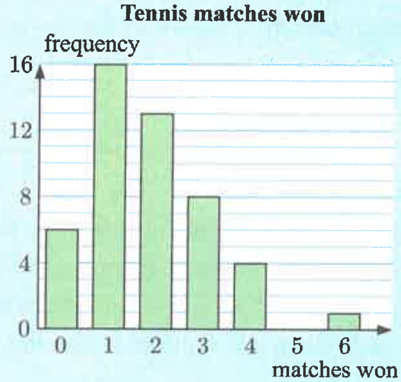
c How many times did the player advance past the second match of a tournament?

d On what percentage of occasions did the player win less than 2 matches?

a

Wins	*Tally*	*Frequency*
0	~~||||~~ |	6
1	~~||||~~ ~~||||~~ ~~||||~~ |	16
2	~~||||~~ ~~||||~~ |||	13
3	~~||||~~ |||	8
4	||||	4
5		0
6	|	1
	Total	48

b

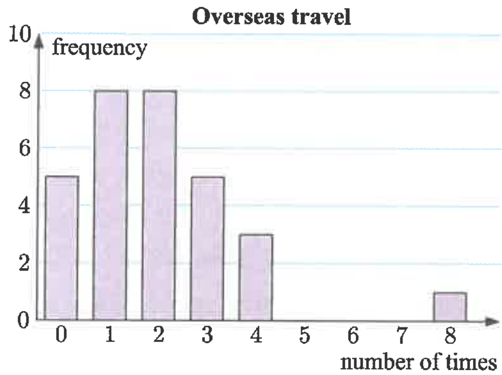


c The player won at least 2 matches on $13 + 8 + 4 + 1 = 26$ occasions.
So, the player advanced past the second match of a tournament 26 times.

d The player won less than 2 matches on $6 + 16 = 22$ occasions.
This corresponds to
$\frac{22}{48} \times 100\% \approx 45.8\%$ of the tournaments.

EXERCISE 18E

1 Workers in an office were asked how many times they had travelled overseas. The responses are displayed in the column graph alongside.



a How many workers have never been overseas?

b How many workers were surveyed?

c What percentage of workers have been overseas at least three times?

d Identify the outlier in the data.

2 Students at a school ran as many laps of the school athletics track as they could in one hour. The results are recorded in this frequency table.

Number of laps	*Students*
10	1
11	2
12	4
13	6
14	3
15	10
16	17
17	8
18	13
19	2

a Draw a column graph for this data.

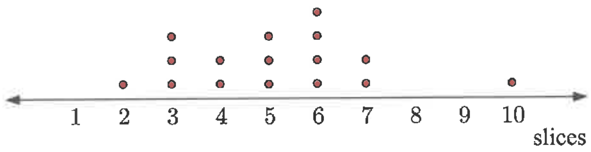
b What was the most common number of laps completed?

c How many students completed 12 laps or less?

d What fraction of the students completed at least 14 laps?

e Are there any outliers in the data?

3 A birthday party was held at an all-you-can-eat pizza restaurant. The number of slices eaten by each person is shown in the dot plot below.



a How many people attended the party?

b What was the least number of slices eaten?

c How many people ate six slices of pizza?

d Are there any outliers in the data?

4 Yvonne counted the number of chocolate chips in each biscuit of a packet, and obtained these results: 4, 7, 5, 5, 6, 4, 7, 8, 2, 5, 6, 6, 5, 5, 7, 5, 7, 5, 4, 6

a Draw a dot plot of her results.

b What was the most frequent number of chocolate chips?

c What was the highest number of chocolate chips?

d How many biscuits contained five chocolate chips?

e What percentage of biscuits contained less than five chocolate chips?

f Are there any outliers in the data?

F STEM-AND-LEAF PLOTS

It is not always practical to draw a dot plot or column graph for a set of numerical data.

For example, consider the numbers of photographs taken by tourists on a bus tour:

21	33	41	17	24	38	40	12	26	39	15	43	23	35	32
29	19	47	38	21	20	35	12	46	37	40	25	32	18	24

Each different value only appears once or twice, so the resulting column graph would have many columns of height 1 or 2. This is not very useful.

Instead, we construct a **stem-and-leaf plot** for the data.

A **stem-and-leaf plot** displays a set of data in order of size.

For each data value, the last digit is used as the **leaf**, and the digits before it determine the **stem** on which the leaf is placed.

We include a **scale** to tell us the place value of each leaf.

CONSTRUCTING A STEM-AND-LEAF PLOT

We use the following steps to construct a stem-and-leaf plot.

On the right we apply the steps to the tourist photograph data.

Step 1: Determine the stem labels.

The smallest value 12 has stem label 1.
The largest value 47 has stem label 4.
∴ the stem labels are 1, 2, 3, and 4.

Step 2: Draw the stem, writing the stem labels under one another in ascending order.

1	
2	
3	
4	

Step 3: For each data value, write the last digit as a leaf on the appropriate stem.
Write the scale to show the place value of each leaf.
This is an **unordered** stem-and-leaf plot.

1	7 2 5 9 2 8
2	1 4 6 3 9 1 0 5 4
3	3 8 9 5 2 8 5 7 2
4	1 0 3 7 6 0

Scale: 1 | 7 means 17 photographs

Step 4: If we wish, we can rewrite each set of leaves in ascending order.
This is an **ordered** stem-and-leaf plot.

1	2 2 5 7 8 9
2	0 1 1 3 4 4 5 6 9
3	2 2 3 5 5 7 8 8 9
4	0 0 1 3 6 7

Scale: 1 | 7 means 17 photographs

EXERCISE 18F

1 A group of schools in a city were surveyed to find out how many Grade 7 students they had. The results are displayed in this stem-and-leaf plot.

4	8
5	4 4 7
6	0 2 5 6 8
7	2 4 7
8	0 1 1
9	5 2
10	5 8
11	1

Scale: 4 | 8 means 48 students

- **a** How many schools were surveyed?
- **b** How many schools had 54 Grade 7 students?
- **c** What was the highest number of Grade 7 students in the schools surveyed?
- **d** How many schools had at least 80 Grade 7 students?

2 The numbers of runs scored by a batsman over a 30 game season were:

27, 7, 12, 74, 30, 11, 42, 19, 29, 51, 62, 14, 49, 22, 2,
35, 43, 12, 62, 22, 28, 37, 59, 40, 5, 13, 69, 32, 16, 21

- **a** Construct an unordered stem-and-leaf plot of the data. Make sure you include a scale.
- **b** Construct an ordered stem-and-leaf plot of the data.
- **c** How many times did the batsman score more than 25 runs?
- **d** Find the batsman's: **i** lowest **ii** highest score.

3 Walter recorded the number of pages in the daily newspaper for 4 weeks:

86 94 78 108 96 112 100 122 92 88 100 96 80 112
78 92 104 124 88 160 116 92 86 94 106 82 114 116

- **a** Construct a stem-and-leaf plot to display the data.
- **b** How many newspapers contained at least 100 pages?
- **c** What percentage of the newspapers contained less than 95 pages?
- **d** Are there any outliers in the data?

DISCUSSION

1. What advantages are there in *ordering* a stem-and-leaf plot?
2. When displaying numerical data, when is it best to use:
   - a dot plot
   - a column graph
   - a stem-and-leaf plot?

G MEASURING THE CENTRE

There are three different numbers which are commonly used to measure the **middle** or **centre** of a set of numerical data. These are the **mean** or **average**, the **median**, and the **mode**.

THE MEAN

The **mean** or **average** is the sum of all data values divided by the number of data values.

$$\textbf{mean} = \frac{\textbf{sum of data values}}{\textbf{number of data values}}$$

Example 7 Self Tutor

Find the mean of this data set: 5, 13, 10, 13, 15, 9, 17, 14

$$\begin{aligned}\text{mean} &= \frac{\text{sum of data values}}{\text{number of data values}}\\ &= \frac{5+13+10+13+15+9+17+14}{8}\\ &= \frac{96}{8}\\ &= 12\end{aligned}$$

THE MEDIAN

The **median** of a set of data is the *middle* value of the *ordered* set of data values.

To find the median of a set of data, we follow these steps:

Step 1: Write the data in order from smallest to largest.

Step 2: Starting at the ends, cross out the data values in pairs, working inwards until you reach the middle.

Step 3:
- If there is an *odd* number of data values, there will be one middle value. This value is the median.
- If there is an *even* number of data values, there will be two middle values. The median is the average of these two values.

Example 8 Self Tutor

Find the median of:

a 9, 7, 6, 14, 10, 4, 11 **b** 2, 5, 9, 4, 12, 3, 7, 4, 10, 7

a The ordered data set is: ~~4, 6, 7,~~ 9, ~~10, 11, 14~~

$\therefore$ median $= 9$

b The ordered data set is: ~~2, 3, 4, 4,~~ 5, 7, ~~7, 9, 10, 12~~

$\therefore$ median $= \dfrac{5+7}{2} = 6$

THE MODE

The **mode** is the score in the data set which occurs most often.

For example, the mode of the data set 0, 2, 3, 3, 4, 5, 5, 5, 6, 7, 9 is 5 since 5 occurs most frequently.

Example 9 Self Tutor

For the data represented by this dot plot, find the:

a mode **b** mean **c** median.

The data values are: 2, 2, 3, 3, 3, 3, 4, 5, 5, 7, 7

a The value 3 occurs most often, so the mode is 3.

b mean $= \dfrac{2+2+3+3+3+3+4+5+5+7+7}{11} = \dfrac{44}{11} = 4$

c The ordered data set is: ~~2, 2, 3, 3, 3,~~ 3, ~~4, 5, 5, 7, 7~~

$\therefore$ median $= 3$

EXERCISE 18G

1 Find the mean of each data set:

a 7, 10, 4, 11

b 12, 9, 6, 11, 17

c 3, 1, 5, 4, 4, 7

d 7, 5, 0, 3, 0, 6, 0, 9, 1, 4

e 2.1, 4.5, 5.2, 7.1, 9.3

f 5, 2.4, 6.2, 8.9, 4.1, 3.4

2 Find the median of each data set:

a 2, 4, 5, 8, 10, 11, 13

b 5, 8, 10, 11, 13, 16, 19, 20

c 2, 1, 1, 3, 4, 3, 2, 1, 5, 4, 3, 3, 0

d 5, 9, 2, 4, 6, 6, 7, 6, 11

e 1.2, 1.9, 2.2, 2.6, 2.9

f 0.5, 5.6, 3.8, 4.9, 2.7, 4.4

3 Consider the data set: 7, 8, 0, 3, 0, 6, 0, 11, 1.

a For this data, find the: **i** mean **ii** median **iii** mode.

b Do you think the mode is a suitable measure of the "centre" of this data set? Explain your answer.

4 Margaret played 10 games of Scrabble in a tournament, and obtained the following scores:

206 120 108 185 219 168 245 295 195 307

For these scores, find the: **a** mean **b** median.

5 The number of text messages that Jim received each day for the last 15 days were:

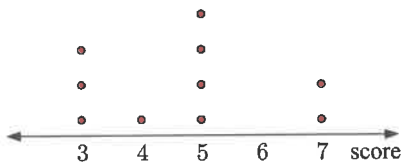
2 3 9 13 4 3 12 1 6 15 3 4 10 2 3

For this data, find the: **a** mean **b** median **c** mode.

6 For the data in each of the following graphs, find the:

i mode **ii** mean **iii** median.

a

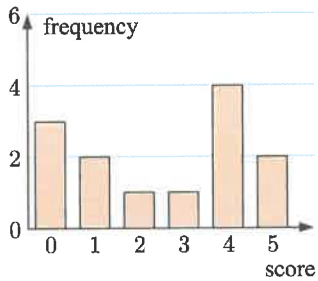


b

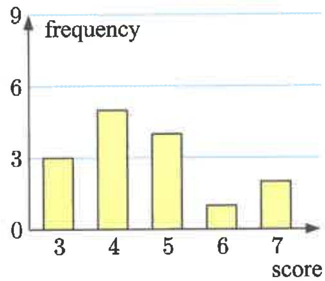
1	3 5 5
2	0 1 2 7
3	4 6 6 6
4	3 5 9

Scale: 1 | 3 means 1.3

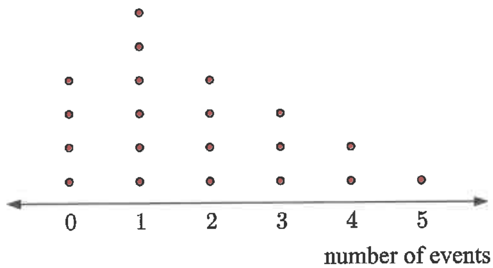
c



d



7 The students in a class were asked how many events they competed in at the school's swimming carnival. The results are displayed on the dot plot.



a How many students did not compete in any events?

b For this data, find the:

i mode **ii** median

iii mean.

c Copy the graph, then locate on it the mode, median, and mean.

8 Consider the data in the **Opening Problem** on page **348**.

a Calculate the mean and median for each boy.

b Who generally catches more fish? Discuss your answer.

9 Two groups of students attempted the same mental arithmetic test. Their scores out of 10 are shown below.

Group X:	7, 6, 6, 8, 6, 9, 7, 5, 4, 7
Group Y:	9, 6, 7, 6, 8, 10, 3, 9, 9, 8, 9

a Calculate the mean mark for each group.

b Since the groups contain different numbers of students, is it unfair to compare the mean scores for these groups?

c Which group performed better at the test?

10 Josh and Eugene each own a hot dog stand. They record the number of hot dogs they sell each day for two weeks. The results are:

Josh:	33, 40, 28, 43, 38, 32, 24, 35, 47, 29, 31, 36, 27, 38
Eugene:	39, 47, 32, 51, 48, 55, 61, 35, 49, 58, 52, 67, 55, 43

a What was the highest number of hot dogs that Josh sold in one day?

b Calculate the mean and median for each data set.

c Who generally sells more hot dogs? Discuss your answer.

H MEASURING THE SPREAD

In addition to the centre of a set of data, it is important to understand how the data is **spread**.

The simplest measure of spread is the **range**.

The **range** of a data set is the difference between the **maximum** or largest data value, and the **minimum** or smallest data value.

range = maximum value − minimum value

Example 10 Self Tutor

17 students were asked how many days they had stayed home sick from school so far this year. The results were:

2, 1, 5, 5, 3, 4, 3, 6, 2, 9, 4, 2, 3, 5, 6, 2, 3

Find the range of this data set.

The minimum value is 1 and the maximum value is 9.

So, the range $= 9 - 1 = 8$ days.

EXERCISE 18H

1 Find the range of each data set:

a 2, 4, 4, 5, 6, 8, 9, 10, 11, 11, 13

b 6, 6, 6, 6, 7, 7, 7, 7, 8

c 7, 9, 12, 9, 4, 8, 11, 6, 10

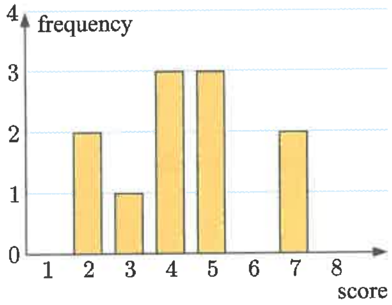
d 6, 8, 15, 4, 11, 18, 14, 10

e 19, 33, 27, 38, 46, 17, 39

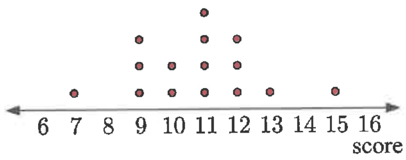
f 8.5, 4.2, 7.6, 7.2, 9.3, 9.1, 5.6

2 Find the range of the data displayed in each graph:

a



b



c

2	5 7 9
3	0 1 2 6 8
4	3 5 6 9
5	0 3 8

Scale: 5 | 0 means 50

d

1	7 8
2	1 3 4 6 6
3	0 2 5
4	1 4

Scale: 2 | 1 means 2.1

3 The numbers of items bought by customers at a convenience store were:

1 3 3 6 3 1 3 7 5 2 3 6 5 5 4

a Draw a dot plot to display the data.

b Calculate the mean, median, and range of the data, and indicate these values on your dot plot.

4 The table below shows the maximum temperatures, in °C, in Australia's capital cities for one week.

City	*Mon*	*Tue*	*Wed*	*Thu*	*Fri*	*Sat*	*Sun*
Adelaide	24	22	23	23	30	30	28
Brisbane	28	29	29	27	28	28	29
Canberra	25	24	21	23	24	23	27
Darwin	34	33	33	33	31	31	31
Hobart	22	18	19	18	19	23	21
Melbourne	26	20	22	22	23	27	30
Perth	27	30	32	34	25	27	32
Sydney	29	26	24	26	24	24	26

a Calculate the range for each city.

b For which city did the maximum temperature have the:

i most variation **ii** least variation?

ACTIVITY

With most goods we buy, we can read the amount we are buying on the packaging. For example, we might buy a 500 g bag of sultanas, a 20 m roll of baking paper, or a 600 mL bottle of water. But how do we know the manufacturer is telling the truth?

Choose a bulk packet that has several of the same item in it. For example:

- a bag containing 8 balls of wool, each weighing 100 g
- a packet containing 12 candles, each 15 cm long
- a 6 pack of fruit juice cartons, each containing 175 mL.

Your task is to analyse whether the manufacturer has made a truthful claim about how much is in their product.

What to do:

1 Choose your item to anaylse. Describe exactly:

a what claim you are testing **b** how you will test it.

2 **a** Measure each item in your packet. Round your data appropriately. Construct a stem-and-leaf plot to display your results.

b For your data, find the:

i median **ii** mean **iii** range.

c Were the results in **b** what you expected? Explain your answer.

3 Calculate the percentage of items that were below the amount stated on the packaging.

4 From your results, can you form any conclusions about the amount of each item in your packet? Do you think the manufacturers are telling the truth? Discuss your answers.

MULTIPLE CHOICE QUIZ

QUICK QUIZ

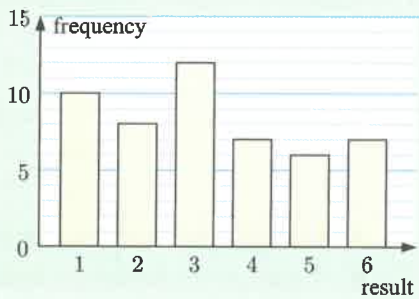
REVIEW SET 18A

1 State whether each of the following is a census or a sample:

a Customers passing through one of the checkouts at a supermarket are asked for the postcode of their home suburb.

b All members of a football club are asked their opinions about the membership fees.

2 Samantha rolled a die 50 times. The results are shown in the column graph alongside.



a How many times did Samantha roll a 2?

b What percentage of the rolls were greater than 4?

c For this data, find the:

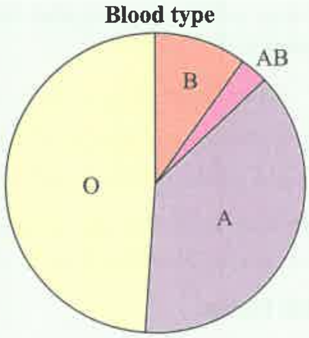
i mode **ii** median.

3 A random sample of people were surveyed about their blood type. The results are displayed in the pie chart opposite.
Decide whether each of these statements is true or false, giving reasons for your answers.

a The most common blood type is type O.

b More than one quarter of the people surveyed have type B blood.

c More than one half of the people surveyed do not have type O blood.



4 The hair colours of the students in a class are shown in the table.

a Construct a horizontal bar chart to display this data.

b For this group of students, which hair colour was least common?

c What percentage of students in the class have blonde hair?

Hair colour	*Frequency*
Red	4
Brown	17
Black	11
Blonde	8

5 40 boys and 40 girls were asked to name their favourite piece of playground equipment.

Equipment	*Boys*	*Girls*
Slide	13	6
Swings	8	9
Monkey bars	7	15
Zip-line	12	10

a How many boys chose the zip-line?

b How many girls chose the slide?

c Draw a side-by-side column graph to display the data.

d Which piece of playground equipment was most popular with:

i boys **ii** girls?

e Did more boys or girls choose the zip-line?

6 Consider the data set: 3, 4, 6, 6, 7, 9, 12, 13, 14, 17, 19.
Find the: **a** mean **b** median **c** mode.

7 During the 24 game netball season, Alyssa played in all games and scored 482 goals. Due to injury, Stephanie only played 19 games, and scored 335 goals. Which player received the award for the highest average goals scored per game?

8 An arithmetic test out of 10 was given to two groups of students. Their scores were:

Group 1: 7, 8, 6, 6, 9, 10, 7, 8, 8, 7
Group 2: 10, 9, 10, 8, 4, 9, 10, 8, 7, 7, 9

a Find the mean and median score for each group.

b Which group performed better? Explain your answer.

9 Monica received £60 for her birthday. She spent £15 on a book, £20 on a necklace, and saved the remaining £25. Draw a pie chart to display this information.

10 Some children were asked how much pocket money they receive each week. The results, in dollars, were:

2, 4, 0, 10, 4, 0, 5, 5, 2, 4, 10, 5, 2,
0, 10, 0, 2, 5, 2, 8, 5, 10, 2, 0, 10

a Draw a dot plot of the data.

b Find the:

i mode **ii** mean **iii** median **iv** range.

c Indicate the values found in **b** on your dot plot.

REVIEW SET 18B

1 State whether a census or a sample would be used to investigate:

a the number of double-yolk eggs in cartons of 12 eggs

b the ages of visitors to a museum

c the sports played by students in a Grade 7 class

d the makes of cars for sale in a particular car yard

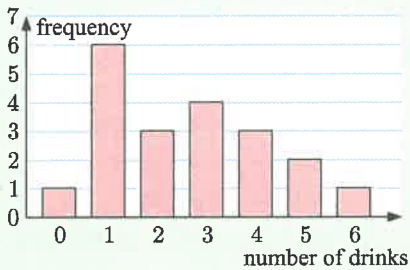
e the weights of newborn lambs.

2 Melanie is conducting a survey of her classmates about their favourite sport. Will the results be categorical or numerical data?

3 The numbers of drinks sold at tables in a café are displayed in this column graph.

a State the mode of the data.

b What percentage of tables ordered 4 or more drinks?

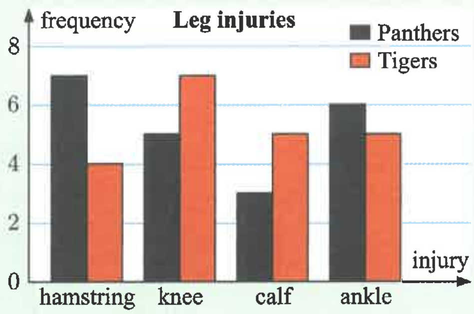


4 This side-by-side column graph shows the leg injuries received by two rugby teams during a season.

a How many knee injuries were received by:

i the Panthers **ii** the Tigers?

b Which team suffered the most ankle injuries?



5 A supermarket puts chocolate bars on sale next to the cash register. They record the number of chocolate bars bought by each customer over an hour. The results were:

0 0 1 1 1 2 1 0 3 1 2 4 0 1 2 7 1 1 0 2 3

a Draw a dot plot to display this information.

b Are there any outliers in the data?

6 The ages of singers in a choir were recorded in the table alongside.

a Copy and complete the tally and frequency table.

b Draw a column graph for this data.

c What is the most common age of the singers?

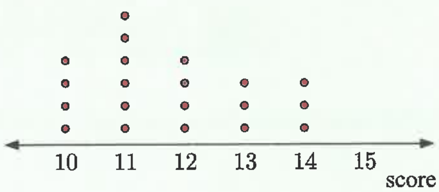
d Find the mean age of the singers.

e What percentage of the singers are at least 25 years old?

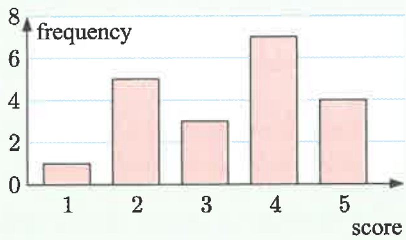
Age	*Tally*	*Frequency*
18	||	
19	||||	
20	𝍸 ||	
21	𝍸 𝍸 |	
22	𝍸 ||||	
23	𝍸 𝍸 |	
24	𝍸 ||	
25	𝍸 ||	
26	|||	
	Total	

7 Find the mean, median, mode, and range of the data represented in each graph:

a



b



8 Jillian recorded the number of pages in the weekly local newspaper over a period of time. The results are shown in the stem-and-leaf plot.

a What percentage of newspapers contained at least 60 pages?

b Find the median of the data.

3	8
4	1 1 2 3 6 7 8 8
5	1 2 7 8 9
6	0 0 1 2
7	1

Scale: 3 | 8 means 38 pages

9 A group of 20 children played a round of mini-golf. Their scores were:

43 32 59 35 60 26 39 41 53 67
39 54 28 46 65 30 45 23 32 65

a Draw a stem-and-leaf plot to display the data.

b How many children scored less than 40?

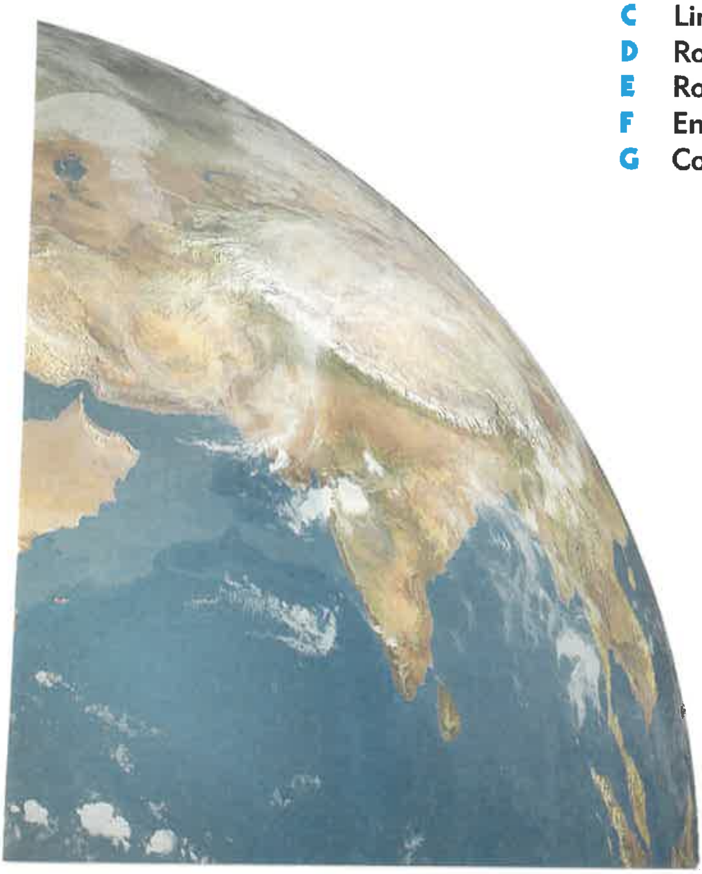
c What percentage of children scored more than 55?

d For this data, find the: **i** mean **ii** median **iii** range.

Chapter 19

Transformations

Contents:



OPENING PROBLEM

Consider the photograph alongside.

Things to think about:

a How is the photograph *transformed* into each image below?

b For which transformation has the *size* of the photograph changed?

c For which transformation has the *orientation* of the photograph changed?

A

B

C

In this course we will study four types of **transformation**:

- translations
- reflections
- rotations
- enlargements and reductions.

When we perform a transformation, the original shape is called the **object**. The shape which results from the transformation is called the **image**.

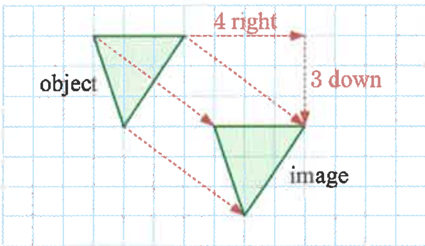
If the object is A then we can label the image A′.

A TRANSLATIONS

In a **translation**, every point on an object is moved the same distance in the same direction to form the image.

We can describe a translation using a horizontal step to the right or left, followed by a vertical step up or down.

For example, in this translation, the object has been translated 4 units right and 3 units down to produce the image.

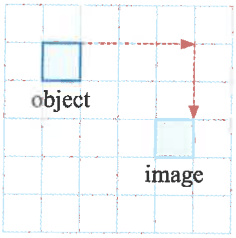


To translate an object, first translate its vertices or other important points, then join the points to form the image.

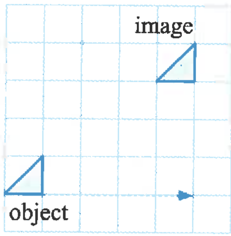
EXERCISE 19A

1 Describe each translation using a horizontal step and a vertical step.

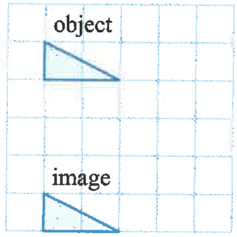
a



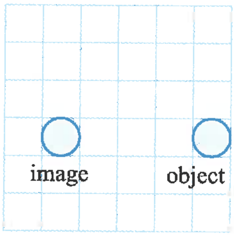
b



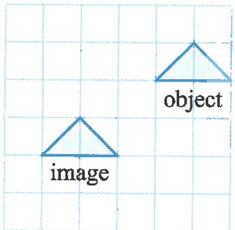
c



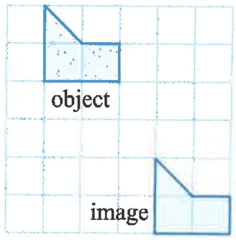
d



e

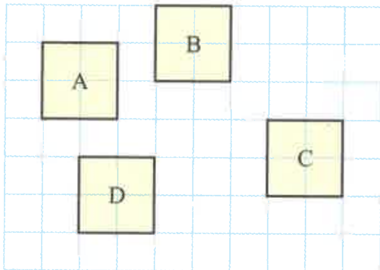


f



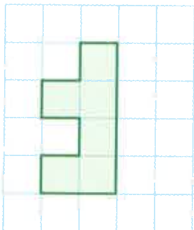
2 Describe the translation from:

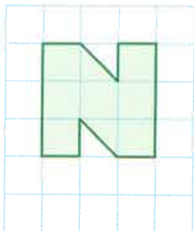
a A to B **b** B to A **c** B to C
d C to B **e** D to C **f** C to D
g B to D **h** D to B.

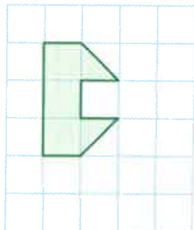


3 Copy each figure onto grid paper and complete the translation given:

a 3 units right, 4 units down **b** 6 units left, 4 units up **c** 2 units right, 5 units up







4 **a** Translate figure A 5 units left and 4 units up to produce figure B.

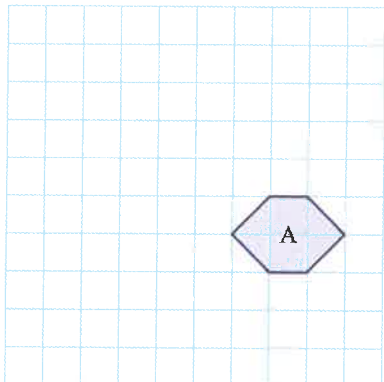
b Translate figure B 1 unit right and 6 units down to produce figure C.

c Describe the translation which is needed to move:

i figure A to figure C

ii figure C to figure A.

PRINTABLE DIAGRAMS



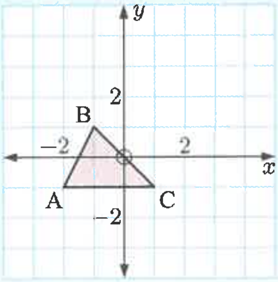
Example 1 **Self Tutor**

Consider the triangle in the Cartesian plane alongside.

a State the coordinates of the vertices of the triangle.

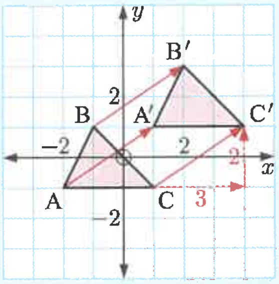
b Translate the triangle 3 units right and 2 units up.

c State the coordinates of the vertices of the image.

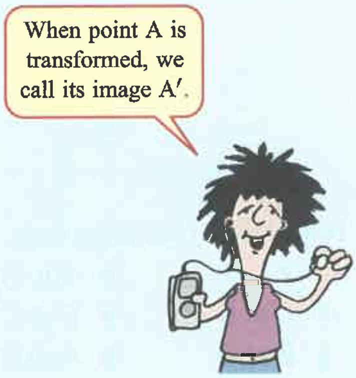


a A is $(-2, -1)$, B is $(-1, 1)$, C is $(1, -1)$.

b



c A′ is $(1, 1)$, B′ is $(2, 3)$, C′ is $(4, 1)$.



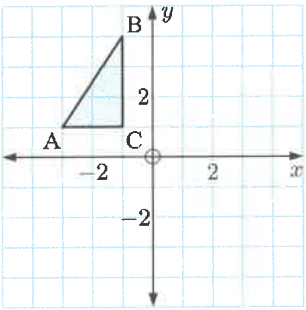
5 For each figure:

i State the coordinates of the vertices.

ii Translate the figure in the direction indicated.

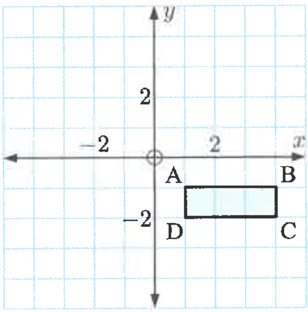
iii State the coordinates of the vertices of the image.

a



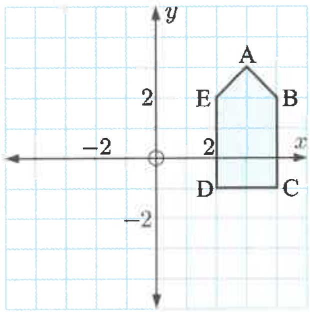
4 units right, 3 units down

b



2 units left, 4 units up

c



3 units left, 4 units down

6 **a** Find the image when each point is translated 2 units right and 5 units up:

i $(0, 0)$ **ii** $(1, 3)$ **iii** $(-3, -5)$ **iv** $(2, -1)$

b Copy and complete:

When $P(a, b)$ is translated 2 units right and 5 units up, the image is $P'(...., ...)$.

7 **a** Find the image when each point is translated 3 units left and 1 unit down:

i (0, 0) **ii** (3, 2) **iii** (−1, 4) **iv** (1, −2)

b Copy and complete:

When $P(a, b)$ is translated 3 units left and 1 unit down, the image is $P'(...., ...)$.

8 **a** Find the coordinates of the image when $P(0, 0)$ is translated:

i 4 units right and 2 units up **ii** 3 units left and 5 units down.

b Find the coordinates of the image when $P(2, 3)$ is translated:

i 2 units right and 1 unit up **ii** 1 unit left and 3 units down

iii 2 units left and 3 units up.

c Copy and complete:

i When $P(a, b)$ is translated m units right and n units up, the image is $P'(...., ...)$.

ii When $P(a, b)$ is translated m units left and n units down, the image is $P'(...., ...)$.

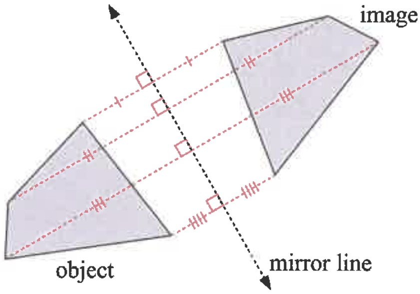
B REFLECTIONS

When we **reflect** an object in a **mirror line**, the image is its **reflection**.

To reflect an object, we draw lines at right angles to the mirror line which pass through important points on the object.

Each image point is the same distance from the mirror line as the object point, but is on the other side of the mirror line.

DEMO

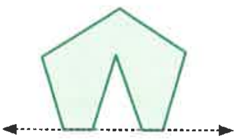


EXERCISE 19B

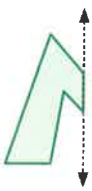
1 Reflect each object in the given mirror line.

PRINTABLE DIAGRAMS

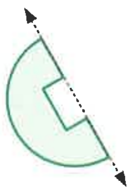
a



b

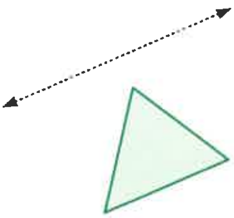


c

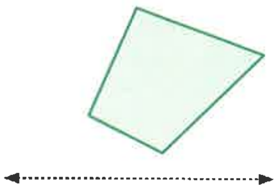


d

e

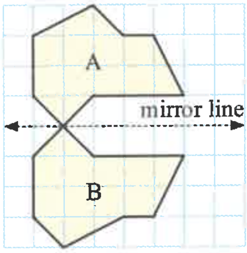


f

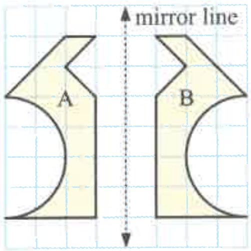


2 Is B a reflection of A in the given mirror line? If not, copy A and the mirror line, then draw the correct reflection.

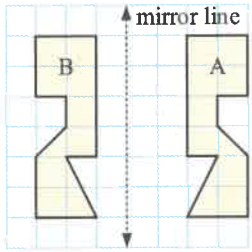
a



b

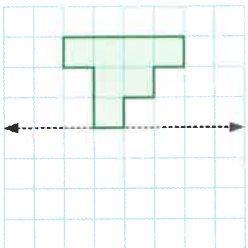


c

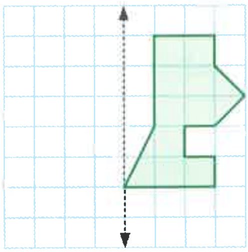


3 Reflect each object in the given mirror line:

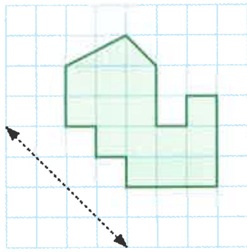
a



b



c

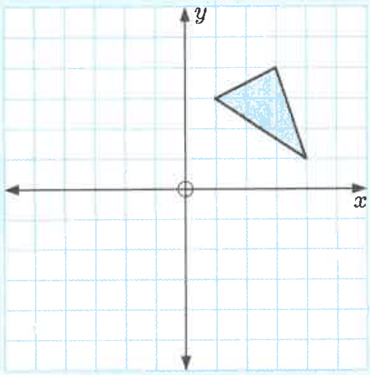


Example 2 **Self Tutor**

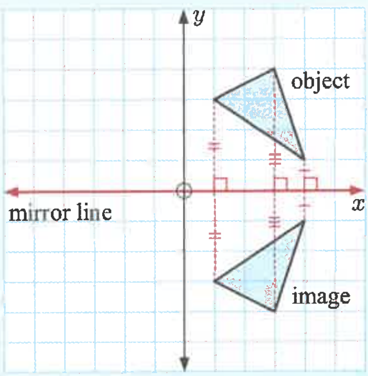
Reflect this figure in:

a the x-axis

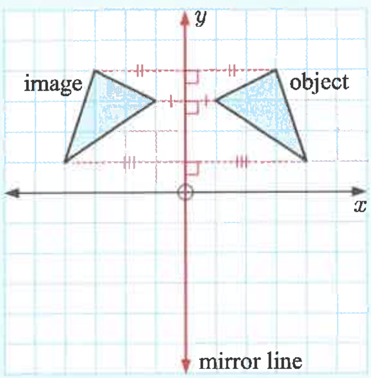
b the y-axis.



a

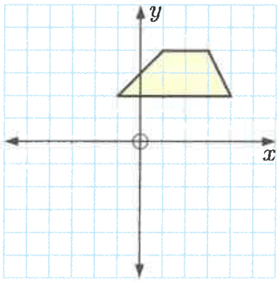


b



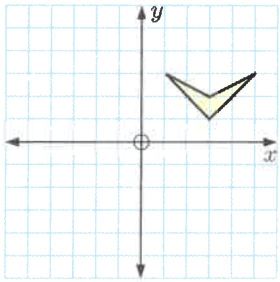
4 Reflect each figure in the axis indicated:

a



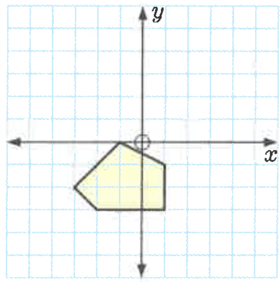
x-axis

b



y-axis

c

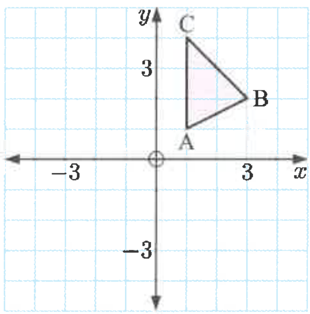


x-axis

5 For each figure:

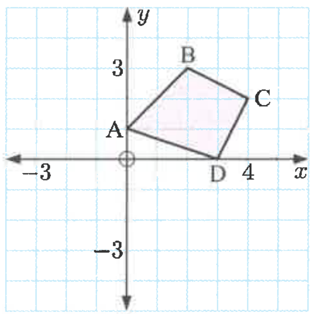
- **i** State the coordinates of the vertices.
- **ii** Reflect the figure in the axis indicated.
- **iii** State the coordinates of the vertices of the image.

a



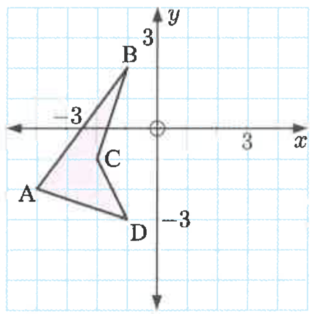
y-axis

b



x-axis

c



y-axis

6 In which quadrant will the image lie when:

- **a** a point in the 1st quadrant is reflected in the x-axis
- **b** a point in the 2nd quadrant is reflected in the y-axis
- **c** a point in the 3rd quadrant is reflected in the x-axis?

7 **a** Reflect each point in the x-axis, and state the coordinates of the image:

i $(3, 2)$ **ii** $(1, 3)$ **iii** $(-4, 5)$ **iv** $(2, -4)$ **v** $(0, -3)$

b Reflect each point in the y-axis, and state the coordinates of the image:

i $(2, 1)$ **ii** $(4, 3)$ **iii** $(5, -1)$ **iv** $(-3, 4)$ **v** $(-5, 0)$

c Copy and complete:

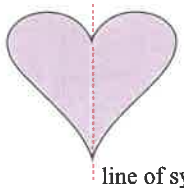
- **i** When $\mathrm{P}(a, b)$ is reflected in the x-axis, the image is $\mathrm{P}'(....., ...)$.
- **ii** When $\mathrm{P}(a, b)$ is reflected in the y-axis, the image is $\mathrm{P}'(....., ...)$.

PUZZLE

Draw *two* line segments on the Cartesian plane so that when they are reflected in the y-axis, the object and image form:

a an isosceles triangle **b** a kite **c** a rhombus **d** a square.

C LINE SYMMETRY



Look at this heart-shaped figure.

The left side of the figure is a reflection of its right side.

We say that the figure is **symmetrical** or has **line symmetry**.

The dashed line is the mirror line for the reflection. We call it the **line of symmetry**.

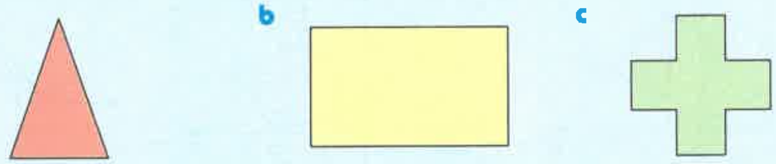
A figure has **line symmetry** if it has at least one line of symmetry.

DEMO

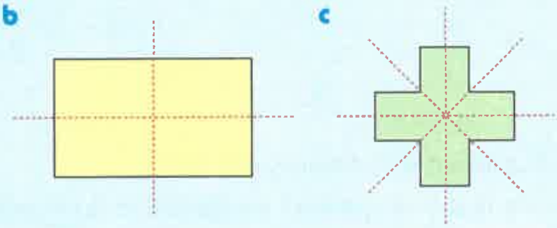
Example 3 — Self Tutor

For each figure, draw any lines of symmetry:

a **b** **c**

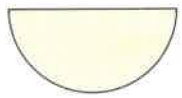


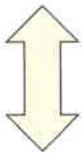
a **b** **c**

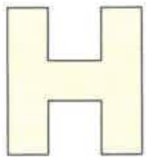


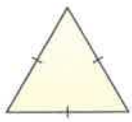
EXERCISE 19C

1 Copy each figure and draw any lines of symmetry:

a


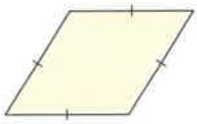
b


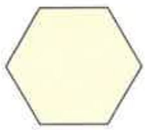
c


d


e

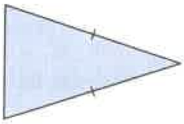
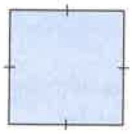
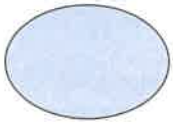
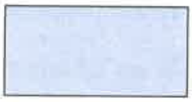

f


g


h


PRINTABLE FIGURES

2 **a** Copy each figure and draw any lines of symmetry.

i  **ii**  **iii**  **iv** 

b Which figure has the most lines of symmetry?

3 How many lines of symmetry does each pattern have?

a  **b** 

4 **a** How many lines of symmetry can a triangle have? Draw all of the possible cases.

b How many lines of symmetry can a quadrilateral have? Draw all of the possible cases.

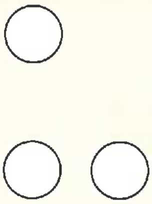
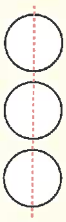
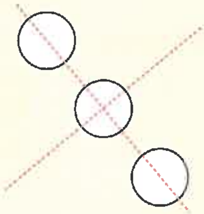
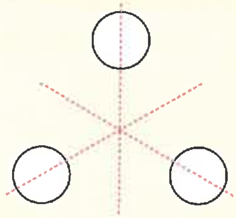
c How many lines of symmetry do you think a circle has?

INVESTIGATION — LINE SYMMETRY

Suppose we are given a group of identical circles.

Can they be positioned, without touching one another, so they together have a particular number of lines of symmetry?

If there are 3 circles, we can position them so the number of lines of symmetry is:

- 0 
- 1 
- 2 
- 3 

What to do:

1 Illustrate how 4 circles can be positioned so the number of lines of symmetry is:

a 0 **b** 1 **c** 2 **d** 3 **e** 4

2 Illustrate how 5 circles can be positioned so the number of lines of symmetry is:

a 0 **b** 1 **c** 2 **d** 4 **e** 5

3 Is it possible to position 5 circles so there are 3 lines of symmetry?

4 Investigate what numbers of lines of symmetry can be produced using:

a 6 circles **b** 7 circles **c** 8 circles **d** 9 circles **e** 10 circles.

5 Discuss any patterns you notice in your answers.
Can you write down a general rule for what numbers of lines of symmetry are possible with n identical circles, where $n \geqslant 3$?

D ROTATIONS

We are all familiar with objects which rotate, such as wheels, propellers, and the hands of a clock. We know that the Earth rotates on its axis once every day.

A **rotation** turns an object about a point and through a given angle.

The point about which the figure rotates is called the **centre of rotation**, and is often labelled O. In any rotation, each point and its image are the same distance from O.

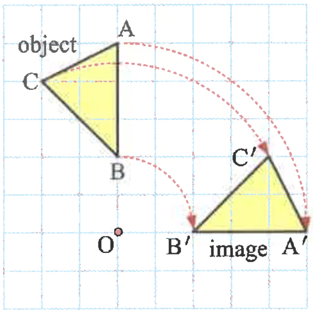
In mathematics we rotate in an *anticlockwise* direction unless we are told otherwise.

When we rotate objects on a grid, it is easiest to start with points or lines on the object which lie on the same horizontal or vertical grid line as O.

For example, the object alongside has been rotated 90° clockwise about O.

Click on the icon to view the rotation.

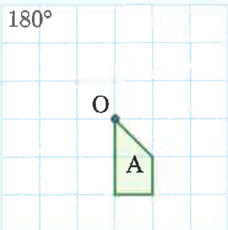
DEMO



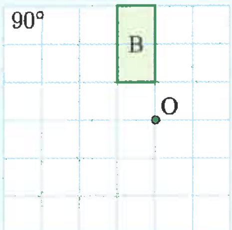
Example 4 Self Tutor

Rotate each figure about O through the angle indicated:

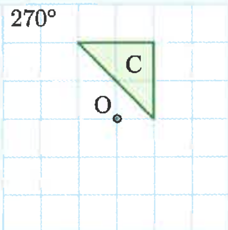
a 180°



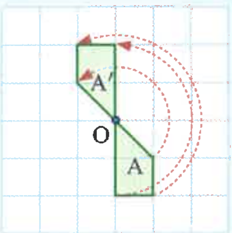
b 90°



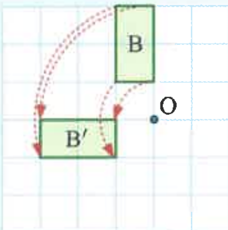
c 270°



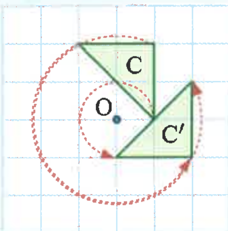
a



b

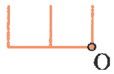


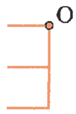
c



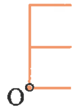
EXERCISE 19D

1 Consider the rotations of  below:

A 

B 

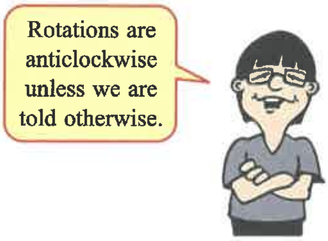
C 

D 

Which of **A**, **B**, **C**, or **D** is a rotation of the object about O through:

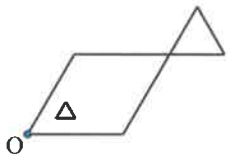
a 180° **b** 360°

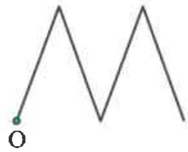
c 90° **d** 270°?



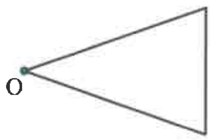
2 Copy each figure then rotate it about the centre of rotation O, through the angle given.

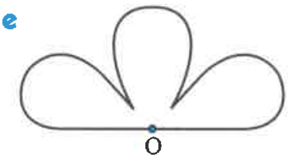
a 
90°

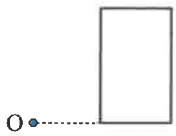
b 
270°

c 
360°

PRINTABLE WORKSHEET

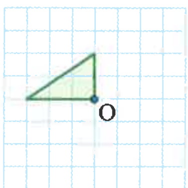
d 
180°

e 
90°

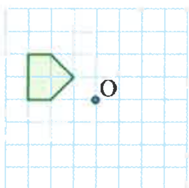
f 
270°

3 Rotate each figure about O as directed:

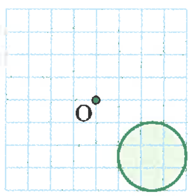
a 90° anticlockwise



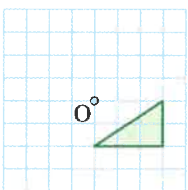
b 180° anticlockwise



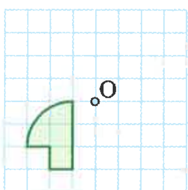
c 90° clockwise



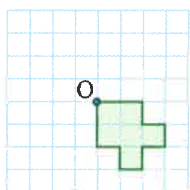
d 180° clockwise



e 90° anticlockwise



f 90° clockwise

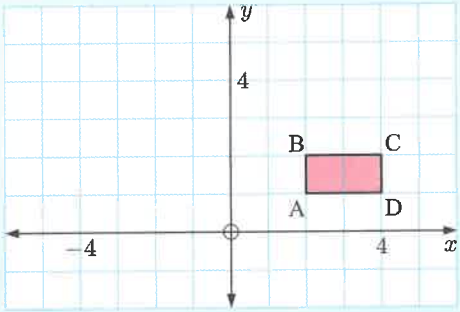


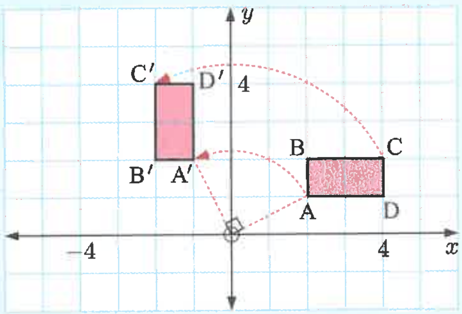
Example 5

Self Tutor

Rotate the rectangle ABCD 90° anticlockwise about the origin O.

State the coordinates of the vertices of the image rectangle.





The image has vertices $A'(-1, 2)$, $B'(-2, 2)$, $C'(-2, 4)$, and $D'(-1, 4)$.

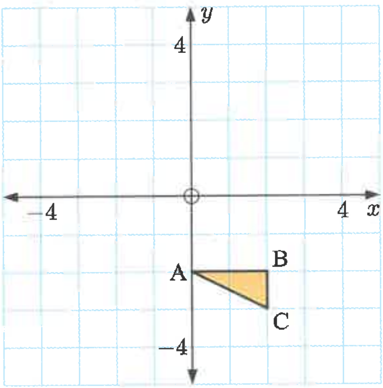
4 For each figure:

i State the coordinates of the vertices.

ii Rotate the figure about the origin O as indicated.

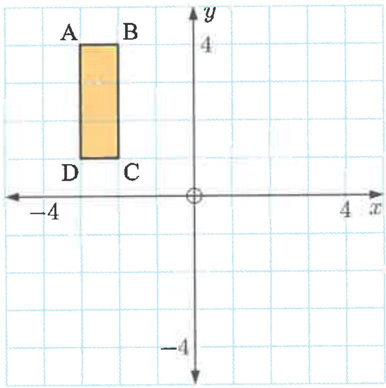
iii State the coordinates of the vertices of the image.

a



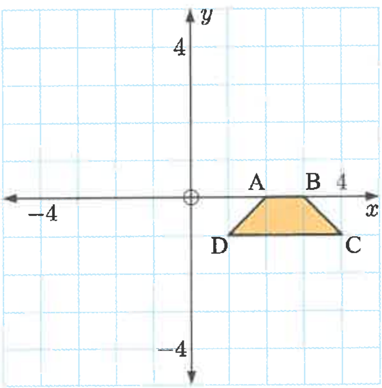
90° anticlockwise

b



180° anticlockwise

c



90° clockwise

5 In which quadrant will the image lie when:

a a point in the 1st quadrant is rotated $90°$ anticlockwise about the origin

b a point in the 4th quadrant is rotated $90°$ clockwise about the origin

c a point in the 2nd quadrant is rotated $180°$ anticlockwise about the origin?

6 Find the coordinates of the image when the point $(2, 5)$ is rotated about the origin O through:

a $90°$ **b** $180°$ **c** $270°$.

7 Copy and complete:

a When $P(a, b)$ is rotated $90°$ about the origin, the image is $P'(....., ...)$.

b When $P(a, b)$ is rotated $180°$ about the origin, the image is $P'(....., ...)$.

c When $P(a, b)$ is rotated $270°$ about the origin, the image is $P'(....., ...)$.

DISCUSSION

How could you rotate point P through $30°$ about the origin?

If P has coordinates which are integers, will P′ have coordinates which are integers?

GLOBAL CONTEXT — CISTERCIAN NUMBERS

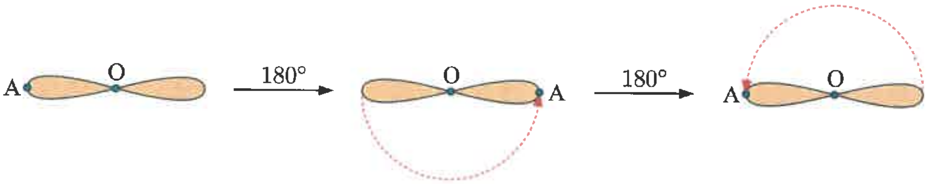
Global context: Personal and cultural expression

Statement of inquiry: Exploring ancient number systems can help us understand the number system we use today.

Criterion: Communicating

E ROTATIONAL SYMMETRY

Suppose we rotate this propeller through $360°$ about O.



The propeller will rotate exactly onto itself after completing $180°$, and again after completing $360°$.

An object has **rotational symmetry** if, when rotated about a point, it is rotated onto itself *before* completing a full turn.

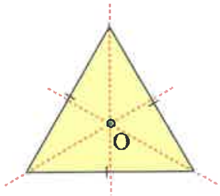
If an object has rotational symmetry:

- The **order of rotational symmetry** is the number of times the object rotates onto itself in a full turn.
- The point about which it rotates is called the **centre of rotational symmetry**.

For example:

- The propeller has rotational symmetry of order 2.

- An equilateral triangle has rotational symmetry of order 3.



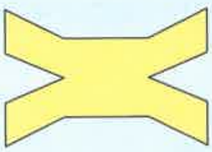
Note that:

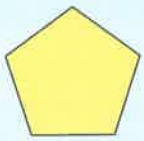
- *Every* figure will map onto itself under a full turn, but this is not rotational symmetry.
- If a figure has more than one line of symmetry then it also has rotational symmetry. The centre of rotational symmetry is the point where the lines of symmetry meet.

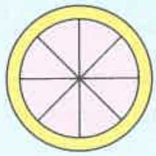
Example 6 **Self Tutor**

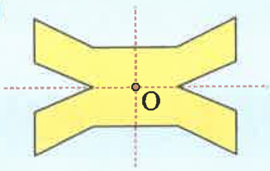
For each figure:

i Mark the centre of rotational symmetry O.
ii State the order of rotational symmetry.

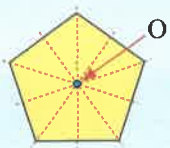
a


b


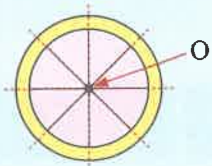
c


a **i**


ii order $= 2$

b **i**


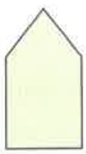
ii order $= 5$

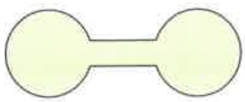
c **i**


ii order $= 8$

EXERCISE 19E

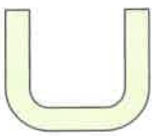
1 Decide whether each figure has rotational symmetry:

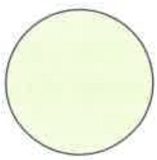
a


b


c


d

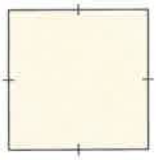
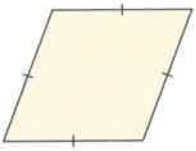

e


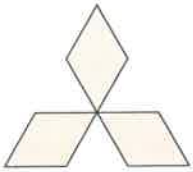
f


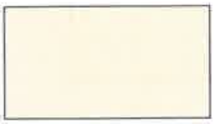
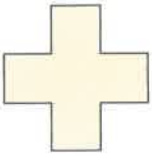
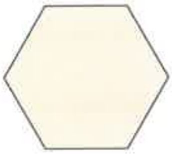
2 For each figure:

- **i** Mark the centre of rotational symmetry O.
- **ii** State the order of rotational symmetry.

PRINTABLE DIAGRAMS

a

b

c

d


e

f

g

h


3 Draw a figure which has order of rotational symmetry: **a** 6 **b** 8.

4 Draw a figure which has:

- **a** line symmetry but not rotational symmetry
- **b** rotational symmetry but not line symmetry.

ACTIVITY

In this Activity we use software to construct a shape that has rotational symmetry.

What to do:

1. Choose a sector angle from the menu.
   Make a simple design with different shapes and colours in the sector, then press $\boxed{\text{Rotate}}$ to see your creation.
2. Experiment with different sector angles. How does the size of the sector angle affect the order of rotational symmetry of the design?

ROTATING FIGURES

F ENLARGEMENTS AND REDUCTIONS

If you want to make a picture bigger or smaller, you can **enlarge** or **reduce** it using a photocopier.

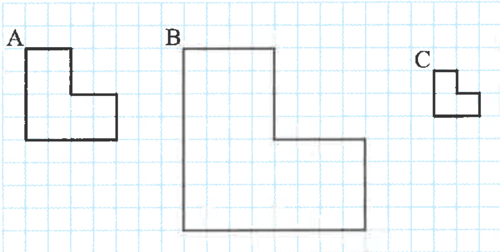
An **enlargement** of an object is **larger** than the original.

A **reduction** of an object is **smaller** than the original.

When an object is enlarged or reduced, all of its lengths are enlarged or reduced by the same **scale factor**.

For example:

- The side lengths in figure B are all twice the corresponding side lengths in figure A.
  We say that B is an **enlargement** of A with scale factor 2.
- The side lengths in figure C are all half the side lengths in figure A.
  We say that C is a **reduction** of A with scale factor $\frac{1}{2}$.



If the scale factor is greater than 1, we have an **enlargement**.

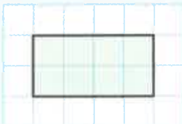
If the scale factor is less than 1, we have a **reduction**.

Example 7 Self Tutor

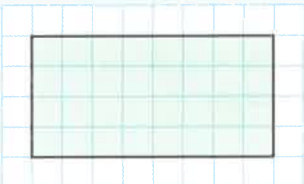
Find the image when this figure is:

a enlarged with scale factor 2

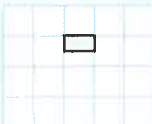
b reduced with scale factor $\frac{1}{4}$.



a In an enlargement with scale factor 2, all lengths are doubled.



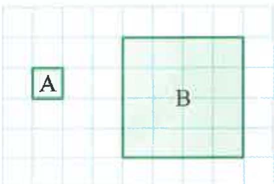
b In a reduction with scale factor $\frac{1}{4}$, all lengths are divided by 4.



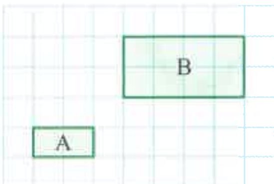
EXERCISE 19F

1 State the scale factor for each enlargement from A to B:

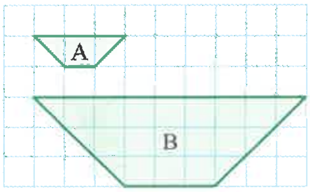
a



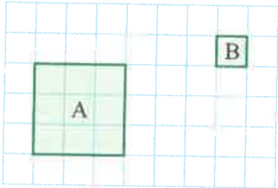
b

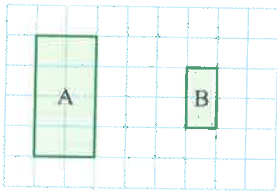


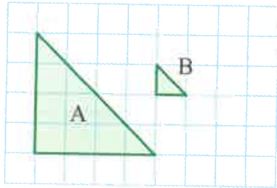
c



2 State the scale factor for each reduction from A to B:

a


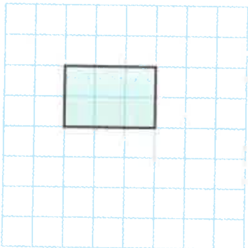
b


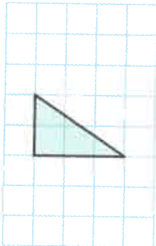
c


3 Describe what happens to the lengths on an object when it is:

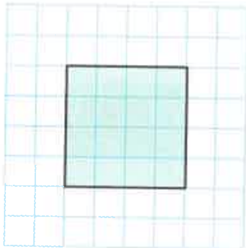
a enlarged with scale factor 5 **b** reduced with scale factor $\frac{1}{3}$.

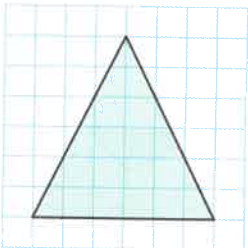
4 Enlarge or reduce each object using the scale factor given:

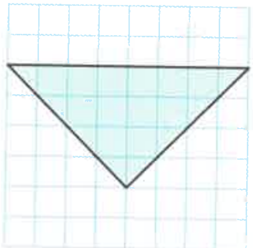
a

scale factor 2

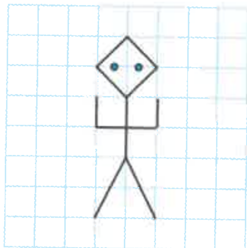
b

scale factor 3

PRINTABLE WORKSHEET

c

scale factor $\frac{1}{2}$

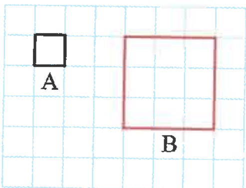
d

scale factor $\frac{1}{3}$

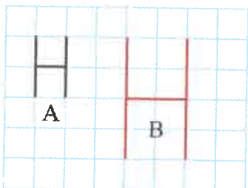
e

scale factor $\frac{1}{4}$

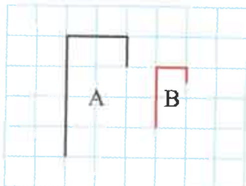
f

scale factor 2

5 Find the scale factor when:

i A is transformed to B **ii** B is transformed to A.

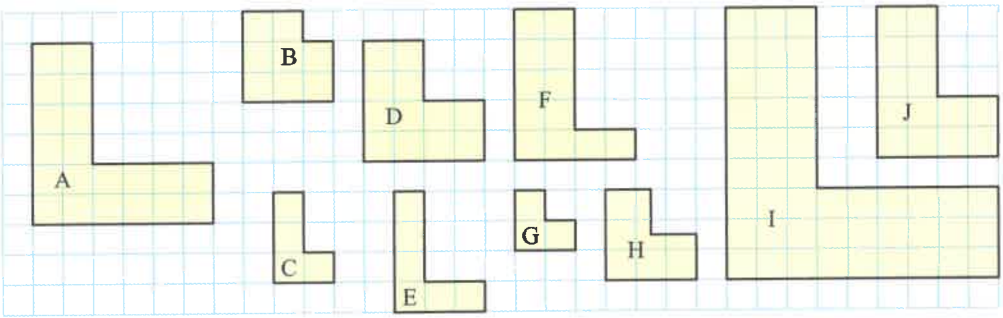
a


b


c


6 An object is enlarged with scale factor k.
What needs to be done to the image to return it to the original size?

7



Copy and complete:

a Figure ...... is obtained by ...... figure A with scale factor ......

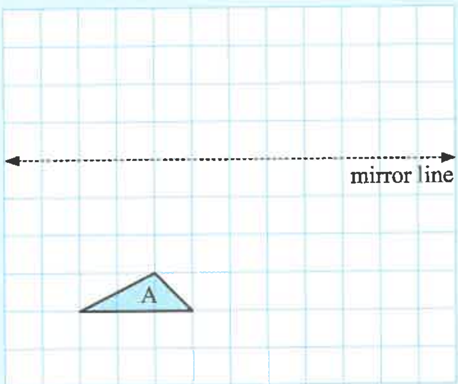
b Figure ...... is obtained by reducing figure ...... with scale factor $\frac{3}{4}$.

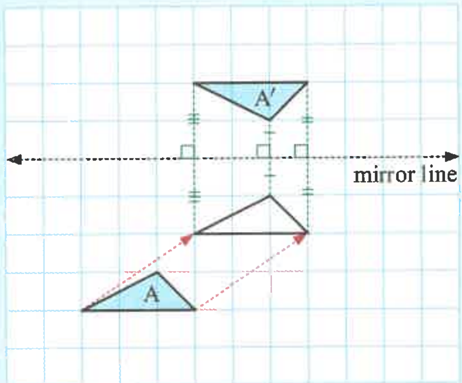
G COMBINATIONS OF TRANSFORMATIONS

In this Section we perform several transformations on a figure, one after another.

Example 8 Self Tutor

Translate figure A 3 units right and 2 units up, then reflect the result in the mirror line.



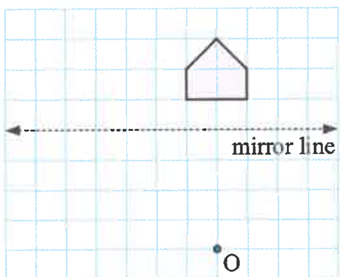


EXERCISE 19G

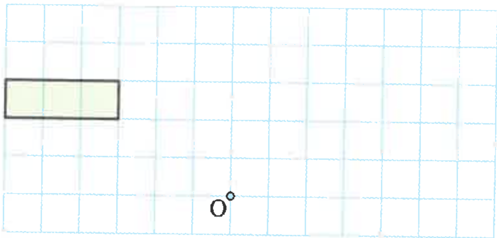
1 Find the image when the figure alongside is:

a translated 3 units left and 1 unit up, then reflected in the mirror line

b reflected in the mirror line, then rotated 90° anticlockwise about O.



2



a Translate the given rectangle 3 units right, then rotate it 90° clockwise about O.

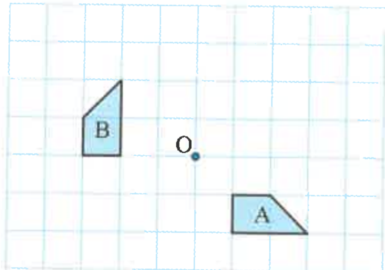
b Rotate the given rectangle 90° clockwise about O, then translate the result 3 units right.

c Does the order in which transformations are performed affect the result?

3 A figure X is reflected in a mirror line to give the image X′. What happens if X′ is reflected in the same mirror line? Illustrate your answer.

4 **a** Describe how to transform figure A to figure B using a translation followed by a rotation about O.

b Hence describe how to transform figure B to figure A.

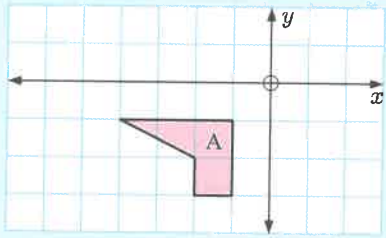


Example 9

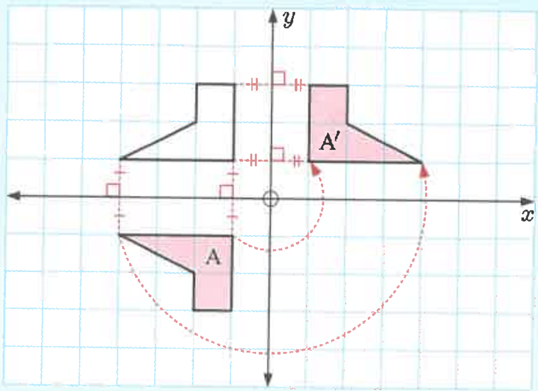
Self Tutor

a Reflect figure A in the x-axis, then reflect the result in the y-axis.

b Describe a single transformation which would give the same result.



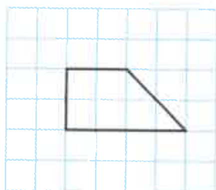
a



b A rotation of 180° about the origin would give the same result.

5 **a** Enlarge this figure with scale factor 2, then reduce the result with scale factor $\frac{1}{4}$.

b Describe a single transformation which gives the same result.

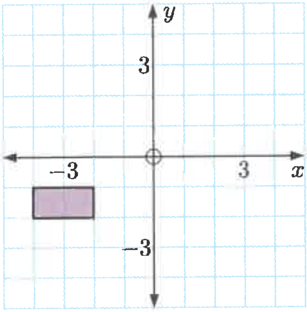


6 For each figure:

  i Perform the combination of transformations indicated.

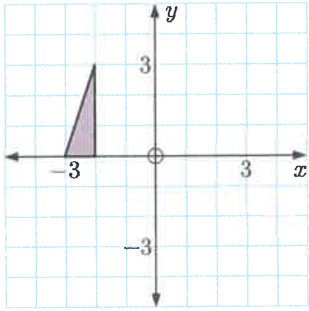
  ii Describe a single transformation which gives the same result.

a



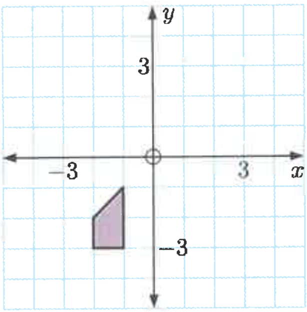
Translate 4 units right and 2 units up, then translate 3 units left and 1 unit up.

b



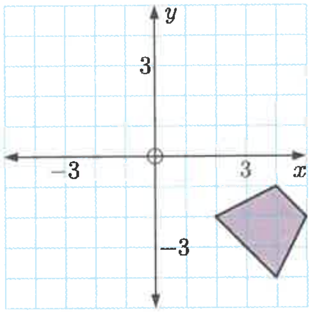
Rotate 180° about O, then rotate 90° about O.

c



Rotate 180° about O, then reflect in the y-axis.

d



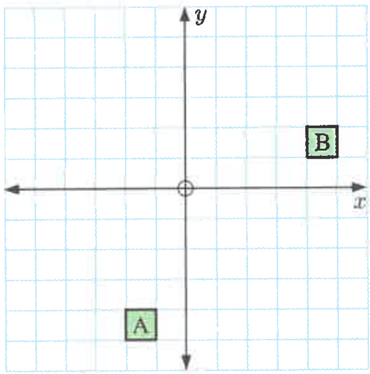
Reflect in the y-axis, then reflect in the x-axis.

7 Describe how square A can be transformed to square B using:

  a a single translation

  b a reflection in the y-axis followed by another transformation

  c a reflection in the x-axis followed by another transformation.



GAME — BATTLEGRID

Click on the icon to play a challenging game involving transformations around a number plane.

BATTLEGRID

MULTIPLE CHOICE QUIZ

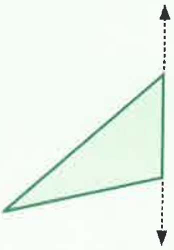
QUICK QUIZ

REVIEW SET 19A

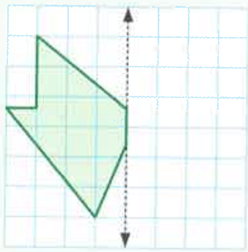
PRINTABLE DIAGRAMS

1 Reflect each object in the given mirror line.

a

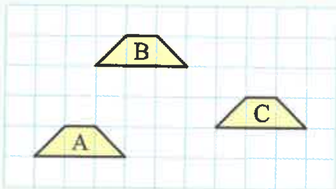


b



2 Describe the translation from:

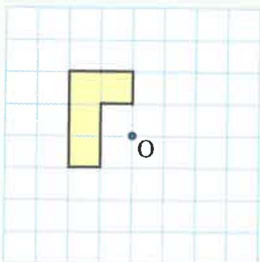
a A to B **b** B to A
c B to C **d** C to A.



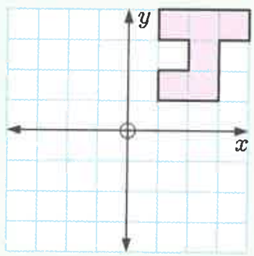
3 Does this boomerang have:

a line symmetry
b rotational symmetry?

4 Rotate the given figure $90°$ anticlockwise about O.

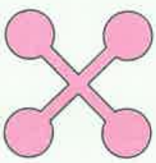


5 Reflect this figure in the x-axis.



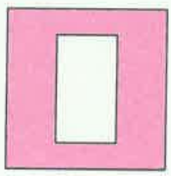
6 Find the order of rotational symmetry for each shape:

a



b

c

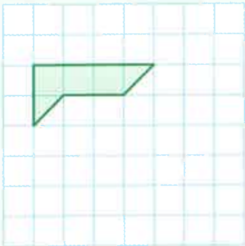


7 **a** Rotate T(2, 3) through 270° about the origin.

b State the coordinates of the image T′.

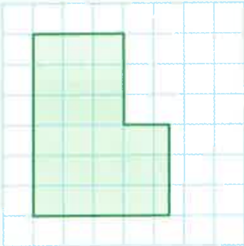
8 Enlarge or reduce each figure using the scale factor given:

a



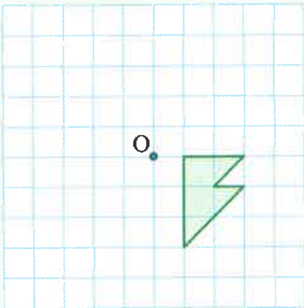
scale factor 2

b



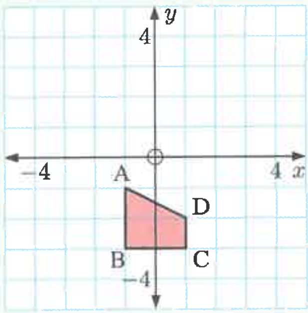
scale factor $\frac{1}{3}$

9 Find the image when the figure alongside is translated 5 units up, then rotated 180° about O.



10 **a** Translate this figure 2 units left and 1 unit up, then reflect the result in the x-axis.

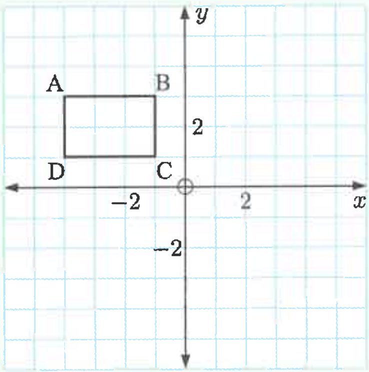
b State the coordinates of the vertices of the image.



11 **a** State the order of rotational symmetry of rectangle ABCD.

b Translate the rectangle 2 units right and 4 units down.

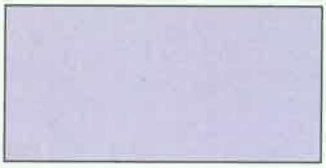
c Suppose ABCD was enlarged with scale factor 3. Find the area of the image.



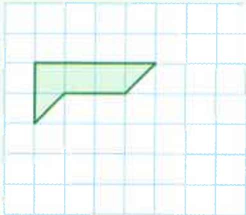
REVIEW SET 19B

PRINTABLE DIAGRAMS

1 Draw the lines of symmetry for this rectangle.



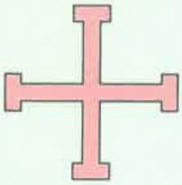
2 Translate this figure 1 unit to the right and 3 units down.



3 For the given figure:

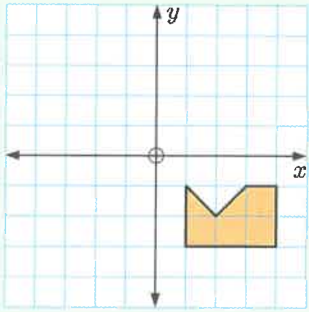
a locate the centre of rotational symmetry

b find the order of rotational symmetry.

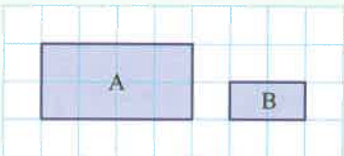


4 Reflect this figure in:

a the x-axis **b** the y-axis.



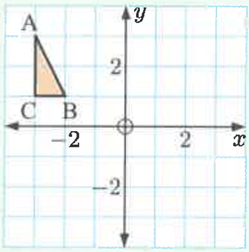
5



In the diagram alongside, figure A has been reduced to figure B.
Find the scale factor.

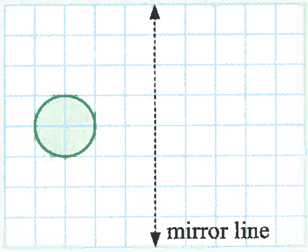
6 **a** Rotate triangle ABC 90° clockwise about O.

b State the coordinates of the vertices of the image.



7 Find the coordinates of the image when $(-5, 3)$ is reflected in the y-axis.

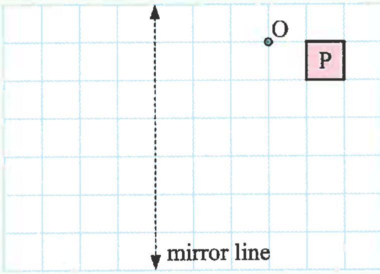
8 A point in the 3rd quadrant is rotated 180° about the origin, then reflected in the x-axis. In which quadrant does the image lie?

9 

a Find the image when the circle is translated 1 unit right, then reflected in the mirror line.

b Find the image if the transformations in **a** are performed in the opposite order.

10 Show the result when P is rotated 90° clockwise about O, then translated 3 units down, then reflected in the mirror line.



11 **a** **i** Which figure is an enlargement of A?

ii State the scale factor for the enlargement.

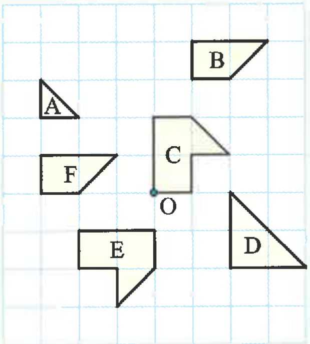
b **i** Which figure is a translation of B?

ii Describe the translation.

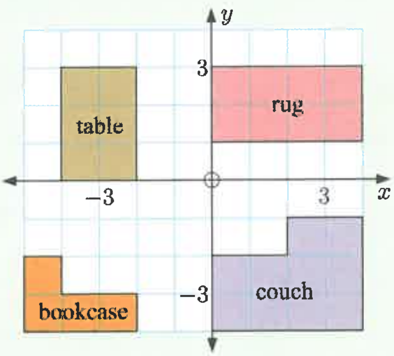
c Describe how to transform figure C to figure E using:

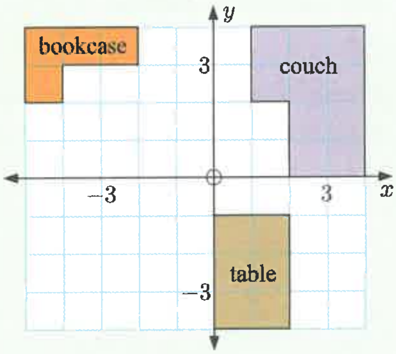
i a rotation about O followed by a translation

ii a translation followed by a rotation about O.



12 Cheryl is rearranging the furniture in her living room. She has drawn a plan of where the furniture was previously, and another plan of where she is moving it to. O is the location of the light fitting.





a Describe the transformation that has been applied to the:

i table **ii** couch **iii** bookcase.

b The rug is to be rotated 90° anticlockwise about O, then translated 3 units down. Write down the coordinates of the corners of the rug in the new layout.

ANSWERS

ERCISE 1A

a 7 b 70 c 7 d 70
e 700 f 7000 g 7 h 700 000
i 70 000 j 7 000 000 k 70 000 000 l 70

a 65 b 721 c 6805 d 20 430
e 109 084 f 506 098 g 7 002 063

a 700 + 30 + 4 b 3000 + 900 + 20 + 8
c 20 000 + 1000 + 80
d 600 000 + 30 000 + 8000 + 400 + 9

a thirty six b four hundred and five
c six thousand, five hundred and one
d eleven thousand and eighty five
e fifty four thousand, seven hundred and sixty
f two hundred and eighty five thousand, four hundred
g thirteen million h eight billion, seven hundred million

5 a 17 b 164 c 328 d 810 e 2901
f 5 402 390 g 305 000 000 h 12 800 000 000

6 100 000 000 000 000 atoms

7 a one hundred and three dollars, $113, $130
b Xiao 109 cm, Kylie 116 cm, Wendy 118 cm, Sarah 126 cm
c giraffe 674 kg, hippopotamus 1872 kg, rhinoceros 2156 kg, elephant 3058 kg
d Milan 107 m, Salisbury 123 m, Rome 138 m, Cologne 157 m
e fourteen pounds, four thousand pounds, £4100, fourteen thousand pounds, forty thousand pounds

8 2457 9 865 320

EXERCISE 1B

1 a 60 b 40 c 70 d 130 e 100
f 230 g 310 h 10 000

2 a 400 b 300 c 100 d 900 e 900
f 2000 g 18 700 h 25 900

3 a 6000 b 2000 c 7000 d 1000 e 14 000
f 10 000 g 26 000 h 254 000

4 a 50 b 200 c 400 d 500 e 900
f 1000 g 9000 h 50 000

5 a 680 b 210 c 590 d 170 e 2000
f 3900 g 9000 h 17 000

6 a 1830 b 26 800 c 484 000 d 3 620 000

7 a $3200 b 300 g c 4600 m
d 68 000 people e $27 000 f 700 flights

8 a 47 000 runners b 47 000 runners c 46 980 runners

EXERCISE 1C

1 a 16 b 17 c 18 d 14 e 32 f 20

2 a 3 b 27 c 9 d 16 e 7 f 53

3 a 16 b 15 4 a 5 b 12 c 8

5 a 45 b 24 c 6 d 48 e 0 f 42
g 0 h 64 i 44 j 44 k 54 l 0
m 40 n 30 o 72 p 150

6 a 3 b 9 c 9 d 29 e 0 f 11 g 0
h 8 i 0 j 11 k undefined l 12

7 a 24 b 0 8 $60 9 8 lollies

EXERCISE 1D

1 a C b D c A d E e B

2 a 4^2 b 6^3 c 11^4 d 2×3^2 e $2^2\times 3\times 5$
f 2×5^3 g $3^2\times 5^3$ h $2^3\times 5\times 7$
i $3^3\times 7^2$ j $3^4\times 5^2$ k $7^5\times 11^3$

3 a $2\times 2\times 2$ b $3\times 3\times 3\times 3\times 3\times 3$
c $2\times 2\times 2\times 2\times 2\times 3\times 3$ d $5\times 5\times 7$
e $2\times 3\times 3\times 3\times 3$ f $2\times 2\times 3\times 5\times 5\times 5$

4 a 16 b 49 c 32 d 50
e 5000 f 400 g 45 h 36
i 180 j 280 k 6300 l 12 000

5 a 11 664 b 42 017 500 c 104 544
d 178 200 e 5 282 739 f 54 925 000

6 a 2^1 b 2^2 c 2^4 d 2^6

7 a 3^1 b 3^3 c 3^4 d 3^6

8 a 10^2 b 10^3 c 10^5 d 10^7

9 a 11^1 b 11^2 c 11^3 d 11^4 10 100 zeros

REVIEW SET 1A

1 nine thousand, six hundred and two 2 7000 3 965 210

4 a 49 550 b 49 600 c 50 000 5 36 000 views

6 a 9 b 14 7 a 0 b 77 c 96

8 8 lollies 9 a $2^3\times 3$ b $3^2\times 5^4$

10 a 300 b 40 c 45 000

11 a 4^1 b 4^2 c 4^3

12 a i 81 ii 729 iii 6561 iv 59 049 v 531 441
b i 9 ii 1

REVIEW SET 1B

1 a 300 + 80 + 4 b 7000 + 900 + 2
c 60 000 + 100 + 50 d 8 000 000 + 50 000 + 70 + 1

2 a 3068 b 4 700 209 c 50 076 300 000

3 two hundred and seven dollars, $217, two hundred and seventy dollars, $277

4 a 570 b 37 000 c 4200 d 120 000

5 a 14 000 km b 14 500 km

6 a 31 b 28 c 23

7 a 8 b 67 c 0 8 48 km

9 a $3\times 3\times 3\times 3$ b $2\times 2\times 2\times 2\times 2\times 7\times 7$
c $2\times 2\times 2\times 2\times 3\times 3\times 3\times 5$

10 a 576 b 15 435 c 2 695 275

11 a $2^{30} = \underbrace{2\times 2\times \ldots\times 2}_{30\text{ times}}$ b 3 000 000
c i 1 000 000 000 bytes
ii We are rounding the number to a particular place value.

12 a 12 packets b 120 biscuits c 30 biscuits

EXERCISE 2A

1

Number	1	2	3	4	5	6	7	8	9	10
Perfect square	1	4	9	16	25	36	49	64	81	100

Number	11	12	13	14	15	16	17	18	19	20
Perfect square	121	144	169	196	225	256	289	324	361	400

2 **a** 625 **b** 1024 **c** 6889

3 64, 81, 100, 121, 144 **4** **a** $31^2 = 961$ **b** $32^2 = 1024$

5 **a** 2 **b** 4 **c** 7 **d** 9 **e** 10 **f** 11 **g** 0 **h** 100 **i** 17 **j** 25 **k** 32 **l** 99

6 **a** $\sqrt{25} = 5$, $\sqrt{25} \times \sqrt{25} = 25$ **b** $\sqrt{64} = 8$, $\sqrt{64} \times \sqrt{64} = 64$

c $\sqrt{144} = 12$, $\sqrt{144} \times \sqrt{144} = 144$ **d** $\sqrt{196} = 14$, $\sqrt{196} \times \sqrt{196} = 196$

7 **a**
$$\begin{aligned} 1^2 &= 1 \\ 11^2 &= 121 \\ 111^2 &= 12\,321 \\ 1111^2 &= 1\,234\,321 \end{aligned}$$
b **i** 123 454 321 **ii** 12 345 654 321

8 **a**
$$\begin{aligned} 1 \times 3 + 1 &= 4 = 2^2 \\ 2 \times 4 + 1 &= 9 = 3^2 \\ 3 \times 5 + 1 &= 16 = 4^2 \\ 4 \times 6 + 1 &= 25 = 5^2 \\ 5 \times 7 + 1 &= 36 = 6^2 \\ 6 \times 8 + 1 &= 49 = 7^2 \\ 7 \times 9 + 1 &= 64 = 8^2 \end{aligned}$$
b **i** 400 **ii** 899

9 **a** Its square is multiplied by 4.

b For example,
$$\begin{aligned} &(5 \times 2)^2 \\ &= (5 \times 2) \times (5 \times 2) \\ &= 5 \times 2 \times 5 \times 2 \\ &= 5 \times 5 \times 2 \times 2 \\ &= 25 \times 4 \end{aligned}$$

10 A perfect square can only end in 0, 1, 4, 5, 6, or 9.

EXERCISE 2B

1

Number	1	2	3	4	5	6	7	8	9	10
Perfect cube	1	8	27	64	125	216	343	512	729	1000

2 **a** 2197 **b** 8000 **c** 1 000 000 **d** 1 000 000 000

3 $21^3 = 9261$ and $22^3 = 10\,648$

$\therefore$ 21 cubic numbers are less than 10 000.

4 8, 9 **5** $46^3 = 97\,336$

6 **a** 1 **b** 4 **c** 6 **d** 9

7 **a** $\sqrt[3]{125} = 5$, $\sqrt[3]{125} \times \sqrt[3]{125} \times \sqrt[3]{125} = 125$

b $\sqrt[3]{343} = 7$, $\sqrt[3]{343} \times \sqrt[3]{343} \times \sqrt[3]{343} = 343$

8 **a**
$$\begin{aligned} 1^3 &= 1 &&= 1 &&= 1^2 \\ 1^3 + 2^3 &= 1 + 8 &&= 9 &&= (1+2)^2 \\ 1^3 + 2^3 + 3^3 &= 1 + 8 + 27 &&= 36 &&= (1+2+3)^2 \\ 1^3 + 2^3 + 3^3 + 4^3 &= 1 + 8 + 27 + 64 &&= 100 &&= (1+2+3+4)^2 \end{aligned}$$

b **i** $(1+2+3+4+5)^2 = 225$

ii $(1+2+3+4+5+6+7+8+9+10)^2 = 3025$

9 216

EXERCISE 2C

1 **a** divisible **b** divisible **c** divisible **d** not divisible **e** not divisible **f** divisible **g** divisible **h** not divisible **i** divisible

2 96 **3** 105

4 **a** True; when any counting number is divided by 1, the res is itself with no remainder.

b True; when any perfect square is divided by its square ro the result is its square root with no remainder.

EXERCISE 2D

1 **a** even **b** odd **c** odd **d** even **e** even **f** odd **g** even **h** odd

2 **a** 24, 30, 36 **b** 33 **c** 22, 26, 30, 34, 38 **d** 23, 25, 29, 31, 35, 37 **e** 36 **f** 27

3 **Note:** Other answers are possible.

a $18 + 42$ **b** $11 + 49$ **c** 2×30 **d** 4×15

4 **a** even **b** odd **c** even **d** even

5 **a**
$$\begin{aligned} 1 &= 1 = 1^2 \\ 1 + 3 &= 4 = 2^2 \\ 1 + 3 + 5 &= 9 = 3^2 \\ 1 + 3 + 5 + 7 &= 16 = 4^2 \\ 1 + 3 + 5 + 7 + 9 &= 25 = 5^2 \end{aligned}$$
b **i** $6^2 = 36$ **ii** $10^2 = 100$

EXERCISE 2E

1 **a** divisible **b** not divisible **c** not divisible **d** divisible **e** not divisible **f** divisible

2 **a** not divisible **b** divisible **c** divisible **d** not divisible

3 **a** divisible **b** divisible **c** not divisible **d** divisible

4 **a** not divisible **b** divisible **c** not divisible **d** divisible

5 **a** true **b** false **c** false **d** true **e** true **f** true **g** false **h** false **i** true **j** false **k** true **l** true

6 **a** divisible by 2 and 5 **b** divisible by 3 and 9 **c** divisible by 3 and 5 **d** not divisible by any of them

7 3, 6, 9, 12, 15, or 18

8 **a** 2, 5, or 8 **b** 2 or 6 **c** 0 or 5 **d** 2 or 8 **e** 2 **f** 1

9 **a** **Note:** Other answers are possible.

i 1485 **ii** 1548

b $1 + 4 + 5 + 8 = 18$, which is divisible by 9. Therefore, any number containing these digits, regardless of their order, is divisible by 9.

10 When the digits of a number are reversed, the difference between the sum of the even digits and the sum of the odd digits will stay the same.
So, if a given number is divisible by 11, then if we reverse its digits, the result will also be divisible by 11.

EXERCISE 2F

1 **a** yes **b** no **c** no **d** yes

2 **a** 1, 2, 4, 8 **b** 1, 2, 4, 8, 16 **c** 1, 2, 4, 5, 10, 20 **d** 1, 2, 11, 22 **e** 1, 5, 25

3 **a** $24 = 6 \times 4$ **b** $28 = 4 \times 7$ **c** $88 = 11 \times 8$ **d** $100 = 5 \times 20$ **e** $143 = 11 \times 13$ **f** $91 = 13 \times 7$

4 $36 = 1 \times 36$, $36 = 2 \times 18$, $36 = 3 \times 12$, $36 = 4 \times 9$, $36 = 6 \times 6$

$\therefore$ the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36.

5 **a** 1, 2, 4, 7, 14, 28 **b** 1, 29 **c** 1, 2, 3, 5, 6, 10, 15, 30 **d** 1, 2, 4, 8, 16, 32 **e** 1, 2, 4, 11, 22, 44 **f** 1, 7, 49

g 1, 2, 5, 10, 25, 50 **h** 1, 2, 4, 7, 8, 14, 28, 56
i 1, 3, 7, 9, 21, 63 **j** 1, 5, 13, 65
k 1, 3, 5, 15, 25, 75 **l** 1, 2, 4, 19, 38, 76

6 **a** 7 **b** 9 **c** 11 **d** 35 **e** 27 **f** 45

7 **a** 12 **b** 30 **c** 105 **d** 210 **8** 63

9 **a** **i** 3 factors **ii** 3 factors **iii** 3 factors **iv** 9 factors
b square numbers
c Every square number has exactly one factor pair which is made up of identical numbers, so this pair contributes 1 to the number of factors. All other factor pairs contribute 2 to the number of factors. So, the total number of factors must be odd.

EXERCISE 2G

1 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

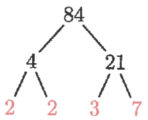
2 **a** 1 only has itself as a factor.
b All even numbers are divisible by 2.
2 is the only even number with exactly 2 distinct factors.

3 **a** 3 **b** 15 **c** 5 **d** 11

4 **a** $6485 = 5 \times 1297$ **b** $9320 = 2 \times 4660$
c $2222 = 2 \times 1111$ **d** $4279 = 11 \times 389$

5 **a** $28 = 2^2 \times 7$ **b** $27 = 3^3$
c $84 = 2^2 \times 3 \times 7$ **d** $160 = 2^5 \times 5$
e $216 = 2^3 \times 3^3$ **f** $528 = 2^4 \times 3 \times 11$
g $784 = 2^4 \times 7^2$ **h** $138 = 2 \times 3 \times 23$
i $250 = 2 \times 5^3$ **j** $189 = 3^3 \times 7$
k $726 = 2 \times 3 \times 11^2$ **l** $9625 = 5^3 \times 7 \times 11$

6 **Note:** Other factor trees may be possible.

a


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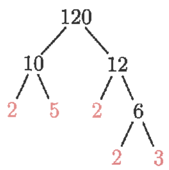

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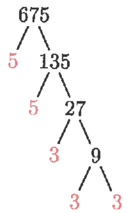


7 **a** $24 = 2^3 \times 3$ **b** $70 = 2 \times 5 \times 7$ **c** $63 = 3^2 \times 7$
d $72 = 2^3 \times 3^2$ **e** $225 = 3^2 \times 5^2$ **f** $88 = 2^3 \times 11$
g $480 = 2^5 \times 3 \times 5$ **h** $1024 = 2^{10}$

8 4^2 is not a product of prime factors, as 4 is not a prime number.

EXERCISE 2H

1 **a** 3 **b** 3 **c** 7 **d** 8 **e** 4 **f** 9
g 2 **h** 22

2 **a** 2 **b** 3 **c** 8

3 **a** 27 **b** 45 **c** 4 **d** 36

4 14 cm squares **5** 40 nails **6** 15 pappadums

EXERCISE 2I

1 **a** 6, 12, 18, 24, 30, 36 **b** 8, 16, 24, 32, 40, 48
c 13, 26, 39, 52, 65, 78 **d** 15, 30, 45, 60, 75, 90
e 25, 50, 75, 100, 125, 150

2 **a** 42 **b** 99 **c** 165 **d** 9900

3 **a** 504 **b** 996 **c** 63, 72, 81, 90, 99, 108
d 169

4 **a, b**

1	2	3	$\boxed{4}$	5	6	7	$\boxed{8}$	9	10
11	$\boxed{12}$	13	14	15	$\boxed{16}$	17	18	19	$\boxed{20}$
21	22	23	$\boxed{24}$	25	26	27	$\boxed{28}$	29	30

c 12, 24

5 **a** 63, 99 **b** 36 **c** 35

6 **a** 30, 60, 90 **b** 72, 96 **c** 36, 54

EXERCISE 2J

1 **a** 20 **b** 15 **c** 24 **d** 60 **e** 30 **f** 28
g 72 **h** 42 **i** 66 **j** 65 **k** 75 **l** 108

2 56 minutes **3** 156 buns

4 The HCF is the smaller number because it is a factor of itself and the larger number.
The LCM is the larger number because it is a multiple of itself and the smaller number.

REVIEW SET 2A

1 **a** 13 **b** $\sqrt{169} \times \sqrt{169} = 169$

2 **a** 4913 **b** 42 875 **c** 3 **d** 10

3 **a** divisible **b** not divisible

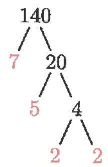
4 **a** 0, 4, or 8 **b** 2, 5, or 8 **c** 0 or 9

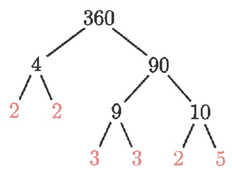
5 **Note:** Other answers are possible.
a $10 + 26$ **b** $9 + 27$ **c** 2×18 **d** 4×9

6 **a** 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
b 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90
c 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63, 126

7 **a** $2950 = 2 \times 1475$ **b** $1863 = 3 \times 621$

8 **a**


b


c


9 **a** $44 = 2^2 \times 11$ **b** $504 = 2^3 \times 3^2 \times 7$
c $693 = 3^2 \times 7 \times 11$

10 **a** 7 **b** 6 **11** 42, 49, 56

12 **a** 12 **b** 40 **c** 60

13 **a** **i** 10, 20, 30, 40, 50, 60, 70, 80, 90, 100
ii 12, 24, 36, 48, 60, 72, 84, 96, 108, 120
b 60 **c** 6 cookie boxes **d** 5 netballs

14 **a** **i** 1, 2, 7, 14 **ii** 1, 2, 4, 5, 10, 20
iii 1, 5, 7, 35 **iv** 1, 2, 7, 14, 49, 98
v 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120
vi 1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150
b 14, 98, and 150
c **i** $14 = 2 \times 7$ **ii** $20 = 2^2 \times 5$ **iii** $35 = 5 \times 7$
iv $98 = 2 \times 7^2$ **v** $120 = 2^3 \times 3 \times 5$
vi $150 = 2 \times 3 \times 5^2$
d A number has the same number of even and odd factors if, when written in prime factored form, its power of two is equal to 1.

15 **a** **i** 4 rectangles **ii** 1 rectangle **iii** 5 rectangles
b **i** 1×120 rectangle **ii** 10×12 rectangle
c 1 rectangle

REVIEW SET 2B

1 225, 256, 289
2 **a** 8 **b** $\sqrt[3]{512} \times \sqrt[3]{512} \times \sqrt[3]{512} = 512$
3 **a** divisible **b** not divisible **c** divisible
d not divisible
4 **a** 94 **b** 88
5 $40 = 1 \times 40$, $40 = 2 \times 20$, $40 = 4 \times 10$, $40 = 5 \times 8$
$\therefore$ the factors of 40 are 1, 2, 4, 5, 8, 10, 20, and 40.
6 No, $3601 = 13 \times 277$ **7** 32 or 72
8 $564 = 2^2 \times 3 \times 47$ **9** 21 **10** 84
11 **a** **i** $9 = 3^2$ **ii** $16 = 2^4$ **iii** $25 = 5^2$
iv $36 = 2^2 \times 3^2$ **v** $81 = 3^4$
b The exponents are all even. **c** $576 = 2^6 \times 3^2$
d $\sqrt{576} = 24$, $24 = 2^3 \times 3$
e The prime bases are the same for **c** and **d**. If we double the exponents for each prime base in 24, we get the exponent values for 576.
f no
12 70 days
13 **a** **i** 1, 2, 3, 6, 9, 18, 27, 54 **ii** 1, 2, 3, 6, 13, 26, 39, 78
b $54 = 2 \times 3^3$ **c** $78 = 2 \times 3 \times 13$
d **i** 6 cm $\times$ 6 cm **ii** 117 pieces
14 **a** **i** Terry closes every even numbered locker.
ii Nick changes every locker that is a multiple of 3.
b 16 lockers; 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96
c 49 lockers
d **i** 1, 4, 9, 16, 25, 36, 49, 64, 81, 100
ii They are square numbers.

EXERCISE 3A

1 **a** The line segment [AB] joins the two points A and B. It is only a part of the line (AB).

b The ray [AB] starts at A, passes through point B, and continues on endlessly.

c A point of intersection is a point where two intersecting lines meet.

point of intersection

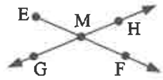
d Parallel lines are lines which are always a fixed distance apart and never meet.

e Collinear points are three or more points which lie on a single straight line.

f Concurrent lines are three or more lines which meet at the same point.

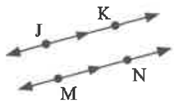
2 **a** (AB) or (BA) **b** (XY), (YX), (XZ), (ZX), (YZ), or (ZY)
3 **a** [MN] **b** [ST] **c** (AC)
4 **a** [PQ], [PR], [QR] **b** [PQ], [PR]
c No, they do not lie on the same line.
5 **a** A **b** C **c** A **d** [AD]
6 **a**

b

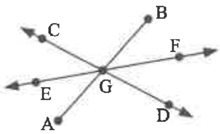


c

d



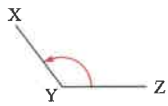
e



7 **a** intersect at F **b** collinear **c** parallel
d concurrent at D

EXERCISE 3B

1 **a** **C** **b** **A** **c** **D** **d** **B**
2 **a** $\widehat{PQR}$, acute **b** $\widehat{AOB}$, straight **c** $\widehat{LMN}$, obtuse
d $\widehat{SOT}$, reflex **e** $\widehat{CDE}$, right
3 **a** **i** b **ii** g **iii** d
b **i** reflex **ii** obtuse **iii** acute
4 **a**



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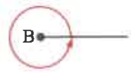

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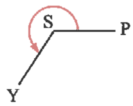

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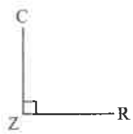

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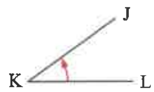
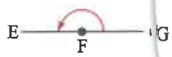
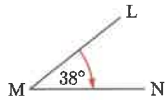


f



5 **a** 70° **b** 80° **c** 65° **d** 105° **e** 105° **f** 70°
6 28° **7** 110°
8 **a**



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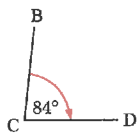

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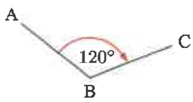

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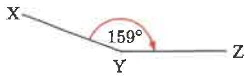

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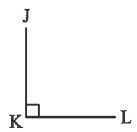
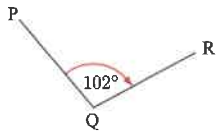


f

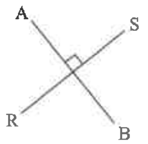


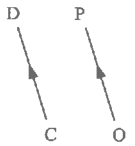
9 **a** 25°, acute **b** 90°, right **c** 45°, acute
d 115°, obtuse

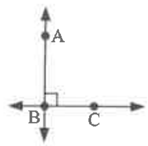
EXERCISE 3C

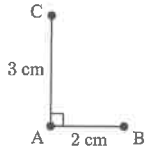
1 **a** [AB] || [EF] **b** [AB] $\perp$ [OC] **c** [MN] $\perp$ [RS]
2 **a** [DA] || [CB], [AB] || [DC]

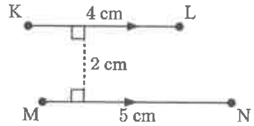
b [AB] ⊥ [BC], [BC] ⊥ [CD], [CD] ⊥ [AD], [AD] ⊥ [AB]

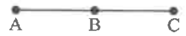
3 **a**


b


c


4 **a**


b


5 A, B, and C are collinear.


EXERCISE 3D

1 **a** 180°, supplementary **b** 184°, neither
c 90°, complementary **d** 186°, neither
e 80°, neither **f** 180°, supplementary

2 **a** 75° **b** 3° **c** 47°

3 **a** 51° **b** 123° **c** 90°

4 **a** supplementary **b** neither **c** complementary
d neither

5 **a** the size of the angle complementary to $x°$ is $(90-x)°$
b the size of the angle supplementary to $y°$ is $(180-y)°$

6 **a** $p=125$ **b** $q=38$ **c** $k=94$ **d** $b=85$
e $a=90$ **f** $g=30$

7 **a** $r=266$ **b** $z=120$ **c** $m=146$ **d** $s=50$
e $b=115$ **f** $m=31$

EXERCISE 3E

1 **a** A and C, B and D
b $A\widehat{O}C$ and $B\widehat{O}D$, $A\widehat{O}D$ and $B\widehat{O}C$

2 **a** $x=76$ **b** $a=111$ **c** $x=33$

3 $x=93$, $y=47$, $z=40$

EXERCISE 3F

1 **a** yes **b** no **c** yes **d** yes

2 **a** S **b** S **c** Q **d** Q

3 **a** no **b** yes **c** yes **d** no

4 **a** C **b** D **c** D **d** B

5 **a** no **b** yes **c** no **d** yes

6 **a** Z **b** Z **c** X **d** Y

7 **a** C and D **b** A and D **c** B and C **d** B and D

8 **a** corresponding **b** alternate **c** co-interior
d corresponding **e** corresponding
f vertically opposite **g** vertically opposite
h co-interior **i** alternate

EXERCISE 3G

1 **a** $x=124$ {equal corresponding angles}
b $b=82$ {supplementary co-interior angles}
c $q=42$ {equal alternate angles}
d $y=57$ {equal corresponding angles}
e $k=62$ {equal alternate angles}
f $a=135$ {equal corresponding angles}
g $x=147$ {equal alternate angles}
h $y=73$ {supplementary co-interior angles}
i $d=15$ {equal corresponding angles}

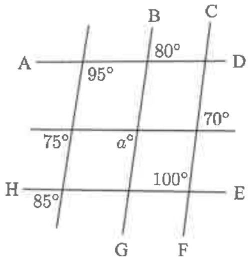
2 **a** $a=76$ {vertically opposite angles}
$b=104$ {supplementary co-interior angles}
b $a=117$ {equal corresponding angles}
$b=117$ {vertically opposite angles}
c $a=38$ {vertically opposite angles}
$b=38$ {equal alternate angles}
d $a=145$ {angles at a point}
$b=35$ {supplementary co-interior angles}
e $m=96$ {supplementary co-interior angles}
$n=84$ {supplementary co-interior angles}
f $a=36$ {equal corresponding angles}
$b=36$ {equal alternate angles}

3 **a** $x=y$ {equal alternate angles}
b $a+b=180$ {supplementary co-interior angles}
c $p=q$ {equal corresponding angles}
d $a+b=c$ {equal alternate angles}

4 $a=40$ {equal alternate angles}
$b=40$ {vertically opposite angles with a}
$c=40$ {equal alternate angles with a}
$d=90$ {equal corresponding angles}
$e=90$ {vertically opposite angles with d}
$f=50$ {angles on a line sum to 180° with a and d}
$g=50$ {equal corresponding angles with f}

EXERCISE 3H

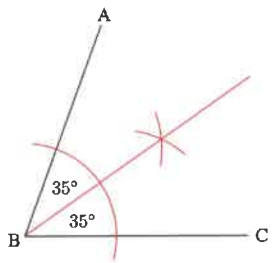
1 **a** parallel {equal alternate angles}
b not parallel {co-interior angles do not sum to 180°}
c not parallel {alternate angles are not equal}
d parallel {equal corresponding angles}
e parallel {angles on a straight line, equal corresponding angles}
f parallel {angles on a straight line, equal corresponding angles}

2 **a** The figure contains a pair of parallel lines.
{co-interior angles sum to 180°}
∴ $a=120$ {supplementary co-interior angles}
b The figure contains a pair of parallel lines.
{angles on a straight line, equal corresponding angles}
∴ $a=115$ {equal corresponding angles, vertically opposite angles}
c


[AD] ∥ [HE] {angles on a straight line, equal corresponding angles}
[BG] ∥ [CF] {equal corresponding angles, co-interior angles sum to 180°}
∴ $a=70$ {equal corresponding angles, vertically opposite angles}

EXERCISE 3I

1



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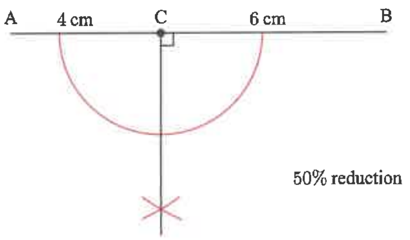
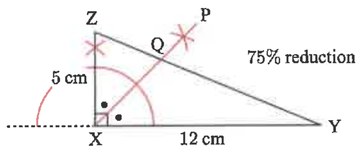
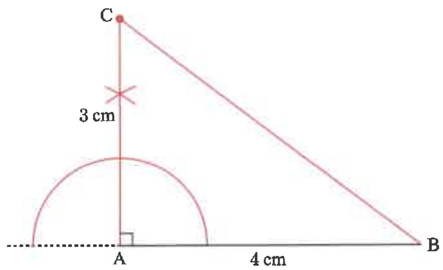


3 **a, c**



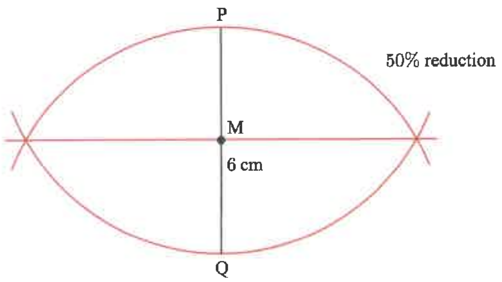
b 13 cm **d** $\approx 112°$

4 **a**



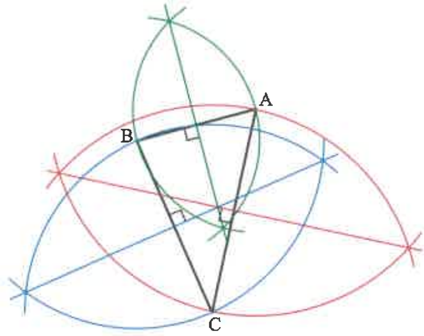
b 5 cm **c** $\approx 37°$

5 **a**



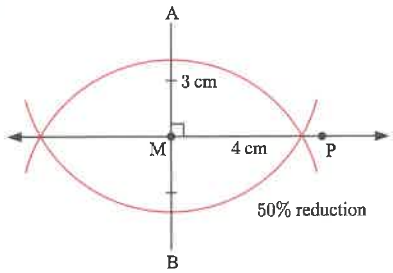
b **i** 3 cm **ii** 3 cm

6 **a**



c "The three perpendicular bisectors of the sides of a triangle are concurrent (meet at the same point)."

7 **a**

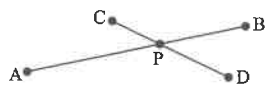


b AP = BP = 5 cm **c** $M\widehat{A}P = M\widehat{B}P \approx 53°$

REVIEW SET 3A

1 **a** (PQ) **b** [BA] **c** [RS]

2



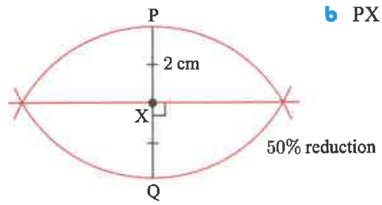
3 **a** $X\widehat{O}Y$, obtuse **b** $C\widehat{A}B$, acute **c** $F\widehat{O}G$, straight

4 **a** [PQ] $\parallel$ [SR] **b** [PQ] $\perp$ [PS], [PS] $\perp$ [SR]

5 **a** 37° **b** 50° **6** **a** $a = 45$ **b** $b = 27$ **c** $c = 120$

7 **a** E **b** A **c** C **d** C

8 **a**



b PX = QX = 2 cm

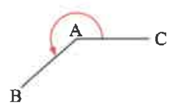
9 **a** $m = 116$ {equal alternate angles}
b $m = 81$ {equal corresponding angles}
c $m = 141$ {supplementary co-interior angles}

10 **a** parallel {vertically opposite angles, equal corresponding angles}
b parallel {angles at a point, supplementary co-interior angles}

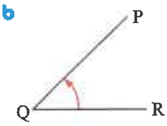
REVIEW SET 3B

1 **a** they are collinear **b** they are concurrent at D

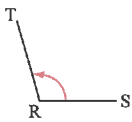
2 **a**



b

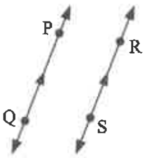


c

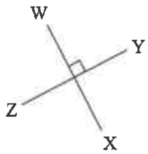


3 **a** 22° **b** 338° {angles at a point}

4 **a**



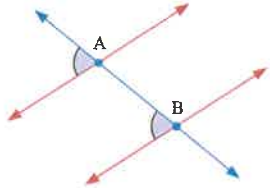
b



5 **a** **i** (AB) **ii** [AB]

b **i** The lines appear to be parallel so we do not expect them to eventually meet.

ii



The shaded corresponding angles are equal.
∴ the red lines are parallel, so they will never meet.

6 **a** supplementary **b** neither **c** complementary **d** neither

7 **a** $a = 35$ **b** $b = 45$

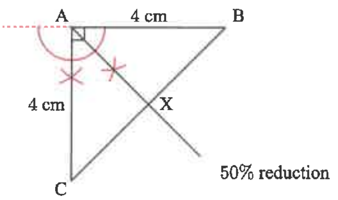
8 **a** $x = 62$ **b** $m = 111$ **c** $k = 51$

9 **a** $x = y$ {vertically opposite angles, equal corresponding angles}

b $a + b = 180$ {supplementary co-interior angles}

10 parallel {vertically opposite angles, supplementary co-interior angles, equal alternate angles}

11 **a, b**



c $\widehat{AXB}$ is a right angle.

EXERCISE 4A

1 **a** 39 **b** 51 **c** 51 **d** 61 **e** 295 **f** 1061
2 **a** 107 **b** 104 **c** 1705 **d** 724 **e** 3311 **f** 12 427
3 **a** 57 **b** 54 **c** 359 **d** 69 **e** 124 **f** 542
4 **a** 195 **b** 149 **c** 115 **d** 234 **e** 514 **f** 373 **g** 341 **h** 1122 **i** 818 **j** 1098 **k** 995 **l** 150
5 628 km **6** 516 km

EXERCISE 4B

1 **a** 54 **b** 47 **c** 71 **d** 25 **e** 83 **f** 177
2 **a** 53 **b** 15 **c** 85 **d** 221 **e** 183 **f** 2222
3 **a** 34 **b** 28 **c** 47 **d** 198 **e** 88 **f** 272
4 **a** 31 **b** 83 **c** 775 **d** 115 **e** 219 **f** 168
5 127 **6** 319 m

EXERCISE 4C

1 **a** 280 **b** 480 **c** 280 **d** 63 000 **e** 7200 **f** 30 000 **g** 20 000 **h** 63 000 **i** 360 000
2 **a** 130 **b** 900 **c** 3100 **d** 600 **e** 2100 **f** 600 **g** 1400 **h** 9000
3 **a** 344 **b** 680 **c** 1134 **d** 770 **e** 2520 **f** 1722 **g** 1584 **h** 3440
4 **a** 144 **b** 1064 **c** 2014 **d** 693 **e** 936 **f** 11 976 **g** 8428 **h** 74 925
5 **a** 7400 **b** 2448 **c** 32 048 **d** 56 000 **e** 3000 **f** 474 **g** 2852 **h** 4158 **i** 2717
6 **a** 370 000 **b** 1 000 000 **c** 1 000 000
7 1365 people **8** \$924 **9** 144 socks

EXERCISE 4D

1 **a** 3 **b** 8 **c** 70 **d** 9 **e** 9 **f** 400
2 **a** 4 **b** 2 **c** 5 **d** 26 **e** 18 **f** 17 **g** 15 **h** 25 **i** 48
3 **a** 41 **b** 62 **c** 49 **d** 31 **e** 22 **f** 54
4 32 bags **5** 22 kg **6** \$9

EXERCISE 4E

1 **a** 12 000 **b** 13 000 **c** 100 000 **d** 300 **e** 3000 **f** 20 000
2 **a** 2200 **b** 23 000
3 **a** 8000 **b** 14 000 **c** 180 000 **d** 40 000 **e** 2 000 000 **f** 800 000 **g** 32 000 000 **h** 4 000 000 **i** 20 000 000
4 **a** 20 **b** 4 **c** 5 **d** 30 **e** 200 **f** 350 **g** 100 **h** 50 **i** 75
5 1200 tests **6** 1000 biscuits **7** \$190
8 15 eggs **9** 54 000 avocados
10 **a** **i** 20 000 people per km^2 **ii** 10 000 people per km^2 **b** Tokyo
11 **a** 5 600 000, reasonable **b** 720 000, not reasonable **c** 500, reasonable **d** 22 500, not reasonable

EXERCISE 4F

1 **a** 8 **b** 10 **c** 0 **d** 6 **e** 6 **f** 24 **g** 11 **h** 15 **i** 7
2 **a** 35 **b** 25 **c** 6 **d** 15 **e** 4 **f** 18 **g** 21 **h** 19 **i** 5 **j** 23 **k** 11 **l** 13
3 **a** 21 **b** 21 **c** 0 **d** 3 **e** 0 **f** 6 **g** 33 **h** 9 **i** 5 **j** 25 **k** 1 **l** 4 **m** 2 **n** 60 **o** 24
4 **a** 50 **b** 68 **c** 9 **d** 17 **e** 1 **f** 225 **g** 9 **h** 19 **i** 17
5 **a** 241 **b** 369 **c** 23 **d** 8 **e** 77 **f** 161
6 **a** $5 + 9 \div 3 = 8$ **b** $7 \times 11 - 21 = 56$ **c** $18 - 16 \div 2 = 10$ **d** $17 - 3^2 = 8$ **e** $13 - 4 \times 2 = 5$ **f** $4 \times 13 - 6 \times 7 = 10$
7 **a** $3 \times (4 + 2) \times 5 = 90$ **b** $(3 \times 4 - 5) \times 4 = 28$ **c** $4 \times (16 - 1) - 6 = 54$ **d** $(6 + 7 \times 2) \div 5 = 4$ **e** $4 + 4 \div (2 + 2) = 5$ **f** $(3 + 11 - 5) \div 3 = 3$

EXERCISE 4G

1 **a** $1080 - (427 + 173) + 769$ dollars **b** \$1249
2 **a** $92 - 2 \times 15$ kg **b** 62 kg
3 **a** $(23\,000 + 12\,500) \div 2$ dollars **b** \$17 750
4 **a** $100 - (3 \times 20 + 30)$ dollars **b** \$10

5 a $70 + (4 + 7) \times 8$ litres b 158 litres
6 a $(60\,000 + 8 \times 15\,000) \div (2 + 4 + 3)$ pounds
b £20 000
7 a $(29 \times 4 + 30 \times 6) \times 21$ books b 6000 books
c 6216 books

REVIEW SET 4A

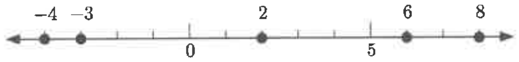
1 a 82 b 181 c 372 d 96 e 806 f 547
2 a 45 b 126
3 a 450 b 1300 c 299 d 28 000 e 171 f 950
4 £660 5 a 50 b 6 c 53
6 35 boxes 7 a 8000 b 1000 c 40
8 $4500 9 a $580 b $563 c $17
10 a 7 b 26 c 90
11 a $2 + 12 \div (4 - 2) = 8$ b $30 \div (5 + 1) + 4 = 9$
12 a $6 \times 15 \div (6 - 1)$ students b 18 students
13 a $6 \times 20 + (11 - 6 - 1) \times 25$ guest rooms
b 220 guest rooms c $65 340

REVIEW SET 4B

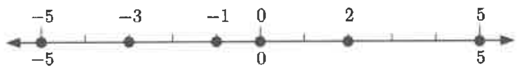
1 a 650 b 7000 c 149 2 $345
3 a 159 b 25 c 356 4 276
5 1955 kg 6 32 000 newspapers
7 a 90 b 8 c 98
8 a 3000 b 15 c 28 000
9 48 000; Scott's answer is not reasonable.
10 a 30 minutes b 21 minutes
11 a 56 b 9
12 a $6 - 1 + 7 = 12$ b $20 - 8 \div 2 - 4 = 12$
13 a $16 \times 3 \div (16 - 4)$ kg b 4 kg c $56

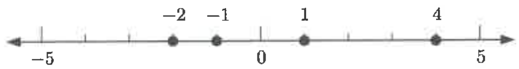
EXERCISE 5A

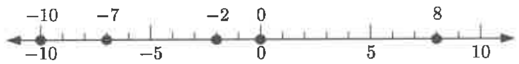
1 a 3 b −8 c 10 d −7 e −19 f 14
2 a A = −3, B = 0, C = −5, D = 6
b A = −2, B = 9, C = 3, D = −8
3 a


b


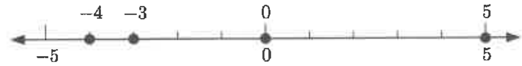
c


d


4 a $3 > -4$ b $-7 < -5$ c $-6 < 1$
d $-2 < 0$ e $-3 > -10$ f $0 > -9$
5 a true b false c true d true e false f true
6 a


−2, −1, 1, 4
b


−10, −7, −2, 0, 8

7 a


5, 0, −3, −4
b


8, 6, −2, −5, −9
8 Moscow −6°C, Oslo −4°C, Tokyo 1°C, Ulaanbaatar 3°C, Melbourne 19°C, Singapore 33°C

EXERCISE 5B

1 a +21 b −17 c −30
2 a −3 b +15 c −250
3 a +9 b −6 c +15 d −5 e −20 f +40
4 a 7 km to the east b 12 m ahead
c 1 floor below ground level
5

	Number	*Opposite of statement*	*Opposite number*
a	+5	losing by 5 goals	−5
b	+25	25 km to the west	−25
c	−3	The clock is 3 minutes fast	+3
d	−200	200 m above sea level	+200
e	−60	60 g overweight	+60

6 a moving 7 steps north b warming by 6°C
c losing $50
7

	Operation	*Opposite of statement*	*Opposite operation*
a	subtract 4 kg	gaining 4 kg	add 4 kg
b	add 3 floors	3 floors downwards	subtract 3 floors
c	add 6°C	cooling by 6°C	subtract 6°C
d	subtract 8 m	walking 8 m right	add 8 m
e	subtract 20 cm	rising 20 cm	add 20 cm

EXERCISE 5C

1 a 3 b −3 c −3 d 3
2 a 0 b 0 c 12 d −12
3 a −2 b −1 c 3 d −5 e −9 f −6
g −2 h −11
4 a −4 b −6 c −10 d 7
5 a −3 b −2 c 3 d 0 e −8 f −6
g −12 h −17
6 −2°C 7 1 floor below 8 at her campsite
9 20 m below the surface
10 a travelling 5 km south b travelling 6 km east
11 a gaining 5 kg b gaining 5 kg
12 a a fall of 3°C b going down 1 floor c spending $30
d a profit of £40 e losing by 14 points
13 walking 2 km east

EXERCISE 5D

1 a 1 b 7 c −7 d −1 e −4 f 8
g −8 h 4 i 5 j 15 k −15 l −5
2 a −1 b −8 c −2 d 4 e −10 f 9
g 3 h −17 i −9 j −12 k 3 l 6
3 a −2 b 2 c −14 d 14 e −14 f −2
g 14 h 2

4 **a** 1 **b** 6 **c** 3 **d** 2 **e** −7 **f** 17
g −17 **h** −27 **i** 16

5 **a** 9 **b** 3 **c** 9 **d** 2 **e** 16 **f** 15
g 9 **h** 21

6 **a** apartment: 7, lift: 0, car park: −5, rubbish skip: −10
b **i** 5 m **ii** 17 m **iii** 12 m **iv** 5 m

7 **a** **i** Monday **ii** Wednesday
b **i** 9°C **ii** 6°C **iii** 5°C

8 **a** **i** The team has scored more goals than they have had scored against them.
ii The team has scored the same number of goals as they have had scored against them.
iii The team has scored less goals than they have had scored against them.
b −5 **c** **i** 21 goals **ii** 42 goals
d sum of goal differences = 0
Every goal *for* one team is a goal *against* another team.

EXERCISE 5E

1 **a** 24 **b** −24 **c** −24 **d** 24 **e** −24 **f** −24
g 24 **h** 24

2 **a** −10 **b** −30 **c** 6 **d** −50 **e** −48 **f** −45
g −88 **h** −33 **i** −81 **j** 24 **k** −55 **l** 42

3 **a** □ = 1 **b** □ = −2 **c** □ = −11 **d** □ = −4
e □ = −6 **f** □ = −2 **g** □ = 4 **h** □ = −1
i □ = 6 **j** □ = −3 **k** □ = −3 **l** □ = −10

4 **a** −3 points **b** −12 points

5 **a** 4 **b** −60 **c** −36 **d** 40 **e** 72 **f** −63
g 80 **h** −27

6 **a** 1 **b** −1 **c** 1 **d** −1 **e** 1 **f** −1
−1 raised to an even power is 1.
−1 raised to an odd power is −1.

7 **a** $6 = 1 \times 6$, $6 = -1 \times -6$, $6 = 2 \times 3$, $6 = -2 \times -3$
b $-9 = 1 \times -9$, $-9 = -1 \times 9$, $-9 = -3 \times 3$
c $15 = 1 \times 15$, $15 = -1 \times -15$, $15 = 3 \times 5$, $15 = -3 \times -5$
d $-14 = 1 \times -14$, $-14 = -1 \times 14$, $-14 = 2 \times -7$, $-14 = -2 \times 7$
e $-12 = 1 \times -12$, $-12 = -1 \times 12$, $-12 = 2 \times -6$, $-12 = -2 \times 6$, $-12 = 3 \times -4$, $-12 = -3 \times 4$
f $13 = 1 \times 13$, $13 = -1 \times -13$
g $-24 = 1 \times -24$, $-24 = -1 \times 24$, $-24 = 2 \times -12$, $-24 = -2 \times 12$, $-24 = 3 \times -8$, $-24 = -3 \times 8$, $-24 = 4 \times -6$, $-24 = -4 \times 6$
h $36 = 1 \times 36$, $36 = -1 \times -36$, $36 = 2 \times 18$, $36 = -2 \times -18$, $36 = 3 \times 12$, $36 = -3 \times -12$, $36 = 4 \times 9$, $36 = -4 \times -9$, $36 = 6 \times 6$, $36 = -6 \times -6$

EXERCISE 5F

1 **a** 5 **b** −5 **c** −5 **d** 5 **e** 5 **f** −5
g −5 **h** 5 **i** 11 **j** −11 **k** −11 **l** 11

2 **a** −2 **b** −5 **c** 7 **d** −5 **e** −7 **f** 9
g −1 **h** −7 **i** 10 **j** −7 **k** −11 **l** 12

3 **a** □ = −4 **b** □ = −6 **c** □ = −4 **d** □ = −25
e □ = −9 **f** □ = −12 **g** □ = −8 **h** □ = −5
i □ = −9 **j** □ = 3 **k** □ = −28 **l** □ = −8
m □ = −120 **n** □ = 144 **o** □ = 12

4 −9°C (drops 9°C per hour)

5 828 m (gains 828 m per minute)

EXERCISE 5G

1 **a** 1 **b** −7 **c** 1 **d** −2 **e** −10 **f** 14
g 4 **h** −15 **i** 5 **j** 16 **k** 8 **l** 4

2 **a** −7 **b** −5 **c** −9 **d** −3 **e** 7 **f** 6
g −45 **h** 144 **i** −32

3 No, $-3^2 = -9$ and $(-3)^2 = 9$.

4 **a** $(18\,000 \times 4 - 45\,000 \times 3) \div (4 + 3)$ dollars
b −$9000 (a loss of $9000 per month)

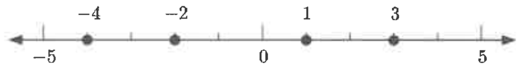
EXERCISE 5H

1 **a** −21 **b** 53 **c** −51 **d** −54 **e** −950
f −4 **g** −24 **h** 140 **i** −340

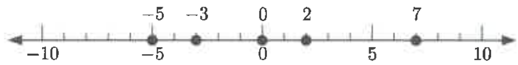
2 8 m below **3** −$500

REVIEW SET 5A

1 **a** −4 **b** 7 **c** −12 **d** 15

2 **a**


3, 1, −2, −4

b


7, 2, 0, −3, −5

3 **a** −4 **b** +5 **c** −7

4 **a** −5 **b** 6 **c** −2 **d** −17

5 **a** travelling 8 km south **b** a loss of $20

6 **a** 5 **b** 6 **c** −7

7 **a** Ying **b** Cathy
c **i** 19 minutes **ii** 21 minutes **iii** 8 minutes

8 **a** −6 **b** −45 **c** 77 **d** 64

9 **a** −8 **b** −2 **c** 9 **d** −9

10 **a** □ = −7 **b** □ = −40 **c** □ = −9

11 **a** 6 **b** 24 **c** −27

12 **a** −532 **b** −459 **c** 705

13 **a** **i** −2 **ii** +6 **iii** −5 **b** **i** −12 **ii** −6

14 **a** **i** 20 **ii** 4 **iii** −12 **iv** −28
b 32 m higher **c** 32 m lower **d** 48 m

REVIEW SET 5B

1 A = 3, B = −1, C = 7, D = 0, E = −4

2 **a** $0 > -6$ **b** $-5 < 3$ **c** $-7 > -10$

3 cooling by 12°C **4** **a** 4 **b** −6 **c** −2

5 4 km west of the harbour

6 **a** 11 **b** −14 **c** 9 **7** **A**

8 **a** −72 **b** 240 **c** −64 **d** −8 **e** −9 **f** 10

9 **a** Amy: 48 points, Sean: −8 points **b** 56 points

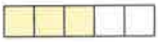
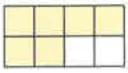
10 −1°C **11** **a** 28 **b** −6 **c** 4 **12** −$222

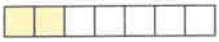
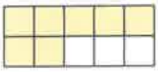
13 **a** 8 floors **b** floor 4 **c** floor 4
d **i** $(3 + 2) - (3 - 2)$ **ii** 4 floors **iii** No

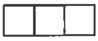
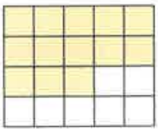
14 **a** −25 **b** −14
c **i** −12 **ii** 2 knots above the top of the plant

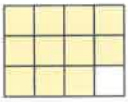
EXERCISE 6A

1 **a** $\frac{1}{4}$ **b** $\frac{5}{6}$ **c** $\frac{5}{8}$ **d** $\frac{7}{15}$ **e** $\frac{7}{10}$ **f** $\frac{17}{20}$ **g** $\frac{80}{100}$ **h** $\frac{57}{100}$

2 **a**  **b** 

c  **d** 

e  **f** 

g  **h** 

3 **a** $\frac{3}{4}$ **b** $\frac{1}{8}$ **c** $\frac{5}{6}$ **d** $\frac{7}{10}$

4 **a** $\frac{9}{23}$ **b** $\frac{5}{23}$ **c** $\frac{7}{23}$ **d** $\frac{16}{23}$ **e** $\frac{18}{23}$ **f** $\frac{16}{23}$

EXERCISE 6B

1 **a** $\frac{1}{2}$ **b** $\frac{1}{5}$ **c** $\frac{4}{7}$ **d** $\frac{8}{9}$ **e** $\frac{2}{3}$ **f** $\frac{9}{10}$ **g** $\frac{9}{6}$ **h** $\frac{20}{4}$

2 **a** $3 \div 5$ **b** $2 \div 7$ **c** $6 \div 10$ **d** $5 \div 8$ **e** $13 \div 3$

3 **a** $\frac{-2}{3}$ **b** $\frac{4}{-5}$ **c** $\frac{-6}{-7}$ **d** $\frac{10}{-12}$ **e** $\frac{-11}{22}$ **f** $\frac{-23}{-5}$ **g** $\frac{16}{-8}$ **h** $\frac{-18}{2}$

4 **a** $-1 \div 8$ **b** $-4 \div -6$ **c** $3 \div -9$ **d** $-10 \div -2$ **e** $-24 \div 6$

5 **a** $8 \div 2 = 4$ **b** $15 \div 5 = 3$ **c** $24 \div 8 = 3$ **d** $10 \div 10 = 1$ **e** $16 \div 4 = 4$ **f** $42 \div 6 = 7$ **g** $60 \div 20 = 3$ **h** $77 \div 11 = 7$ **i** $72 \div 9 = 8$ **j** $132 \div 11 = 12$

6 **a** $25 \div 5 = 5$ **b** $25 \div -5 = -5$ **c** $-25 \div 5 = -5$ **d** $-25 \div -5 = 5$ **e** $27 \div 9 = 3$ **f** $-27 \div 9 = -3$ **g** $27 \div -9 = -3$ **h** $-27 \div -9 = 3$

7 $-\frac{4}{2} = -(4 \div 2) = -2$

$\frac{-4}{2} = -4 \div 2 = -2$

$\frac{4}{-2} = 4 \div -2 = -2$

$\therefore \quad -\frac{4}{2} = \frac{-4}{2} = \frac{4}{-2}$

8 **a** $-36 \div 9 = -4$ **b** $72 \div -8 = -9$ **c** $-63 \div 7 = -9$ **d** $-60 \div -12 = 5$ **e** $108 \div -9 = -12$

9 **a** 2 **b** 2 **c** -3 **d** 2 **e** 2 **f** 3 **g** -10 **h** 4 **i** 2 **j** -4 **k** 5 **l** 7

EXERCISE 6C

1 **a** $\frac{5}{2}$ **b** $\frac{5}{4}$ **c** $\frac{14}{5}$ **d** $\frac{35}{8}$ **e** $\frac{37}{7}$ **f** $\frac{33}{10}$ **g** $\frac{31}{16}$ **h** $\frac{67}{12}$ **i** $\frac{83}{20}$ **j** $\frac{607}{100}$

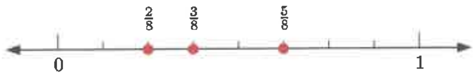
2 **a** $1\frac{4}{5}$ **b** $1\frac{1}{4}$ **c** $2\frac{1}{10}$ **d** $10\frac{1}{3}$ **e** $3\frac{5}{6}$ **f** $3\frac{4}{7}$ **g** $7\frac{1}{9}$ **h** $10\frac{3}{10}$ **i** $9\frac{1}{8}$ **j** $7\frac{7}{12}$

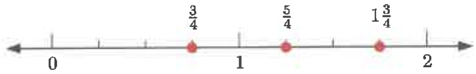
3 **a** $-1\frac{1}{5}$ **b** $-2\frac{2}{3}$ **c** $-1\frac{3}{4}$ **d** $-2\frac{1}{5}$ **e** $-4\frac{1}{3}$

4 **a** 11 cartons **b** $\frac{5}{6}$ **c** $11\frac{5}{6}$

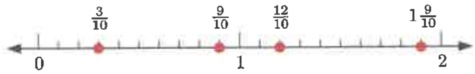
5 **a** 7 cartons **b** $\frac{5}{12}$ **c** $7\frac{5}{12}$ **6** $3\frac{3}{4}$ m **7** $4\frac{5}{6}$ m

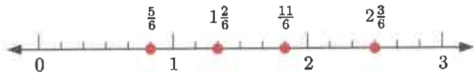
EXERCISE 6D

1 **a**


b


c


d


e


f


2 **a** $A = \frac{10}{6}$ or $1\frac{4}{6}$, $B = \frac{3}{6}$, $C = \frac{7}{6}$ or $1\frac{1}{6}$, $D = \frac{1}{6}$

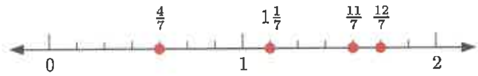
b $A = \frac{6}{4}$ or $1\frac{2}{4}$, $B = \frac{9}{4}$ or $2\frac{1}{4}$, $C = \frac{13}{4}$ or $3\frac{1}{4}$, $D = \frac{11}{4}$ or $2\frac{3}{4}$

c $A = \frac{2}{7}$, $B = \frac{10}{7}$ or $1\frac{3}{7}$, $C = \frac{6}{7}$, $D = \frac{8}{7}$ or $1\frac{1}{7}$

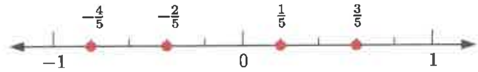
d $A = \frac{11}{5}$ or $2\frac{1}{5}$, $B = \frac{4}{5}$, $C = \frac{13}{5}$ or $2\frac{3}{5}$, $D = \frac{7}{5}$ or $1\frac{2}{5}$

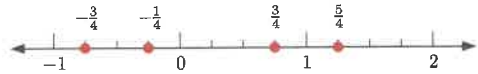
3 **a**

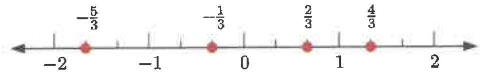

b $\frac{10}{9}, 1\frac{4}{9}, \frac{15}{9}, 2\frac{2}{9}$

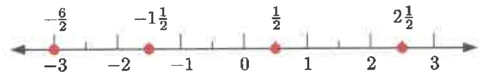
4 **a**


b $\frac{12}{7}, \frac{11}{7}, 1\frac{1}{7}, \frac{4}{7}$

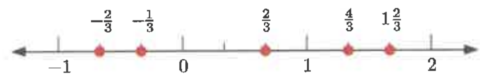
5 **a**


b


c


6 **a**


b $-\frac{6}{2}, -1\frac{1}{2}, \frac{1}{2}, 2\frac{1}{2}$

7 **a**


b $1\frac{2}{3}, \frac{4}{3}, \frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$

EXERCISE 6E

1 a true b false c true d false e false

2 a $\frac{4}{10}$ b $\frac{8}{20}$ c $\frac{20}{50}$ d $\frac{40}{100}$

3 a $\frac{1}{3}$ b $\frac{20}{60}$ c $\frac{5}{15}$ d $\frac{100}{300}$

4 a $\frac{3}{12}$ b $\frac{8}{12}$ c $\frac{10}{12}$ d $\frac{54}{12}$ e $\frac{10}{12}$

5 a $\frac{4}{20}$ b $\frac{15}{20}$ c $\frac{26}{20}$ d $\frac{13}{20}$ e $\frac{9}{20}$

6 a $\frac{30}{100}$ b $\frac{80}{100}$ c $\frac{44}{100}$ d $\frac{86}{100}$ e $\frac{70}{100}$

EXERCISE 6F

1 a $\frac{1}{3}$ b $\frac{1}{4}$ c $\frac{4}{5}$ d $\frac{2}{3}$ e $\frac{1}{2}$
f $\frac{3}{4}$ g $\frac{4}{5}$ h $\frac{6}{7}$ i $\frac{4}{5}$ j $\frac{1}{3}$
k $\frac{7}{10}$ l $\frac{2}{5}$ m $\frac{5}{9}$ n $\frac{16}{25}$ o $\frac{3}{8}$

2 a $\frac{3}{2}$ b $\frac{4}{3}$ c $\frac{8}{5}$ d $\frac{7}{4}$ e $\frac{5}{3}$
f $\frac{7}{3}$ g $\frac{5}{3}$ h $\frac{7}{2}$ i $\frac{5}{4}$ j $\frac{12}{7}$

3 a $-\frac{1}{3}$ b $-\frac{4}{5}$ c $\frac{-2}{7}$ d $\frac{2}{-5}$ e $-\frac{3}{2}$
f $\frac{-3}{5}$ g $\frac{1}{-2}$ h $-\frac{9}{14}$ i $\frac{-7}{3}$ j $\frac{5}{-3}$

4 a $\frac{2}{3}$ b $-\frac{3}{8}$ c -3 d $\frac{2}{3}$ e 1
f $\frac{1}{2}$ g $\frac{5}{3}$ h -2

EXERCISE 6G

1 a $\frac{2\times\cancel{3}}{5\times\cancel{3}}=\frac{2}{5}$ b $\frac{9\times\cancel{5}}{2\times\cancel{5}}=\frac{9}{2}$ c $\frac{3\times\cancel{2}}{\cancel{2}\times 7}=\frac{3}{7}$
d $\frac{-4\times\cancel{3}}{5\times\cancel{3}}=\frac{-4}{5}$ e $\frac{\cancel{3}\times\cancel{4}}{\cancel{4}\times\cancel{3}}=1$ f $\frac{\cancel{-1}\times 5}{\cancel{-1}\times 6}=\frac{5}{6}$

2 a 7 b 5 c -3 d 2 e 12 f 12

3 a $\frac{1}{2}$ b $\frac{1}{4}$ c $\frac{2}{3}$ d $\frac{2}{-3}$ e $\frac{3}{4}$ f $\frac{-6}{7}$
g $\frac{1}{3}$ h $\frac{3}{-4}$ i $\frac{2}{3}$ j $\frac{-4}{5}$ k $\frac{2}{3}$ l $\frac{5}{9}$

4 Yes, since the numerator and denominator are different prime numbers, it is not possible for them to share common factors other than 1.

EXERCISE 6H

1 a $\frac{1}{5}$ b $\frac{1}{5}$ c $\frac{3}{5}$ d $\frac{1}{8}$ e $\frac{3}{16}$ f $\frac{1}{4}$

2 a $\frac{1}{5}$ b $\frac{4}{7}$ c $\frac{3}{8}$ d $\frac{3}{5}$ e $\frac{1}{5}$ f $\frac{1}{9}$

3 a $\frac{1}{3}$ b $\frac{1}{4}$ c $\frac{2}{5}$

4 a $\frac{1}{8}$ b $\frac{1}{6}$ c $\frac{2}{3}$ 5 $\frac{7}{10}$ 6 $\frac{2}{5}$ 7 $\frac{3}{20}$

EXERCISE 6I

1 a $\frac{5}{8}>\frac{3}{8}$ b $\frac{12}{5}>\frac{9}{5}$ c $-\frac{2}{7}<\frac{1}{7}$
d $-\frac{3}{8}>-\frac{5}{8}$ e $-\frac{2}{5}>-\frac{4}{5}$ f $\frac{13}{6}>\frac{11}{6}$

2 a $\frac{3}{5}<\frac{11}{15}$ b $\frac{2}{9}<\frac{1}{3}$ c $-\frac{1}{6}>-\frac{2}{11}$
d $\frac{7}{10}<\frac{19}{25}$ e $-\frac{8}{11}>-\frac{3}{4}$ f $\frac{7}{12}>\frac{9}{16}$
g $-\frac{11}{8}<-\frac{8}{6}$ h $\frac{16}{5}<\frac{10}{3}$

3 a $2\frac{2}{3}>\frac{7}{3}$ b $\frac{11}{4}>2\frac{1}{2}$ c $-1\frac{2}{3}>-\frac{7}{4}$
d $\frac{32}{9}>3\frac{1}{3}$ e $-1\frac{7}{8}<-\frac{11}{6}$ f $\frac{27}{10}<2\frac{3}{4}$

4 Lex

5 a $\frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{3}{4}$ b $-\frac{2}{3}, -\frac{2}{5}, \frac{1}{4}, \frac{1}{3}$ c $\frac{5}{8}, \frac{2}{3}, \frac{11}{15}, \frac{7}{9}$
d $1\frac{5}{6}, \frac{-12}{-6}, 2\frac{1}{8}, \frac{11}{5}$

EXERCISE 6J

1 a $\frac{7}{9}$ b $\frac{2}{5}$ c $\frac{9}{7}$ d 1 e $\frac{4}{5}$ f 4
g $\frac{1}{2}$ h -1 i $\frac{9}{5}$ j $\frac{5}{4}$ k 0 l $\frac{1}{5}$

2 a $\frac{3}{8}$ b $\frac{3}{10}$ c $\frac{1}{14}$ d $\frac{1}{2}$ e $\frac{1}{6}$ f $\frac{27}{20}$
g $\frac{5}{12}$ h $\frac{1}{21}$ i $\frac{7}{12}$ j $\frac{13}{24}$ k $-\frac{11}{10}$ l $\frac{11}{20}$
m $\frac{29}{18}$ n $\frac{7}{40}$ o $\frac{3}{4}$ p $\frac{11}{12}$

3 a $4\frac{2}{5}$ b $1\frac{2}{3}$ c $2\frac{1}{6}$ d $2\frac{13}{14}$ e $\frac{9}{10}$ f $3\frac{17}{18}$
g $\frac{5}{6}$ h $\frac{5}{12}$

4 a $\frac{3}{4}$ b $1\frac{1}{3}$ c $-1\frac{4}{7}$ d $3\frac{3}{10}$

5 a $\frac{53}{35}$ b $1\frac{7}{8}$ c $\frac{2}{9}$ d $2\frac{2}{5}$

6 a the second day b $\frac{43}{60}$ 7 $\frac{1}{40}$ 8 $\frac{23}{90}$

9 $4\frac{1}{4}$ hours

EXERCISE 6K

1 a $\frac{3}{4}$ b $\frac{4}{7}$ c $\frac{12}{5}$ d $\frac{15}{8}$

2 a 4 b 12 c 6 d 36

3 a $\frac{3}{2}$ b $\frac{4}{3}$ c $\frac{3}{2}$ d $\frac{15}{2}$ e $\frac{21}{2}$
f $\frac{28}{3}$ g $\frac{20}{3}$ h $\frac{77}{3}$

4 a \$42 b 15 kg c 14 km

5 \$81 6 45 questions 7 \$50

8 a $-\frac{3}{5}$ b $\frac{-3}{5}$ c $-\frac{3}{5}=\frac{-3}{5}=\frac{3}{-5}$

EXERCISE 6L

1 a $\frac{1}{6}$ b $\frac{3}{10}$ c $\frac{4}{9}$ d $\frac{4}{15}$ e $\frac{4}{27}$ f $\frac{5}{12}$
g $\frac{10}{9}$ h $\frac{63}{100}$ i $\frac{15}{16}$ j $\frac{8}{15}$ k $4\frac{1}{12}$ l $9\frac{5}{8}$

2 a $\frac{1}{4}$ b $\frac{1}{3}$ c 1 d $\frac{5}{9}$ e $\frac{9}{14}$ f $\frac{3}{2}$
g $\frac{1}{4}$ h $\frac{1}{12}$ i $\frac{4}{3}$ j 1 k $\frac{3}{4}$ l $1\frac{1}{2}$

3 a $\frac{1}{8}$ b $\frac{3}{2}$ c 20 d $\frac{33}{1000}$

4 a $\frac{3}{10}$ b $\frac{1}{16}$ c $\frac{1}{3}$ d $\frac{3}{4}$ e $2\frac{1}{4}$ f 2

5 a $\frac{1}{6}$ b $\frac{1}{6}$ c $\frac{1}{14}$ d $\frac{1}{10}$ e $\frac{1}{5}$ f $\frac{6}{5}$

6 a $-\frac{5}{12}$ b $-\frac{1}{6}$ c $-\frac{20}{21}$ d $\frac{9}{20}$

7 5 hours 8 $\frac{3}{10}$ 9 $\frac{91}{200}$ litres

EXERCISE 6M

1 a $\frac{4}{3}$ b $\frac{3}{2}$ c $\frac{6}{5}$ d $\frac{7}{4}$ e $\frac{3}{8}$ f $\frac{5}{18}$

2 a $\frac{2}{3}$ b $\frac{3}{8}$ c $\frac{5}{11}$ d $\frac{4}{19}$ e $\frac{8}{15}$ f $\frac{6}{31}$

3 a $-\frac{4}{3}$ b -3 c $-\frac{6}{5}$ d $-\frac{5}{12}$ e $-\frac{8}{9}$ f $-\frac{5}{14}$

EXERCISE 6N

1 a 3 b 2 c $\frac{5}{3}$ d $\frac{1}{4}$

2 a $\frac{4}{3}$ b $\frac{12}{5}$ c $\frac{9}{10}$ d $\frac{14}{15}$

3 a $\frac{5}{16}$ b $\frac{3}{8}$ c $\frac{1}{5}$ d $\frac{5}{18}$ e $\frac{1}{10}$ f $1\frac{3}{7}$
g $1\frac{4}{5}$ h $4\frac{1}{8}$

4 a $\frac{9}{16}$ b $3\frac{1}{2}$

5 a $-\frac{8}{9}$ b $-\frac{4}{9}$ c $-\frac{4}{15}$ d $\frac{3}{10}$ 6 b $\frac{9}{5}$

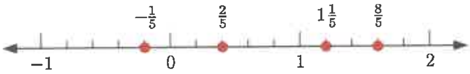
7 80 packets 8 2880 bottles 9 \$18 000 10 $\frac{1}{6}$

REVIEW SET 6A

1 **a** $\frac{4}{15}$ **b** $\frac{7}{8}$ **2** **a** $\frac{5}{6}$ **b** $\frac{8}{2}$ **c** $\frac{30}{7}$

3 **a** $9\frac{2}{3}$ **b** $7\frac{3}{5}$ **c** $-3\frac{1}{4}$

4



5 **a** $\frac{12}{28}$ **b** $\frac{3}{8}$ **6** **a** $\frac{1}{6}$ **b** $-\frac{6}{11}$ **c** $\frac{9}{5}$

7 **a** $\frac{2}{5} > \frac{7}{20}$ **b** $\frac{5}{9} < \frac{7}{11}$ **c** $-\frac{3}{8} > -\frac{5}{12}$

8 **a** $\frac{1}{3}$ **b** $\frac{17}{100}$ **c** $\frac{9}{100}$

9 **a** $\frac{16}{15}$ **b** $\frac{1}{30}$ **c** $6\frac{1}{3}$ **d** $2\frac{2}{5}$

10 **a** $\frac{5}{21}$ **b** $\frac{3}{4}$ **c** $3\frac{1}{2}$ **d** $\frac{12}{55}$

11 **a** $\frac{15}{22}$ **b** $\frac{35}{48}$ **12** 9 students **13** **a** $\frac{8}{15}$ **b** $\frac{7}{15}$

14 **a** $\frac{1}{20}$ **b** $\frac{1}{50}$ **c** $\frac{7}{100}$
d yes (there will be 200 g remaining)

15 **a** 627 apricots **b** 456 apricots **c** 96 apricots

16 **a** $\frac{5}{12}$ **b** $\frac{7}{12}$ **c** $\frac{7}{24}$ **d** 340 apples

REVIEW SET 6B

1 **a** $\frac{7}{20}$ **b** $\frac{1}{5}$ **c** $\frac{1}{2}$ **2** **a** 3 **b** 25 **c** -3

3 **a** $\frac{19}{10}$ **b** $\frac{36}{5}$ **4** $A = 2\frac{3}{5}$, $B = -\frac{2}{5}$, $C = 1\frac{1}{5}$

5 **a** $\frac{8}{24}$ **b** $\frac{15}{24}$ **c** $\frac{14}{24}$

6 $\frac{7}{20}$ **7** $\frac{2}{7}$ **8** $\frac{4}{15}, \frac{1}{3}, \frac{2}{5}$ **9** $2\frac{1}{14}$

10 **a** $\frac{35}{8}$ **b** $\frac{15}{2}$ **c** 12

11 **a** $\frac{1}{12}$ **b** $4\frac{1}{15}$ **c** $-\frac{5}{28}$ **d** $2\frac{4}{5}$ **12** 250 pots

13 $\frac{4}{15}$ **14** **a** $\frac{11}{12}$ bottles **b** $\frac{11}{36}$ **c** $3\frac{1}{8}$ L

15 **a** 300 video files **b** **i** $\frac{3}{200}$ GB **ii** 8000 music files

16 **a** $\frac{7}{15}$ **b** $\frac{4}{15}$ **c** \$85

EXERCISE 7A

1 **a** 7 and 8 **b** 15 and 16 **c** 9 and 10 **d** 22 and 23

2 **a** $\frac{5}{10}$ **b** $\frac{4}{100}$ **c** $\frac{1}{1000}$ **d** $\frac{6}{10\,000}$

3 **a** 700 **b** $\frac{7}{10}$ **c** $\frac{7}{100}$ **d** $\frac{7}{1000}$
e $\frac{7}{100}$ **f** 70 000 **g** $\frac{7}{10}$ **h** $\frac{7}{10\,000}$

4 **a** $4 + \frac{2}{10}$ **b** $7 + \frac{5}{10} + \frac{3}{100}$ **c** $9 + \frac{1}{10} + \frac{8}{100}$
d $3 + \frac{3}{100}$ **e** $\frac{2}{10} + \frac{3}{100} + \frac{4}{1000}$ **f** $1 + \frac{5}{100} + \frac{9}{1000}$
g $5 + \frac{6}{1000} + \frac{1}{10\,000}$ **h** $\frac{7}{10\,000} + \frac{1}{100\,000}$
i $2 + \frac{5}{10} + \frac{1}{1000}$ **j** $\frac{7}{100} + \frac{7}{1000} + \frac{1}{10\,000}$
k $10 + 1 + \frac{9}{10} + \frac{1}{100} + \frac{2}{1000}$ **l** $\frac{1}{100} + \frac{1}{10\,000}$

5 **a** 0.7 **b** 0.15 **c** 0.549 **d** 0.03 **e** 0.105
f 0.067 **g** 0.084 **h** 0.0039 **i** 0.6155

6 **a** 0.71 **b** 0.13 **c** 0.54 **d** 0.267
e 0.506 **f** 0.097 **g** 0.803 **h** 0.022

7 **a** $\frac{3}{100}$ **b** $\frac{17}{100}$ **c** $\frac{39}{100}$ **d** $\frac{72}{100}$ **e** $\frac{93}{100}$

8 **a** $\frac{6}{1000}$ **b** $\frac{80}{1000}$ **c** $\frac{92}{1000}$ **d** $\frac{205}{1000}$ **e** $\frac{713}{1000}$

9 **a** 7.6 **b** 3.67 **c** 12.17 **d** 2.05
e 1.461 **f** 6.039 **g** 12.001 **h** 3.0007
i 5.39 **j** 7.0203 **k** 7.21 **l** 3.723

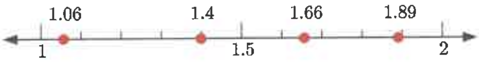
EXERCISE 7B

1 **a** 28.3 cm **b** 38.5°C **c** 74.6 kg

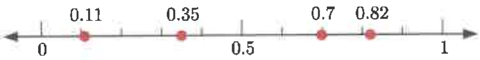
2 **a** P: 7.3, Q: 7.9 **b** P: -0.6, Q: 0.2
c P: -1.3, Q: 0.5 **d** P: -4.4, Q: -2.3

3 **a** P: 5.61, Q: 5.68 **b** P: 2.02, Q: 2.14
c P: -0.08, Q: 0.06 **d** P: -1.79, Q: -1.58

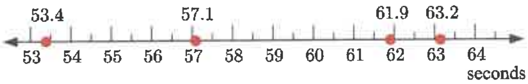
4 **a**



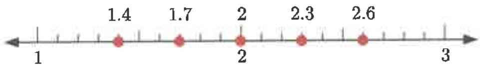
b



5



6 **a**



b 1.4, 1.7, 2, 2.3, 2.6

7 **a**

b 0.14, 0.1, 0.04, -0.03, -0.1

EXERCISE 7C

1 **a** $0.339 < 0.393$ **b** $5.05 > 0.55$ **c** $0.6 = 0.60$
d $2.62 > 2.6$ **e** $0.39 < 0.4$ **f** $12.121 < 21.121$
g $0.123 < 0.132$ **h** $\frac{150}{1000} = 0.15$ **i** $2.4 = 2.400$
j $0.902 > 0.209$ **k** $0.008\,76 < 0.0876$
l $3.20 < 3.201$

2 **a** 1.036, 1.3, 1.36 **b** 8.6, 8.67, 8.76
c 0.052, 0.495, 0.5 **d** 32.7, 32.71, 33.17
e 7.999, 8.066, 8.1 **f** 6.043, 6.304, 6.34, 6.403
g 9.009, 9.09, 9.1, 9.2 **h** 0.09, 0.099, 0.9, 0.99

3 **a** 4.44, 4.4, 4.2 **b** 7.1, 7.03, 6.97
c 0.6, 0.576, 0.56 **d** 32.51, 23.59, 23.5
e 16.4, 16.053, 16.05 **f** 10.4, 10.046, 9.49
g 3.111, 3.11, 3.101, 3.011 **h** 5.911, 5.901, 5.9, 5.19

4 16.91 seconds, 16.98 seconds, 17.1 seconds, 17.19 seconds

5 **a** Thursday **b** Tuesday

EXERCISE 7D

1 **a** 3.4 **b** 5.2 **c** 26.8 **d** 36.4
e 7.8 **f** 9.1 **g** 41.7 **h** 53.0

2 **a** 6.76 **b** 0.28 **c** 4.10 **d** 11.97
e 8.06 **f** 24.18 **g** 0.07 **h** 108.61

3 **a** 5.183 **b** 7.076 **c** 0.267 **d** 3.501
e 40.097 **f** 63.490 **g** 8.116 **h** 0.080

4 **a** 56 **b** 106 **c** 243 **d** 430

5 **a** 2.5 **b** 6.1 **c** 0.74 **d** 18 **e** 0.044 **f** 8.5

6 **a** 37.1 **b** 5.21 **c** 0.109 **d** 68.6
e 0.759 **f** 0.0305

7 **a** 23 **b** 23.06 **c** 23.1

8 **a** 8 **b** 8.0 **c** 8.0424

9 83.1 km/h **10** 5.2 goals per game **11** \$57.29

12 **a** 0.348 **b** 1.43 **c** 0.444 **d** 19.2
e 1.73 **f** 6.56 **g** 8.60 **h** 10.3

EXERCISE 7E

1 **a** $\frac{7}{10}$ **b** $\frac{2}{5}$ **c** $1\frac{1}{10}$ **d** $2\frac{3}{5}$ **e** $\frac{19}{100}$ **f** $\frac{29}{100}$
g $\frac{1}{4}$ **h** $\frac{4}{25}$ **i** $\frac{17}{20}$ **j** $\frac{24}{25}$ **k** $\frac{3}{20}$ **l** $\frac{1}{20}$
m $\frac{7}{100}$ **n** $3\frac{13}{100}$ **o** $5\frac{2}{25}$ **p** $7\frac{11}{20}$

2 **a** $\frac{101}{1000}$ **b** $\frac{23}{500}$ **c** $\frac{41}{200}$ **d** $\frac{1}{8}$
e $\frac{73}{500}$ **f** $\frac{7}{8}$ **g** $1\frac{3}{8}$ **h** $4\frac{19}{250}$

3 **a** $-\frac{3}{5}$ **b** $-\frac{9}{10}$ **c** $-\frac{7}{20}$ **d** $-1\frac{1}{5}$
e $-\frac{6}{25}$ **f** $-2\frac{1}{4}$ **g** $-\frac{31}{500}$ **h** $-3\frac{3}{8}$

EXERCISE 7F

1 **a** 0.5 **b** 0.6 **c** 0.55 **d** 0.25
e 0.34 **f** 1.45 **g** 0.92 **h** 0.02
i 0.124 **j** 0.515 **k** 2.56 **l** 0.414
m 0.95 **n** 0.016 **o** 0.625 **p** 3.175

2 **a** $\frac{1}{25} = 0.04$, $\frac{19}{500} = 0.038$, $\frac{1}{40} = 0.025$
b 0.044, $\frac{1}{25}$, $\frac{19}{500}$, 0.03, $\frac{1}{40}$

3 **a** -0.8 **b** -0.75 **c** -0.45 **d** -0.84
e -0.67 **f** -1.25 **g** -0.74 **h** -2.875

EXERCISE 7G

1 **a** 0.9 **b** 0.93 **c** 1.75 **d** 1.13
e 19.633 **f** 19.81 **g** 0.548 **h** 0.6638
i 4.01 **j** 5.24 **k** 6.266 **l** 20.263

2 **a** 2.3 **b** 2.13 **c** 5.13 **d** 0.7
e 4.2 **f** 0.01 **g** 1.29 **h** 1.008
i 6.55 **j** 0.0739 **k** 6.454 **l** 0.9766

3 **a** 20.06 **b** 104.642 **c** 311.349 **d** 130.295

4 **a** 11.9 **b** 5.44 **c** 11.947 **d** 49.065

5 4.27 m **6** **a** 11.05 km **b** 1.55 km farther

7 **a** 50.97 s **b** **i** 0.49 s **ii** 1.74 s **c** 48.25 s

8 \$695.54

EXERCISE 7H

1 **a** 32.71 **b** 327.1 **c** 3271 **d** 32 710

2 **a** 76 **b** 7600 **c** 76 000 **d** 7 600 000

3 **a** 271 **b** 486 **c** 22.5 **d** 3.4
e 16 400 **f** 2 **g** 79.41 **h** 392.6
i 810 **j** 905 **k** 0.17 **l** 16 700
m 10.08 **n** 36 **o** 76.1 **p** 33 800

4 **a** -4.8 **b** -0.57 **c** -60.32

EXERCISE 7I

1 **a** 8.46 **b** 0.846 **c** 0.0846 **d** 0.008 46

2 **a** 0.07 **b** 0.0007 **c** 0.000 07 **d** 0.000 007

3 **a** 0.6 **b** 9.2 **c** 52.9 **d** 5.29
e 0.529 **f** 0.0529 **g** 0.03 **h** 0.0003
i 0.0097 **j** 0.006 **k** 0.0006 **l** 0.000 022
m 0.000 77 **n** 0.002 963 **o** 0.000 035 **p** 0.005 16

4 **a** -1.25 **b** -0.0044 **c** $-0.000\,079\,3$

EXERCISE 7J

1 **a** 2.8 **b** 7.2 **c** 3 **d** 0.24
e 18 **f** 0.24 **g** 21 **h** 18

2 **a** 0.08 **b** 0.56 **c** 0.054 **d** 0.015
e 0.0024 **f** 0.108 **g** 0.112 **h** 0.0147
i 0.11 **j** 0.0234 **k** 0.0247 **l** 0.405

3 **a** 0.04 **b** 0.105 **c** 0.0144

4 **a** 1036.2 **b** 10.362 **c** 103.62 **d** 10.362
e 0.103 62 **f** 1.0362 **g** 1.0362 **h** 103.62
i 0.010 362

5 **a** 3.6 (4) **b** 5.89 (6) **c** 37.38 (36)
d 67.16 (63) **e** 274.06 (280) **f** 30.527 (30)

6 **a** -6.3 **b** -0.48 **c** -0.006 **d** 0.0132

7 31.5 kg **8** \$16.79 (rounded) **9** \$60.50

10 116.25 km

EXERCISE 7K

1 **a** 3.6 **b** 10.9 **c** 20.1 **d** 0.09
e 0.025 **f** 0.26 **g** 7.7 **h** 0.059
i 0.146 **j** 0.020 55 **k** 0.9575 **l** 0.049 875

2 **a** 2 **b** 7 **c** 5 **d** 8
e 2 **f** 11 **g** 30 **h** 300
i 11 **j** 800 **k** 90 **l** 1200

3 **a** 0.5 **b** 0.2 **c** 1.9 **d** 44
e 0.029 **f** 7.3 **g** 839.2 **h** 8.675

4 **a** -0.16 **b** -10 **c** -1.67 **d** 63

5 1.56 L **6** 26 minutes **7** 25 cans

8 550 lengths of pipe

REVIEW SET 7A

1 **a** $\frac{5}{10}$ **b** $\frac{6}{1000}$

2 **a** $2 + \frac{1}{10} + \frac{2}{1000} + \frac{3}{10\,000}$ **b** $\frac{1}{250}$

3 **a** 5.4 **b** 2.38

4 **a** 28.55 **b** 0.5 **c** 46 **d** 0.539

5 **a** 2.05, 2.5, 2.55 **b** 4.3, 4.304, 4.34, 4.403

6 **a** 0.8 **b** -0.36 **c** 0.065

7 **a** 1.006 **b** 6.19 **c** 23.13

8 **a** 859 **b** 8590 **c** 859 000

9 **a** 0.64 **b** 0.48 **c** 0.085 **d** 2.9 **e** -7 **f** 13.6

10 14.74 (14) **11** 40 laps **12** 10.17 tonnes

13 **a** \$31.50 **b** \$7.55 more **c** No, as the total is \$103.50

REVIEW SET 7B

1 **a** 0.43 **b** 0.701 **c** 0.0208

2 **a**


b 0.62, 0.6, 0.53, 0.5, 0.48

3 **a** $3.03 < 3.303$ **b** $0.514 < 0.541$ **c** $2.404 > 2.044$

4 3.3 steals per game **5** **a** $\frac{43}{50}$ **b** $2\frac{6}{25}$ **c** $-\frac{9}{200}$

6 **a** 62.54 **b** 6.87 **c** 0.000 39

7 **a** 0.476 **b** 0.004 76 **c** 0.0476

8 **a** 6.74 **b** 0.0674 **c** 0.006 74

9 **a** 0.013 **b** $-0.000\,72$ **c** 0.32

10 8 cups **11** 176.8 cm

12 **a** 18.95 hours **b** £227.40

13 **a** 1.5 g **b** 4.9 g more protein **c** 182 g **d** 5 slices

EXERCISE 8A

1 **a** **i** 4 puppies **ii** 7 puppies
b **i** $(d-1)$ puppies **ii** $(d+2)$ puppies

2 **a** $(3 \times 8 + 7)$ blueberries **b** $(3 \times 12 + 7)$ blueberries
c $(3 \times b + 7)$ blueberries

3 **a** $(6 \times 2 + 5)$ horses **b** $(6 \times 4 + 5)$ horses **c** $(6 \times h + 5)$ horses

EXERCISE 8B

1 **a** $p+3$ **b** $2p+1$ **c** $3p+2$ **d** $2p+4$

2 **a** $2a$ **b** $3x$ **c** $5p$ **d** $2y+3$ **e** $2x+2$ **f** $3n+3$

3 **a** $2a$ **b** $2a$ **c** $5x$ **d** $5x$ **e** $6n$ **f** $6a$ **g** $20c$ **h** $10d$

4 **a** $b+1$ **b** $b+2p$ **c** $p+3$ **d** $b+2p+3$ **e** $2b+p$ **f** $3b+2$

5 **a** $2b+c$ **b** $x+2y+2$ **c** $3x+2y$ **d** $2m+n+1$ **e** $n+3x+3$ **f** $2p+2q+4$

6 **a** $2a-b$ **b** $3a-c$ **c** $2x+2$ **d** $3x-3$

7 **a** $6-2t$ **b** $x-2y$ **c** $2s-3t$ **d** $n-4m$ **e** $3p-3q$ **f** $4a-2b$

8 **a** $3x+2y$ **b** $3a+2b$ **c** $5m+2n$ **d** $u-7w$ **e** $9c-4d$ **f** $6a-5b$

9 **a** mn **b** mn **c** de **d** $2ab$ **e** $4cd$ **f** $16cd$ **g** $9xy$ **h** xyz **i** bhk

10 **a** $xz+2y$ **b** $5-4xy$ **c** $2a+3bc$

EXERCISE 8C

1 **a** x^3 **b** $2x^2$ **c** p^4 **d** $5b^3$ **e** $3ab^2$ **f** f^2g^3h **g** $4t^2$ **h** $9y^3$ **i** $2a^2b^3$

2 **a** c^2+d **b** $3+a^2$ **c** f^2-f **d** w^3+7 **e** e^3-2e^2 **f** $5a^3+b^2$ **g** $4xy^2+z^2$ **h** x^2-3xy **i** $6xy-x+x^2$

3 **a** $x \times x$ **b** $y \times y \times y$ **c** $3 \times x \times x$ **d** $4 \times m \times m \times m$ **e** $8 \times x \times x \times x \times y$ **f** $5 \times p \times q \times q$ **g** $c \times c + 4 \times d \times d \times d$ **h** $3 \times v \times v - 5 \times w \times w$ **i** $2 \times x \times y \times y + x \times x$ **j** $5 - 3 \times x \times x \times x \times y$ **k** $a \times b \times b + a \times a \times b$ **l** $2 \times p - 3 \times p \times p \times q$

4 **a** $3x^2$ **b** $3x^2$ **c** $20x^2$ **d** $4x^3$ **e** $6x^3$ **f** $2x^4$ **g** $2x^2y$ **h** $6x^2y^2$

EXERCISE 8D

1 **a** 2 plus a **b** a minus 2 **c** 4 times x **d** a on 2 **e** p plus q **f** 7 minus m **g** x times y **h** 3 on a **i** a plus the product of 3 and b **j** 2 times a, minus c **k** a times b, plus 4 **l** a on the product of 2 and b

2 **a** $p+1$ **b** $2q$ **c** $p-q$ **d** $\dfrac{3}{p}$ **e** $4-pq$ **f** $3p+4q$ **g** $\dfrac{p}{q}$

3 **a** 5 times a squared **b** 3 plus b squared **c** a squared minus b squared **d** a squared on 4

4 **a** $p+q^2$ **b** $\dfrac{q^2}{p}$ **c** $3q^2$ **d** $\dfrac{q}{p^2}$

5 **a** B **b** C **c** F **d** E **e** A **f** D

EXERCISE 8E

1 **a** $x, -3, 2y$ **b** $2x, -y, 4$ **c** $-x, 3y, 2x, 1$

2 **a** 4 **b** 3 **c** -2

3 **a** 5 **b** 4 **c** 8 **d** 4 **e** 1 **f** -2 **g** 1 **h** -1 **i** -7

4 **a** 4 terms **b** -7 **c** -2 **d** $5y$ and $-2y$

5 **a** $2x$ and $-3x$ **b** $2x$ and $5x$, 3 and 5 **c** x and $5x$, y and $-y$ **d** $2x$ and $3x$ **e** 3 and 7, q^2 and $4q^2$ **f** $-2b$ and $3b$

6 **a** true **b** false **c** true **d** true **e** false **f** false

7 No, they have different variable forms.

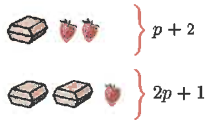
EXERCISE 8F

1 **a** **i** $2b$ blueberries **ii** $3b$ blueberries **iii** $5b$ blueberries **b** $2b+3b=5b$

2 **a** $p+1+p+2=2p+3$ **b** $2p+1+2p+1=4p+2$ **c** $2p+3+p+6=3p+9$ **d** $3p+2+p+2=4p+4$

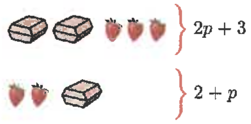
3 **a** $p+1+b+2=b+p+3$ **b** $b+1+b+p+1=2b+p+2$

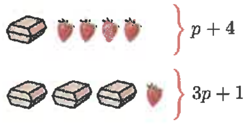
4 **a** $p+2+2p+1$ is: $3p+3$ is:



The expressions are equal.

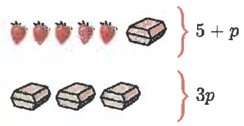
b $2p+3+2+p$ is: $p+4+3p+1$ is:

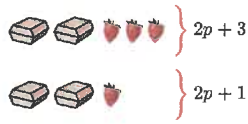




The expressions are not equal.

c $5+p+3p$ is: $2p+3+2p+1$ is:





The expressions are not equal.

5 **a** $4p$ **b** $6p$ **c** $2p+3$ **d** $4p+5$ **e** $2p+7$ **f** $4+5p$

6 **a** $2p$ **b** $3p$ **c** 0 **d** $2p+3$ **e** $2p+4$ **f** $2p+2$

EXERCISE 8G

1 **a** $2a$ **b** $3b$ **c** $2a+3b$ **d** $3+2x+y$ **e** $3f+3$ **f** 5 **g** $2q$ **h** $x-2$ **i** $5+2n$

2 **a** $4a$ **b** $4y$ **c** cannot be simplified **d** $3x+y$ **e** $12b$ **f** $3r$ **g** cannot be simplified **h** $6n$ **i** $2p$ **j** cannot be simplified **k** 0 **l** cannot be simplified **m** $5w$ **n** cannot be simplified **o** $11x$

3 **a** $-2z$ **b** $-4b$ **c** cannot be simplified **d** $-2m$ **e** $-2x$ **f** $-4f$ **g** $-6y$ **h** $-4k$ **i** $-4k+4$ **j** cannot be simplified **k** $-20r$ **l** $-4t$ **m** $-v$

4 **a** $8x+6y$ **b** $3p+8q$ **c** $7a+6b$ **d** $2d$ **e** $-v-4$ **f** $4x+2z$ **g** $5h$ **h** $4r+t+5$ **i** $12x-13y$

5 $4-(p+p+p)=4-3p$, $4-p+p+p=4+p$

EXERCISE 8H.1

1 **a** 9 **b** 11 **c** 9 **d** 20 **e** 10 **f** 4
g 21 **h** 25 **i** 7 **j** -8

2 **a** $3p+10$ **b** **i** 25 **ii** 46 **iii** 85

3 **a** 6 **b** 15 **c** 28 **d** 4 **e** 11 **f** -3
g -3 **h** 6 **i** $\frac{3}{2}$ **j** 1 **k** -5 **l** -11

4 **a** 7 **b** 40 **c** 16 **d** 8 **e** 3 **f** 2
g -2 **h** 12 **i** 18 **j** 30 **k** -9 **l** 48

5 **a** 14 **b** 23 **c** -4 **d** 9 **e** 36 **f** -6
g 0 **h** 0 **i** 52 **j** 46 **k** 1 **l** -18

EXERCISE 8H.2

1 **a** 3 **b** -32 **c** -3 **d** -15 **e** 8 **f** -7
g -8 **h** -10 **i** 12 **j** 6 **k** -4 **l** 14

2 **a** -14 **b** -20 **c** 9 **d** -5 **e** 52 **f** 6
g 9 **h** 1 **i** 324 **j** 93 **k** -88 **l** 6400

EXERCISE 8I

1 **a**

n	1	2	3	4	5
S	4	5	6	7	8

b

b	1	3	5	7	9
L	4	12	20	28	36

c

d	-1	1	4	8	10
C	5	9	15	23	27

d

t	-3	0	2	5	9
P	-13	-4	2	11	23

e

x	2	5	8	12
A	0	9	18	30

f

x	1	2	4	6
A	4	10	28	54

2 **a** $y=-3$ **b** $y=2$ **c** $y=22$ **d** $y=47$

3 **a** $y=15$ **b** $y=5$ **c** $y=29$ **d** $y=35$

4 **a** $A=16$ **b** $A=12$ **c** $A=7$ **d** $A=0$

5 **a** $V=3$ **b** $V=16$ **c** $V=45$ **d** $V=96$

6 **a** \$230 **b** \$425 **c** \$1010

7 **a** 150 km **b** 560 km

8 **a** 12.5 mL **b** 20 mL **c** 25 mL

9 **a** $R=2b$ **b** **i** \$20 **ii** \$50

10 **a** $40h$ dollars **b** $C=20+40h$
c **i** \$60 **ii** \$180 **iii** \$90

11 **a** $C=7n+9$ **b**

Number of wool balls purchased	*Total cost (\$)*
1	16
2	23
3	30
4	37
5	44
6	51
10	79
15	114

12 **a** \$50 per hour **b** \$30 callout fee **c** $C=50h+30$
d **i** \$430 **ii** \$630

REVIEW SET 8A

1 **a** 13 strawberries **b** 19 strawberries
c $(2s+5)$ strawberries

2 **a** $4m$ **b** $7k$ **c** $20y$ **d** $18pq$

3 **a** $4c^3$ **b** x^2-3y^2 **c** $8x^2$

4 **a** r plus s **b** 5 on m **c** x times y, minus 3
d z squared minus the product of 2 and y

5 4 terms **6** -7

7 **a** $2x^2$ and $3x^2$ **b** $5a$ and a, $-3b$ and $-2b$
c $3c$ and $3c$ **d** $2e$ and $-4e$, ef and $2ef$

8 **a** $12x+5$ **b** $5p-2$ **c** $8x$ **d** $5d+cd-3c$

9 **a** 27 **b** -5 **c** 12 **d** -2

10 **a** 30 **b** -45 **c** -63 **d** -16

11 **a**

k	1	2	3	4	5
P	9	13	17	21	25

b

m	1	4	6	10	15
N	-1	5	9	17	27

c

a	1	2	3	4	5
A	4	$\frac{11}{2}$	7	$\frac{17}{2}$	10

d

x	1	2	3	5	8
M	0	2	6	20	56

12 **a** 125 minutes **b** 197 minutes

13 **a** 28 loose cards **b** 5 boxes **c** $5c+28$ **d** 178 cards

REVIEW SET 8B

1 **a** 11 birds **b** $(3b+2)$ birds **c** $(3b+f)$ birds

2 **a** $21x$ **b** $3a+2b$ **c** $4x+2y$

3 **a** k^4 **b** c^2+5c^3 **c** $10m^3$

4 **a** $p\times p\times p\times p$ **b** $2\times q\times q\times q$
c $3\times x\times x\times x-4\times y\times y$

5 **a** ad^2 **b** $\frac{p}{q}-7$ **6** -3

7 $6x$ and $2x$, $-6y$ and y **8** **a** 4 **b** 1 **c** 2

9 **a** $7x+3y$ **b** $-6q+2$ **c** $10a+b$

10 **a** -5 **b** -2 **c** -3 **11** **a** \$80 **b** \$180

12 **a** **i** $10c+5$ **ii** $8c+15$ **iii** $18c+20$
b Warehouse A: $10\times5+5=55$ printers
Warehouse B: $8\times5+15=55$ printers
c **i** Warehouse B (2 more printers)
ii Warehouse A (6 more printers)

13 **a** $35b$ dollars **b** $C=35b+15$
c **i** \$67.50 **ii** \$155

EXERCISE 9A

1 **a** 20% **b** 35% **c** 53% **d** 86%

2 **a** $\frac{13}{100}$ **b** $\frac{37}{100}$ **c** $\frac{6}{100}$ **d** $\frac{92}{100}$ **e** $\frac{79}{100}$

3 **a** 17% **b** 38% **c** 90% **d** 125% **e** 1%

4 **a** **B** **b** **C** **c** **D** **d** **A**

5 **a** Joe's **b** Toby's **c** Katy's **d** Lily's

6 **a** 55% **b** 30% **c** 85%

7 **a** **i** 9% **ii** 5%
b gasoline, diesel and other fuel, other products, jet fuel, heavy fuel oil, asphalt, lubricants
c 100%; this represents all petroleum products.

EXERCISE 9B

1 **a** 0.6 **b** 0.35 **c** 0.18 **d** 0.08
e 0.46 **f** 0.89 **g** 1.25 **h** 2
i 0.495 **j** 0.167 **k** 0.375 **l** 0.777
m 0.3801 **n** 0.001 **o** 1.298 **p** 0.000 02

2 **a** $\frac{2}{5}$ **b** $\frac{1}{5}$ **c** 1 **d** $\frac{3}{20}$ **e** $\frac{3}{50}$ **f** $\frac{11}{20}$
g $\frac{3}{2}$ **h** $\frac{1}{25}$ **i** $\frac{22}{25}$ **j** $\frac{7}{4}$ **k** $\frac{31}{50}$ **l** $\frac{6}{5}$
m 3 **n** $\frac{49}{20}$ **o** $\frac{46}{25}$ **p** $\frac{1}{8}$

3 **a** $\frac{21}{25}$ **b** $\frac{4}{25}$

4 **a**

Animal	Sales %	Decimal	Fraction
Lion	13%	0.13	$\frac{13}{100}$
Giraffe	24%	0.24	$\frac{6}{25}$
Cheetah	14%	0.14	$\frac{7}{50}$
Meerkat	32%	0.32	$\frac{8}{25}$
Zebra	17%	0.17	$\frac{17}{100}$

b lion, cheetah, zebra, giraffe, meerkat

EXERCISE 9C

1 **a** 38% **b** 93% **c** 15% **d** 31.7%
e 54.6% **f** 80.2% **g** 7% **h** 158%
i 90% **j** 0.4% **k** 5.9% **l** 40.73%
m 160% **n** 420% **o** 300% **p** 0.26%

2 **a**

Fraction	Percentage	Decimal
$\frac{1}{4}$	25%	0.25
$\frac{2}{4}$	50%	0.5
$\frac{3}{4}$	75%	0.75
$\frac{4}{4}$	100%	1

b

Fraction	Percentage	Decimal
$\frac{1}{5}$	20%	0.2
$\frac{2}{5}$	40%	0.4
$\frac{3}{5}$	60%	0.6
$\frac{4}{5}$	80%	0.8

3 **a** 70% **b** 36% **c** 55% **d** 68%
e 66% **f** 20.5% **g** 34.1% **h** 70.9%

4 **a**

Fraction	Percentage	Decimal
$\frac{1}{3}$	$\approx 33.3\%$	≈ 0.333
$\frac{2}{3}$	$\approx 66.7\%$	≈ 0.667
$\frac{3}{3}$	100%	1

b

Fraction	Percentage	Decimal
$\frac{1}{8}$	12.5%	0.125
$\frac{2}{8}$	25%	0.25
$\frac{3}{8}$	37.5%	0.375
$\frac{4}{8}$	50%	0.5
$\frac{5}{8}$	62.5%	0.625
$\frac{6}{8}$	75%	0.75
$\frac{7}{8}$	87.5%	0.875

c

Fraction	Percentage	Decimal
$\frac{1}{6}$	$\approx 16.7\%$	≈ 0.167
$\frac{2}{6}$	$\approx 33.3\%$	≈ 0.333
$\frac{3}{6}$	50%	0.5
$\frac{4}{6}$	$\approx 66.7\%$	≈ 0.667
$\frac{5}{6}$	$\approx 83.3\%$	≈ 0.833

5 **a** 22.5% **b** 18.75% **c** 8.75% **d** 56.25%
e 67.5% **f** 8.4%

6 **a** $\approx 11.1\%$ **b** $\approx 71.4\%$ **c** $\approx 44.4\%$ **d** $\approx 41.7\%$
e $\approx 76.9\%$ **f** $\approx 48.6\%$

7 **a** $\frac{9}{20}$ **b** 25% **c** 70%

8 **a** $\frac{5}{8}$ **b** 62.5% **c** 37.5%

EXERCISE 9D

1 **a** 40% **b** 65% **c** 10% **d** 50%
e 40% **f** 75% **g** 90% **h** 80%
i 20% **j** 86% **k** 70% **l** 6%
m 47% **n** 40% **o** 62.5% **p** 75%

2 **a** 40% **b** 45% **c** 40% **d** 25%
e 50% **f** 20% **g** 20% **h** $16\frac{2}{3}\%$
i $66\frac{2}{3}\%$ **j** 22% **k** 42% **l** 55%
m 87.5% **n** 0.0125% **o** $\frac{1}{60}\%$ or $\approx 0.0167\%$

3 **a** 24% **b** 20% **c** 44%

4 yes (average is $55\frac{5}{9}\%$) **5** **a** 86% **b** 14%

6 **a**

	Forest area (km^2)	Land area (km^2)	Forest as % of land area
Bangladesh	14 238	130 170	10.9%
Colombia	584 483	1 038 700	56.3%
Finland	222 180	303 815	73.1%
Indonesia	896 412	1 811 570	49.5%
Madagascar	124 410	581 540	21.4%
Niger	11 172	1 266 700	0.9%
Philippines	85 200	298 170	28.6%
Spain	184 861	498 980	37.0%

b Finland

EXERCISE 9E

1 **a** 9 **b** 7 **c** 9 **d** £1512 **e** $750
f 4.75 tonnes **g** 3.8 m **h** 5600 mL **i** 120°

2 **a** 9 minutes **b** 720 m **c** 580 g **d** 448 mL

3 7 sweets **4** 132 seeds **5** 54 laps

6 **a** 14 students **b** 13 students **c** 17 students
d 19 students

7 3.15 kg **8** **a** 180 mL **b** 1.2 L

9 **a** 12 tonnes **b** 108 tonnes **c** $214 620

EXERCISE 9F

1 **a** 0.52 m or 52 cm **b** 3.12 m

2 **a** €80 **b** €4080 **3** **a** $108 **b** $348

4 **a** 23.4 m **b** 144 L **c** $368 **d** 890.4 g
e €4275.60 **f** 65.8692 kg

5 **a** $2.76 **b** $24.84 **6** **a** 200 g **b** 600 g

7 **a** 480 people **b** 5520 people

8 **a** 96 cm **b** 480 mL **c** \$1760 **d** 155.8 ha **e** £371.67 **f** 107.97 kg

EXERCISE 9G

1 **a** \$24 **b** \$216 **2** **a** \$245 **b** \$245
3 **a** £225 **b** £525
4 **a** \$144 **b** \$36 **c** Yes, the total price is \$108.
5 **a** \$29.55 **b** \$13 455 **c** \$92.64
6 **a** **i** €1692 **ii** €1692 **b** no

EXERCISE 9H

1 **a** a 3 m increase **b** an 8 kg decrease **c** a £6 increase **d** a 235 mL decrease **e** a 3 minute decrease **f** a \$26 increase
2 **a** 30% increase **b** 20% decrease **c** 50% increase **d** 40% decrease **e** 60% increase **f** $\approx$ 20.8% decrease **g** 40% increase **h** 25% decrease **i** $\approx$ 9.21% decrease
3 $\approx$ 57.9% decrease **4** $\approx$ 18.2% discount
5 **a** Lucas ($\approx$ 43.4% compared with $\approx$ 40.3%)
b **i** from age 14 to 15 ($\approx$ 14.9% increase)
ii from age 12 to 13 ($\approx$ 16.8% increase)
6 **a** 90% **b** **i** \$31.92 **ii** \$47.88 **iii** yes

REVIEW SET 9A

1 **a** B **b** C **c** A
2 **a** 0.47 **b** 0.06 **c** 0.927 **d** 1.65
3 **a** $\frac{29}{100}$ **b** $\frac{37}{50}$ **c** $\frac{9}{20}$ **d** $\frac{19}{10}$
4 **a** 56% **b** 68% **c** 260% **d** 27.5%
5 **a** 54% **b** 25% **c** 40.9% **6** 65%
7 **a** 7% **b** 536 students
8 **a** 192 fish **b** 3008 fish
9 **a** 28 points **b** 168 points
10 **a** \$625 **b** 36 km **11** **a** \$23.80 **b** \$214.20
12 **a** 75% increase **b** $\approx$ 20.2% decrease
13 **a** 120 seedlings **b** 90 mature plants **c** \$3070
14 **a** 320 girls **b** 200 boys **c** $\approx$ 51.8%

REVIEW SET 9B

1 **a** $\frac{3}{8}$ **b** 37.5%
2 **a** 23.9% **b** 0.22% **c** 17.5% **d** 12.5%
3 **a** £180 **b** 98 m **c** 24 minutes
4 **a** 0.725 **b** 2.38 **5** **a** $\frac{3}{20}$ **b** $\frac{9}{5}$ **c** $\frac{1}{150}$
6 43.75% **7** 12 150 spectators
8 No, only 62.5% are present.
9 **a** 5% **b** **i** 1.75 L **ii** 1.25 L **c** 700%
10 **a** 154.5 cm **b** 76.5 L
11 **a** a 25% increase **b** an 18.75% decrease
12 35% discount
13 **a** **i** 0.12 **ii** $\frac{3}{25}$ **b** **i** \$31.20 **ii** \$228.80 **c** \$290.40

EXERCISE 10A

1 **a** true **b** false **c** true **d** false **e** false **f** true
2 **a** true **b** false **c** false **d** false **e** true **f** true
3 **a** $x = 6$ **b** $x = 12$ **c** $x = 6$ **d** $x = 9$ **e** $x = 5$ **f** $x = 7$
4 **a** C **b** G **c** A **d** H **e** D **f** B **g** F **h** E

EXERCISE 10B

1 **a** $x = 3$ **b** $x = 8$ **c** $x = 9$ **d** $x = 9$ **e** $x = 7$ **f** $x = 30$ **g** $x = 9$ **h** $x = 7$ **i** $x = 7$ **j** $x = 6$ **k** $x = 12$ **l** $x = 0$
2 **a** $x = -2$ **b** $x = 3$ **c** $x = 12$ **d** $x = -5$ **e** $x = -4$ **f** $x = -2$ **g** $x = -4$ **h** $x = -5$ **i** $x = -13$

EXERCISE 10C

1 **a**

b

2 **a**

b

3 **a**

b


4 **a** $x + 3 = 7$ **b** $x + 5 = 9$ **c** $x = 11$ **d** $2x = 14$ **e** $7 - x = 8$ **f** $8 - 2x = 6$
5 **a** $x + 1 = 9$ **b** $x = 3$ **c** $-x = 9$ **d** $2x = -2$ **e** $5x = -2$ **f** $2x - 4 = 2$
6 **a** $2x = 4$ **b** $8x = 12$ **c** $x = 18$ **d** $2(x + 1) = 6$ **e** $x - 1 = 6$ **f** $2 - x = -6$
7 **a** $x = 2$ **b** $x = 5$ **c** $x + 1 = 4$ **d** $x - 2 = 6$ **e** $x = \frac{8}{5}$ **f** $x = \frac{13}{6}$ **g** $x = -7$ **h** $1 - x = 6$

EXERCISE 10D

1 **a** -4 **b** $\div 2$ **c** $\times 3$ **d** $+1$ **e** -0.7 **f** $+\frac{1}{3}$ **g** $\div \frac{1}{4}$ **h** $+11$ **i** $-\frac{3}{4}$ **j** $\div -1$ **k** $\div 7$ **l** $\times -1$ **m** $+0.6$ **n** $\times \frac{1}{2}$ **o** $\div -4$
2 **a** x **b** x **c** x **d** x **e** x **f** x **g** x **h** x **i** $3x$
3 **a** $x = 7$ **b** $x = 9$ **c** $x = 11$ **d** $x = -5$ **e** $x = -3$ **f** $x = -10$ **g** $x = 8$ **h** $x = 9$ **i** $x = -4$ **j** $x = -17$ **k** $x = 0.2$ **l** $x = \frac{2}{3}$
4 **a** $y = 9$ **b** $y = 25$ **c** $y = 7$ **d** $y = 10$ **e** $y = -7$ **f** $y = 0$ **g** $y = 33$ **h** $y = 3.9$ **i** $y = 4\frac{1}{2}$
5 **a** $a = 5$ **b** $a = 4$ **c** $a = 8$ **d** $a = 4$ **e** $a = -4$ **f** $a = 5$ **g** $a = -7$ **h** $a = 8$ **i** $a = \frac{7}{10}$
6 **a** $x = 6$ **b** $x = 12$ **c** $x = 28$ **d** $x = -5$ **e** $x = 54$ **f** $x = -22$ **g** $x = 12$ **h** $x = -10$ **i** $x = 18$
7 **a** $a = 8$ **b** $b = 13$ **c** $c = 6$ **d** $d = 44$ **e** $e = -7$ **f** $f = -5$ **g** $g = 41$ **h** $h = 21$ **i** $i = -8$ **j** $j = -7$ **k** $k = 1\frac{1}{2}$ **l** $l = -2$ **m** $m = -4$ **n** $n = 8$ **o** $o = -5$ **p** $p = 0.5$

q $q = -50$ **r** $r = 5$ **s** $s = \frac{2}{5}$ **t** $t = -28$
u $u = \frac{5}{9}$

EXERCISE 10E

1 **a** $\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{-3} \boxed{2x-3}$

b $\boxed{x} \xrightarrow{+1} \boxed{x+1} \xrightarrow{\times 4} \boxed{4(x+1)}$

c $\boxed{x} \xrightarrow{-2} \boxed{x-2} \xrightarrow{\div 3} \boxed{\frac{x-2}{3}}$

d $\boxed{x} \xrightarrow{\div 2} \boxed{\frac{x}{2}} \xrightarrow{+5} \boxed{\frac{x}{2}+5}$

2 **a** $\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{+2} \boxed{3x+2}$

b $\boxed{x} \xrightarrow{\div 3} \boxed{\frac{x}{3}} \xrightarrow{-1} \boxed{\frac{x}{3}-1}$

c $\boxed{x} \xrightarrow{-7} \boxed{x-7} \xrightarrow{\div 2} \boxed{\frac{x-7}{2}}$

d $\boxed{x} \xrightarrow{+4} \boxed{x+4} \xrightarrow{\times 2} \boxed{2(x+4)}$

3 **a** Build up: $\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+4} \boxed{2x+4}$

Undo: $\boxed{2x+4} \xrightarrow{-4} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$

b Build up: $\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{-1} \boxed{3x-1}$

Undo: $\boxed{3x-1} \xrightarrow{+1} \boxed{3x} \xrightarrow{\div 3} \boxed{x}$

c Build up: $\boxed{x} \xrightarrow{\times 4} \boxed{4x} \xrightarrow{+3} \boxed{4x+3}$

Undo: $\boxed{4x+3} \xrightarrow{-3} \boxed{4x} \xrightarrow{\div 4} \boxed{x}$

d Build up: $\boxed{x} \xrightarrow{\times 5} \boxed{5x} \xrightarrow{-12} \boxed{5x-12}$

Undo: $\boxed{5x-12} \xrightarrow{+12} \boxed{5x} \xrightarrow{\div 5} \boxed{x}$

e Build up: $\boxed{x} \xrightarrow{\div 2} \boxed{\frac{x}{2}} \xrightarrow{+4} \boxed{\frac{x}{2}+4}$

Undo: $\boxed{\frac{x}{2}+4} \xrightarrow{-4} \boxed{\frac{x}{2}} \xrightarrow{\times 2} \boxed{x}$

f Build up: $\boxed{x} \xrightarrow{\times -1} \boxed{-x} \xrightarrow{-3} \boxed{-x-3}$

Undo: $\boxed{-x-3} \xrightarrow{+3} \boxed{-x} \xrightarrow{\div -1} \boxed{x}$

g Build up: $\boxed{x} \xrightarrow{\times -2} \boxed{-2x} \xrightarrow{+5} \boxed{-2x+5}$

Undo: $\boxed{-2x+5} \xrightarrow{-5} \boxed{-2x} \xrightarrow{\div -2} \boxed{x}$

h Build up: $\boxed{x} \xrightarrow{\times \frac{2}{3}} \boxed{\frac{2}{3}x} \xrightarrow{-1} \boxed{\frac{2}{3}x-1}$

Undo: $\boxed{\frac{2}{3}x-1} \xrightarrow{+1} \boxed{\frac{2}{3}x} \xrightarrow{\div \frac{2}{3}} \boxed{x}$

4 **a** Build up: $\boxed{x} \xrightarrow{\div 3} \boxed{\frac{x}{3}} \xrightarrow{-1} \boxed{\frac{x}{3}-1}$

Undo: $\boxed{\frac{x}{3}-1} \xrightarrow{+1} \boxed{\frac{x}{3}} \xrightarrow{\times 3} \boxed{x}$

b Build up: $\boxed{x} \xrightarrow{-1} \boxed{x-1} \xrightarrow{\div 3} \boxed{\frac{x-1}{3}}$

Undo: $\boxed{\frac{x-1}{3}} \xrightarrow{\times 3} \boxed{x-1} \xrightarrow{+1} \boxed{x}$

c Build up: $\boxed{x} \xrightarrow{+5} \boxed{x+5} \xrightarrow{\div 3} \boxed{\frac{x+5}{3}}$

Undo: $\boxed{\frac{x+5}{3}} \xrightarrow{\times 3} \boxed{x+5} \xrightarrow{-5} \boxed{x}$

d Build up: $\boxed{x} \xrightarrow{\div 3} \boxed{\frac{x}{3}} \xrightarrow{+5} \boxed{\frac{x}{3}+5}$

Undo: $\boxed{\frac{x}{3}+5} \xrightarrow{-5} \boxed{\frac{x}{3}} \xrightarrow{\times 3} \boxed{x}$

e Build up: $\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{+8} \boxed{3x+8}$

Undo: $\boxed{3x+8} \xrightarrow{-8} \boxed{3x} \xrightarrow{\div 3} \boxed{x}$

f Build up: $\boxed{x} \xrightarrow{+8} \boxed{x+8} \xrightarrow{\times 3} \boxed{3(x+8)}$

Undo: $\boxed{3(x+8)} \xrightarrow{\div 3} \boxed{x+8} \xrightarrow{-8} \boxed{x}$

g Build up: $\boxed{x} \xrightarrow{-6} \boxed{x-6} \xrightarrow{\times 2} \boxed{2(x-6)}$

Undo: $\boxed{2(x-6)} \xrightarrow{\div 2} \boxed{x-6} \xrightarrow{+6} \boxed{x}$

h Build up: $\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{-6} \boxed{2x-6}$

Undo: $\boxed{2x-6} \xrightarrow{+6} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$

5 **a** Build up:
$\boxed{x} \xrightarrow{-1} \boxed{x-1} \xrightarrow{\div 2} \boxed{\frac{x-1}{2}} \xrightarrow{+5} \boxed{\frac{x-1}{2}+5}$

Undo:
$\boxed{\frac{x-1}{2}+5} \xrightarrow{-5} \boxed{\frac{x-1}{2}} \xrightarrow{\times 2} \boxed{x-1} \xrightarrow{+1} \boxed{x}$

b Build up: $\boxed{x} \xrightarrow{+1} \boxed{x+1} \xrightarrow{\times -2} \boxed{-2(x+1)}$
$\xrightarrow{+3} \boxed{3-2(x+1)}$

Undo: $\boxed{3-2(x+1)} \xrightarrow{-3} \boxed{-2(x+1)}$
$\xrightarrow{\div -2} \boxed{x+1} \xrightarrow{-1} \boxed{x}$

c Build up:

$$\boxed{x} \xrightarrow{-1} \boxed{x-1} \xrightarrow{\div -4} \boxed{-\frac{x-1}{4}} \xrightarrow{+1} \boxed{1-\frac{x-1}{4}}$$

Undo:

$$\boxed{1-\frac{x-1}{4}} \xrightarrow{-1} \boxed{-\frac{x-1}{4}} \xrightarrow{\times -4} \boxed{x-1} \xrightarrow{+1} \boxed{x}$$

d Build up:

$$\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{-2} \boxed{3x-2} \xrightarrow{\div 5} \boxed{\frac{3x-2}{5}}$$

Undo:

$$\boxed{\frac{3x-2}{5}} \xrightarrow{\times 5} \boxed{3x-2} \xrightarrow{+2} \boxed{3x} \xrightarrow{\div 3} \boxed{x}$$

e Build up:

$$\boxed{x} \xrightarrow{\times -3} \boxed{-3x} \xrightarrow{+2} \boxed{2-3x} \xrightarrow{\times 3} \boxed{3(2-3x)}$$

Undo:

$$\boxed{3(2-3x)} \xrightarrow{\div 3} \boxed{2-3x} \xrightarrow{-2} \boxed{-3x} \xrightarrow{\div -3} \boxed{x}$$

f Build up:

$$\boxed{x} \xrightarrow{\times -2} \boxed{-2x} \xrightarrow{+5} \boxed{5-2x} \xrightarrow{\div 3} \boxed{\frac{5-2x}{3}} \xrightarrow{+2} \boxed{\frac{5-2x}{3}+2}$$

Undo:

$$\boxed{\frac{5-2x}{3}+2} \xrightarrow{-2} \boxed{\frac{5-2x}{3}} \xrightarrow{\times 3} \boxed{5-2x} \xrightarrow{-5} \boxed{-2x} \xrightarrow{\div -2} \boxed{x}$$

EXERCISE 10F

1 **a** $x=5$ **b** $x=1$ **c** $x=3$ **d** $x=4$ **e** $x=-3$ **f** $x=\frac{1}{3}$ **g** $x=2$ **h** $x=\frac{5}{6}$ **i** $x=0$ **j** $x=4$ **k** $x=\frac{1}{7}$ **l** $x=\frac{1}{2}$

2 **a** $x=1$ **b** $x=5$ **c** $x=4$ **d** $x=-2$ **e** $x=-3$ **f** $x=1$

3 **a** $x=4$ **b** $x=21$ **c** $x=-30$ **d** $x=-20$ **e** $x=0$ **f** $x=-80$ **g** $x=-4$ **h** $x=6$

4 **a** $x=12$ **b** $x=3$ **c** $x=1$ **d** $x=32$ **e** $x=2$ **f** $x=-6$ **g** $x=1$ **h** $x=0$

5 **a** $x=5$ **b** $x=4$ **c** $x=8$ **d** $x=-3$ **e** $x=1$ **f** $x=4$ **g** $x=5$ **h** $x=-5$ **i** $x=2$ **j** $x=4$ **k** $x=-2$ **l** $x=2\frac{2}{3}$

6 **a** $x=3$ **b** $x=3\frac{1}{3}$ **c** $x=2$ **d** $x=2\frac{1}{3}$ **e** $x=-1\frac{1}{2}$ **f** $x=1\frac{2}{3}$

7 **a** $x=3$ **b** $x=10$ **c** $x=5$ **d** $x=23$ **e** $x=8$ **f** $x=6$ **g** $x=-9$ **h** $x=-4$ **i** $x=-7$ **j** $x=5$ **k** $x=-14$ **l** $x=-14$ **m** $x=-4$ **n** $x=-5$ **o** $x=-5$ **p** $x=-5$ **q** $x=-10$ **r** $x=40$ **s** $x=3$ **t** $x=-2\frac{3}{4}$ **u** $x=1\frac{1}{2}$

8 **a** $x=5$ **b** $x=2$ **c** $x=-1$ **d** $x=6$ **e** $x=-3$ **f** $x=-7$

EXERCISE 10G

1 **a** $x=6$ **b** $x=12$ **c** $x=-9$ **d** $x=8$ **e** $x=\frac{1}{2}$ **f** $x=3$

2 **a** $x=70$ **b** $x=70$ **c** $x=30$ **d** $x=80$ **e** $x=40$ **f** $x=50$

3 **a** $x=5$ **b** $x=2$

EXERCISE 10H

1 **a** $q=26$ **b** $t=45$ **c** $s=21$ **d** $p=75$ **e** $j=161$

2 **a** $x=100$ **b** $x=34$ **c** $x=50$

EXERCISE 10I

1 **a** $5x=30$ **b** $x+10=23$ **c** $\frac{x}{4}+6=8$ **d** $\frac{11-x}{3}=2$

2 **a** $x+12=27$ **b** $x-150=80$ **c** $\frac{x}{3}=12$ **d** $2x+10=31$

EXERCISE 10J

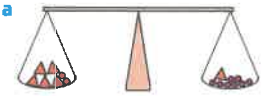
1 The number is 6. **2** The number is 5. **3** The number is 10. **4** 24 chocolates **5** 9 singers **6** \$5 **7** 32 cars **8** 5 balloons **9** 4 boxes **10** £1200

REVIEW SET 10A

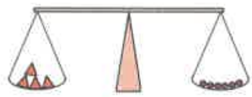
1 **a** true **b** false **c** true

2 $x=7$ **3** **a** $x=7$ **b** $x=8$ **c** $x=-6$

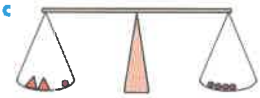
4 **a**



b



c



5 **a** $+7$ **b** $\times 8$ **c** -13 **d** $\div \frac{1}{2}$

6 **a** $x=13$ **b** $x=10$ **c** $x=-5$ **d** $x=24$

7 **a** $\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{-9} \boxed{2x-9}$

b $\boxed{x} \xrightarrow{\div 3} \boxed{\frac{x}{3}} \xrightarrow{+4} \boxed{\frac{x}{3}+4}$

8 **a** Build up: $\boxed{x} \xrightarrow{-4} \boxed{x-4} \xrightarrow{\times 3} \boxed{3(x-4)}$

Undo: $\boxed{3(x-4)} \xrightarrow{\div 3} \boxed{x-4} \xrightarrow{+4} \boxed{x}$

b Build up: $\boxed{x} \xrightarrow{+2} \boxed{x+2} \xrightarrow{\div 3} \boxed{\frac{x+2}{3}}$

Undo: $\boxed{\frac{x+2}{3}} \xrightarrow{\times 3} \boxed{x+2} \xrightarrow{-2} \boxed{x}$

9 **a** $x=2$ **b** $x=-13$ **c** $x=27$ **d** $x=-10$ **e** $x=4$ **f** $x=22$

10 **a** $x = 4$ **b** $x = -15$ **c** $x = 1\frac{1}{3}$

11 **a** $x = 7$ **b** $x = 4$ **c** $x = 65$

12 **a** $a = 16$ **b** $b = 55$ **c** $c = 120$

13 **a** $7x = 35$ **b** $\dfrac{6+x}{2} = 8$ **14** $6 **15** 34 lollies

16 **a** **i** $(x + 40)$ pavers **ii** $(x - 8)$ pavers
b $x + x + 40 + x - 8 = 200$ or $3x + 32 = 200$
c first pallet has 56 pavers, second pallet has 96 pavers, third pallet has 48 pavers

REVIEW SET 10B

1 **a** true **b** false

2 **a** $x = 22$ **b** $x = 3$ **c** $x = -9$

3 **a** $4x = 16$ **b** $x - 2 = 5$ **4** dividing by 8

5 **a** $x = 4$ **b** $x = 15$ **c** $x = -6$ **d** $x = -12$

6 **a** $\boxed{x} \xrightarrow{\times 4} \boxed{4x} \xrightarrow{-7} \boxed{4x - 7}$

b $\boxed{x} \xrightarrow{+9} \boxed{x + 9} \xrightarrow{\times 2} \boxed{2(x + 9)}$

7 **a** Build up: $\boxed{x} \xrightarrow{-9} \boxed{x - 9} \xrightarrow{\div 4} \boxed{\dfrac{x-9}{4}}$

Undo: $\boxed{\dfrac{x-9}{4}} \xrightarrow{\times 4} \boxed{x - 9} \xrightarrow{+9} \boxed{x}$

b Build up: $\boxed{x} \xrightarrow{+5} \boxed{x + 5} \xrightarrow{\times 2} \boxed{2(x + 5)}$

Undo: $\boxed{2(x + 5)} \xrightarrow{\div 2} \boxed{x + 5} \xrightarrow{-5} \boxed{x}$

8 **a** $x = 6$ **b** $x = 2$ **c** $x = 12$ **d** $x = -11$
e $x = -18$ **f** $x = -10$

9 **a** $x = -8$ **b** $x = 8$ **c** $x = -7$

10 **a** $x = 9$ **b** $x = 25$

11 **a** $p = 30$ **b** $q = 26$ **c** $x = 89$

12 $16 - 2x = 4$ **13** The number is 36. **14** 10 friends

15 **a** Each of the six friends pay $\$x$, and Ray's mother gives them \$30. $\therefore$ total amount is $(6x + 30)$ dollars.
b Each ride costs \$15, so the total cost is (6×15) dollars.
$\therefore$ $6x + 30 = 6 \times 15$
c We can use inverse operations to find the exact value of x.
d \$10 each

16 **a** 29 m
b **i** $4l + 1 = 61$
ii $l = 15$ $\therefore$ the fence is 15 m long.

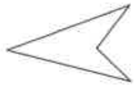
EXERCISE 11A

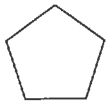
1 **a** polygon
b not a polygon as a side is curved, crossed, and not closed
c not a polygon as sides are not straight
d not a polygon as figure is not closed
e not a polygon as sides are not straight **f** polygon
g not a polygon as sides cross over **h** polygon

2 **a** convex quadrilateral **b** convex triangle
c non-convex decagon **d** convex pentagon
e non-convex quadrilateral **f** non-convex decagon
g non-convex heptagon **h** convex nonagon

3 **a** hexagon **b** octagon **c** dodecagon

4 **a**

b


5 **a**

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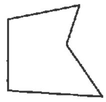
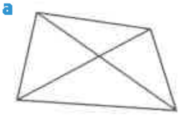


6 **a** all angles not equal **b** all angles not equal
c all sides not equal in length **d** all sides not equal in length

7 **a** true **b** false **c** true **d** false **e** false

8 **a**

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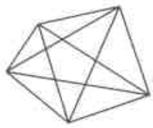
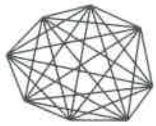

c


EXERCISE 11B

1 **a** scalene **b** isosceles **c** equilateral **d** isosceles

2 **a** obtuse **b** right angled **c** acute **d** acute

3 **a** right angled, scalene **b** obtuse, isosceles
c acute, equilateral **d** right angled, isosceles

4 Yes, an equilateral triangle has at least 2 sides which are equal in length.

EXERCISE 11C

1 **a** $x = 79$ **b** $x = 21$ **c** $x = 54$ **d** $x = 73$
e $x = 38$ **f** $x = 41$

2 **a** $t = 50$ **b** $x = 40$ **c** $y = 30$

3 **a** $n = 60$
b The interior angles of any equilateral triangle always measure 60°.

4 **a** The sum of two obtuse angles is greater than 180°.
b The sum of a right angle and an obtuse angle is greater than 180°.
c The sum of three angles less than 60° is less than 180°.

5 **a** $x = 30$ **b** $x = 36$ **c** $x = 55$ **d** $x = 50$
e $x = 65$ **f** $x = 63$

6 $\widehat{QAC} = c°$ {equal alternate angles}
$\widehat{PAB} = b°$ {equal alternate angles}
Now $\widehat{PAB} + \widehat{BAC} + \widehat{QAC} = 180°$ {angles on a line}
$\therefore$ $a + b + c = 180$

7 **a** $a = 18$, $b = 108$ **b** $a = 36$, $b = 36$, $c = 64$

EXERCISE 11D

1 **a** $a = 137$ **b** $a = 48$ **c** $a = 61$ **d** $a = 99$
e $a = 120$ **f** $a = 30$

2 **a** $c = 65$ **b** $a = 28$ **c** $a = 65$, $b = 45$

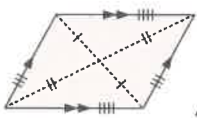
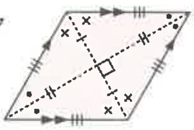
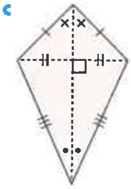
EXERCISE 11E

1 **a** $x = 75$ **b** $x = 64$ **c** $x = 75.5$ **d** $p = 90$
e $x = 5$ **f** $x = 80$ **g** $q = 108$ **h** $r = 48$
i $x = 135$ **j** $x = 70$ **k** $x = 25$ **l** $y = 30$

2 **a** $x = 37$, $\triangle ABC$ is isosceles with $AC = BC$
b $x = 45$, $\triangle KLM$ is isosceles with $KL = LM$
c $x = 20$, $\triangle PQR$ is isosceles with $PR = QR$

3 **a** C **b** E **c** G **d** B **e** F **f** D **g** A

EXERCISE 11F

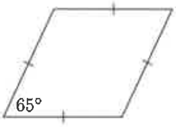
1 a  b  c 

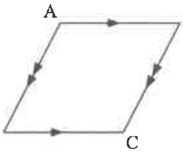
2 a rectangle b kite c rhombus
d square e trapezium f parallelogram

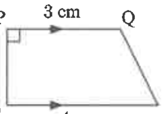
3 a true b false c true d false e false f true

4 a [AB] || [DC] b [KM] ⊥ [LN]
c [WX] || [ZY], [WZ] || [XY], [WZ] ⊥ [ZY], [ZY] ⊥ [YX], [YX] ⊥ [XW], [XW] ⊥ [WZ]

5 a rhombus {all sides equal in length}
b kite {AB = AD and BF = DF}
c $\widehat{BED} = 90°$ {angle sum of triangle, angles at a point} and all sides equal in length
∴ BEDF is a square.

6 a 

b 

c 

7 a $x = 5$ {equal sides}
$y = 90$ {opposite angles of a rhombus}
b $x = 130$ {opposite angles of a kite}
$y = 4$ {equal adjacent sides of a kite}
c $x = 2$ {diagonals of a rhombus bisect each other}
$y = 90$ {diagonals of a rhombus meet at 90°}
d $x = 6$ {diagonals of a rectangle are equal in length}
e $x = 30$ {diagonals of a rhombus bisect angles}
$y = 60$ {diagonals meet at 90°, angle sum of triangle}
f $x = 60$ {co-interior angles supplementary}
g $x = 10$ {opposite sides of a rectangle}
$y = 7$ {diagonals of a rectangle bisect each other}
h $x = 5$ {equal adjacent sides of a kite}
$y = 90$ {diagonals of a kite intersect at right angles}
i $x = 80$ {opposite angles of parallelogram}
$y = 100$ {opposite angles of parallelogram}

EXERCISE 11G

1 a $x = 122$ b $a = 100$ c $a = 70,\ b = 100$
d $a = 105,\ b = 75$ e $a = 98$ f $a = 76,\ b = 104$

2 a $a = 82$ b $a = 97,\ b = 83,\ c = 121$
c $a = 118,\ b = 92,\ c = 94,\ d = 86$
d $a = 84,\ b = 27$ e $a = 102,\ b = 83,\ c = 60$

REVIEW SET 11A

1 a non-convex quadrilateral b convex hexagon
c non-convex octagon

2 a right angled b obtuse

3 a $x = 58$ b $x = 40$ c $x = 70$

4 a $x = 137$ b $x = 55$ c $x = 69$

5 a $a = 68$ b $b = 130$ c $c = 110$

6 a true b false

7 a right angled scalene triangle; one angle is 90°, no equal sides
b parallelogram; opposite angles are equal
c square; diagonals bisect each other at 90° and are equal in length

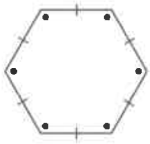
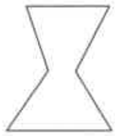
8 a $a = 90$ b $b = 34$ c $c = 34$

9 a $x = 125$ b $x = 70,\ y = 65$
c $a = 80,\ b = 55,\ c = 120$

10 a $x = 104$
b i $\widehat{ABD} = 38°$ ii $\widehat{ADB} = 38°$ iii $\widehat{CBD} = 66°$
iv $\widehat{CDB} = 66°$
c kite; one pair of opposite angles is equal in size, AB = AD and BC = CD

11 a $\widehat{PQR} = 36°$
b i △AXP, △BXR ii △AXQ, △BXQ
iii △CXP, △CXR iv △AXP, △BXR

REVIEW SET 11B

1 a  b  c 

2 a isosceles b scalene

3 a $x = 75$ b $b = 58$ c $x = 140$

4 a $x = 30$ b $a = 60,\ b = 30$ c $x = 115$

5 $x = 70$; △ABC is isosceles with AB = BC

6 parallelogram; diagonals bisect each other but not at 90° nor with equal lengths

7 $a = 65,\ b = 70,\ c = 45$ 8 $x = 90,\ y = 64,\ z = 30$

9 a parallelogram b $x = 88,\ y = 92$

10 The interior angles would sum to less than 360°.

11 a i $\widehat{ABD} = 30°$ ii $\widehat{ADB} = 30°$
{△ABD is isosceles}
b $\widehat{DBC} = 30°$ {angle sum of a triangle}
c a trapezium; $\widehat{ADC} = \widehat{DCB} = 90°$ ∴ [AD] || [BC]

12 a i $a = 130,\ b = 110,\ c = 120$ ii 360°
b i $a = 60,\ b = 60,\ c = 140,\ d = 100$ ii 360°
c The sum is 360°.

EXERCISE 12A

1 a km b mm c m d cm e m f mm

2 a **B** b **C** c **C**

3 a 25 mm b 26 mm c 8 mm d 59 mm
e 5 mm f 18 mm

4 a 4 cm b 6.3 cm

5 a 400 cm b 780 cm c 90 mm d 139.2 mm
e 8000 m f 1090 m g 159 m h 5000 mm
i 3900 mm

6 **a** 8 cm **b** 23.7 cm **c** 6 m **d** 9.03 m **e** 0.568 m **f** 2 km **g** 4.27 km **h** 4 m **i** 1.84 m

7 **a** 61 cm = 610 mm **b** 720 cm = 7.2 m **c** 3020 m = 3.02 km **d** 88 mm = 8.8 cm **e** 11 m = 1100 cm **f** 0.4 km = 400 m **g** 3.27 cm = 32.7 mm **h** 249 cm = 2.49 m **i** 3800 m = 3.8 km **j** 1.7 mm = 0.17 cm **k** 7.8 m = 7800 mm **l** 35 m = 0.035 km

8 **a** 634 cm **b** 454 cm **c** 459.5 cm **d** 512.7 cm

9 **a** 2.08 m **b** 3432.2 m **c** 8926.5 m **d** 75.335 m

10 **a** 2.19 km **b** 0.51 km **11** 19.6 cm

12 10.8 km **13** 4000 pipes **14** 200 lengths

15 92 cm

EXERCISE 12B

1 **a** 16 cm **b** 49 cm **c** 25 m **d** 18.7 cm **e** 93 mm **f** 42.9 m

2 **a** 14 cm **b** 35 m **c** 16 cm **d** 36 cm **e** 24.6 m **f** 56.8 m

3 **a** 29 mm or 2.9 cm **b** 114 mm or 11.4 cm **c** 1020 cm or 10.2 m **d** 440 cm or 4.4 m **e** 11 800 m or 11.8 km **f** 5200 m or 5.2 km

4 **a** 112 mm **b** 125 mm **c** 89 mm

5 **a** **i** 20 m **ii** 28 cm **iii** 60 mm **b** $P = 4s$

6 **a** **i** 16 cm **ii** 18 cm **iii** 34 m **b** $P = 2a + 2b$

7 **a** 480 cm **b** 544 cm **8** $13 770 **9** 540 cm

10 34.58 km **11** **a** **i** 26 cm **ii** 35 cm **b** 8.2 m

12 22 cm

EXERCISE 12C

1 **a** m^2 **b** cm^2 **c** km^2 **d** mm^2 **e** ha

2 **a** B **b** C **c** B **3** **a** 14 cm^2 **b** 12 cm^2

4 35.75 m^2 **5** **a** 34 ha **b** 340 tonnes

EXERCISE 12D

1 **a** 40 cm^2 **b** 36 cm^2 **c** 225 m^2 **d** 220 mm^2 **e** 7 m^2 **f** 23.2 cm^2 **g** 4 m^2 **h** 8.64 km^2

2 **a** 72 mm^2 **b** 1.6 m^2 **c** 1.26 m^2 **d** 1.98 km^2

3 67.5 m^2 **4** **a** 2400 m^2 **b** 80 minutes

5 **a** 0.225 m^2 **b** 16.2 m^2 **c** 72 floorboards **d** $1548

6 6 cm **7** 12 m **8** 4.8 m **9** 8 cm^2

EXERCISE 12E

1 **a** 18 cm^2 **b** 66 m^2 **c** 24 m^2 **d** 37 cm^2 **e** 49.5 m^2 **f** 7.5 cm^2 **g** 13 m^2 **h** 43.5 cm^2 **i** 210 m^2

2 36 m^2 **3** 36.75 m^2 **4** 6 cm

EXERCISE 12F

1 **a** 48 m^2 **b** 66 cm^2 **c** 80 cm^2 **d** 20 cm^2 **e** 24 cm^2 **f** 84 m^2

2 720 cm^2 **3** 45.24 cm^2 **4** 6.3 m^2

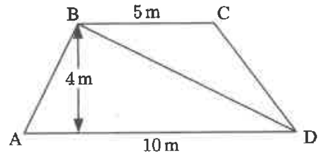
EXERCISE 12G

1 **a** 16 cm^2 **b** 63 m^2 **c** 25 cm^2 **d** 42 cm^2 **e** 13 m^2 **f** 42 cm^2

2 **a** 0.704 m^2 **b** $11.26 **3** 5 cm

4 **a** 30 m^2

b



i 20 m^2 **ii** 10 m^2 **iii** 20 m^2 + 10 m^2 = 30 m^2 ✓

REVIEW SET 12A

1 **a** 129 mm **b** 3950 m **c** 243 cm **d** 5.7 cm **e** 11.4 m **f** 0.64 km

2 2511.9 m

3 **a** 28 m **b** 38 cm **c** 310 cm or 3.1 m

4 800 triangles **5** **a** m^2 **b** km^2 **c** cm^2

6 **a** 5.29 m^2 **b** 60 m^2 **c** 54 mm^2

7 **a** 0.75 m^2 **b** 12 m^2 **c** 16 slabs **8** 144 m^2

9 **a** 35 m^2 **b** 48 cm^2 **c** 85.5 cm^2 **10** 57 cm^2

11 **a** **i** 20.9 m **ii** 167.2 m **b** **i** 168.3 m **ii** 1.1 m

12 **a** 330 m **b** $14 850 **c** **i** 110 trees **ii** $5500 **d** 5850 m^2

REVIEW SET 12B

1 **a** m **b** cm **2** 2.2 km

3 **a** 30 m **b** 21 cm **c** 44 cm

4 15.6 m **5** C

6 **a** 26 m^2 **b** 150 cm^2 **c** 420 m^2 **7** $375

8 **a** 54 cm^2 **b** 18 cm^2 **c** 2.1 cm^2

9 30 000 m^2 **10** 8 m^2

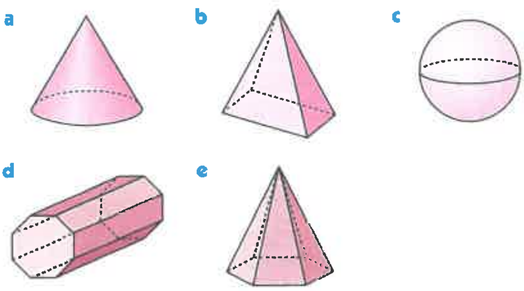
11 **a** 44.5 m^2 **b** 0.0625 m^2 **c** 712 pavers **d** $2492

12 **a** 150 m^2 **b** 12 m **c** 192 m^2 **d** 42 m^2 **e** **Hint:** Compare the areas of triangles ABC and ABD.

EXERCISE 13A

1 **a** cylinder **b** triangular-based pyramid or tetrahedron **c** pentagonal prism

2 **a** **b** **c** **d** **e**



3 **a** a cylinder **b** a sphere **c** a rectangular prism **d** a cone **e** a triangular-based pyramid **f** a cylinder

4 **a** rectangle **b** triangle

5 **a** **b** **c**

4 **a** 10 370 g **b** 0.64 t **c** 5.249 g
d 2630 mg **e** 0.285 kg **f** 43 mg
5 519 g **6** 1411 kg **7** 120 kg **8** 12.58 kg
9 19 kg **10** 4000 bricks **11** 22 m^2

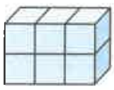
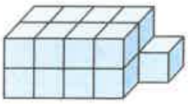
EXERCISE 14F

1 6 g **2** 4 kg **3** 3.45 kg
4 **a** 9.6 L **b** 9.6 kg **c** 8.4 kg
5 **a** 16 m^2 **b** 4800 L
c **i** If the fountain is completely full, the mass of water is 4.8 t.
$12.2 \text{ t} + 4.8 \text{ t} = 17 \text{ t} > 15 \text{ t}$
ii 17.5 cm

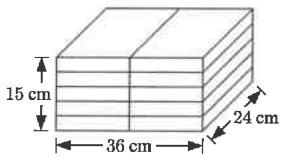
REVIEW SET 14A

1 **a** cm^3 **b** m^3
2 **a** 140 cm^3 **b** 93.6 cm^3 **c** 108 cm^3 **3** \$4438.72
4 **a** 3700 mL **b** 19.85 kL **c** 0.043 ML
5 **a** 3500 cm^2 **b** **i** 315 000 mL **ii** 315 L
6 **a** 5400 g **b** 0.475 g **c** 32 g
7 4500 kg **8** 412.5 kg
9 **a** 92 boxes **b** 8000 cm^3 **c** 736 000 cm^3
d Yes, as it needs to lift 495 kg.
10 **a** 6000 cm^3 **b** 6 L **c** 600 minutes or 10 hours
d 6 kg

REVIEW SET 14B

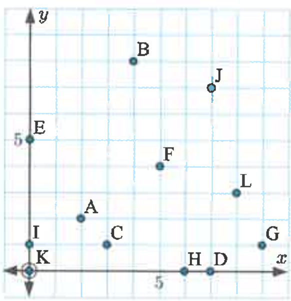
1 **a**

b


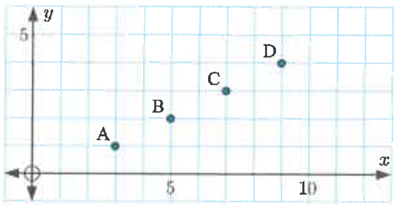
2 **a** 450 m^3 **b** 1.28 m^3 **3** 15.3 cm^3 **4** 16.25 L
5 480 L **6** **a** 12 130 kg **b** 0.043 kg
7 150 times **8** **a** 7 kg **b** 7.95 kg
9 **a** 1296 cm^3
b **i** 12 960 cm^3
ii 10 books

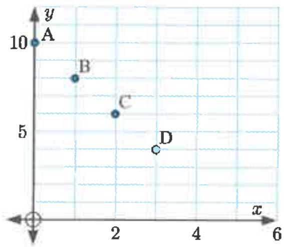


iii 12.5 kg
10 **a** **i** 19 000 000 kL **ii** 19 000 000 000 L
b 19 000 000 m^3 **c** 19 000 000 t

EXERCISE 15A

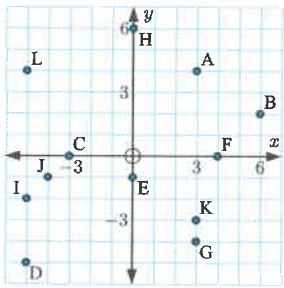
1


2 **a** The y-coordinate of any point on the x-axis is 0.
b The x-coordinate of any point on the y-axis is 0.
3 **a** **i** 6 **ii** 2 **iii** 9 **iv** 6
b **i** 2 **ii** 1 **iii** 5 **iv** 6
c A(2, 8), B(6, 4), C(9, 9), D(2, 3), E(8, 2), F(5, 7), G(6, 6), H(4, 1), I(9, 5), J(2, 5), K(0, 6)
d O(0, 0)
4 **a** C and G. They lie on the same vertical line.
b E and F. They lie on the same horizontal line. **c** E(7, 7)
5 The x-coordinates and y-coordinates of the points are not the same.
6 **a** D(2, 3) **b** D(7, 8)
7 **a** **i** (7, 5) **ii** (9, 2) **iii** (7, 7) **iv** (2, 1) and (6, 6)
b **i** Treasure Trove **ii** Lion's Den **iii** Mt. Ogre
iv Oasis
8 **a**


b E(11, 5)
9 **a**


b E(4, 2) and F(5, 0)
10 **a** move 3 units to the right **b** climb 5 units up
c move 2 units to the right, then climb 4 units up
d move x units to the right, then climb y units up

EXERCISE 15B

1 **a** **i** 0 **ii** 3 **iii** −5 **iv** 6
b **i** 1 **ii** −5 **iii** 0 **iv** 4
c A(−1, 1), B(3, 1), C(−2, −5), D(0, 4), E(1, −4), F(−3, 0), G(6, −2), H(−5, −3), I(4, 4), J(−5, 2)
d **i** B, I **ii** A, J **iii** C, H **iv** E, G
e **i** F **ii** D
2


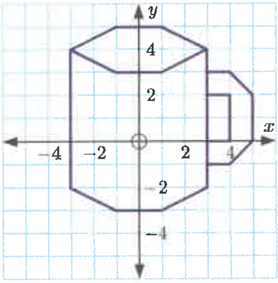
3 **a** **i** (2, 1) **ii** (−3, −2) **iii** (−1, 4) **iv** (4, −3)
v (1, 5) **vi** (−4, 2) **vii** (5, −1) **viii** (−2, −5)

b **i** dog, house **ii** flower garden, carrot patch
iii tree, toolshed **iv** car, letterbox

4 **a** first quadrant **b** third quadrant
c second quadrant **d** fourth quadrant

5 **a** first **b** fourth **c** third **d** second
e fourth **f** fourth **g** third **h** second

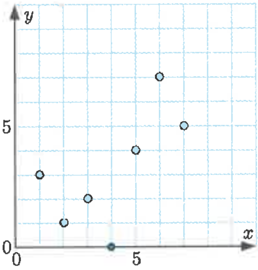
6



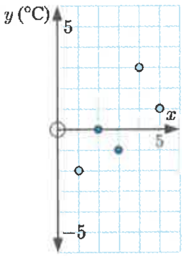
7 **a** climb 2 units down **b** climb 4 units to the left
c climb 3 units to the right, then 1 unit down
d climb 2 units to the left, then 4 units up
e climb 1 unit to the left, then 4 units down
f climb x units to the right, then y units up
Note: If x is negative, Spiros climbs *left*, and if y is negative, he climbs *down*.

EXERCISE 15C

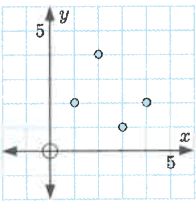
1



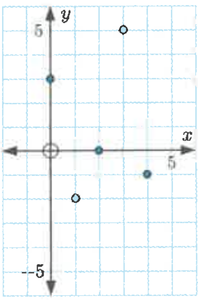
2



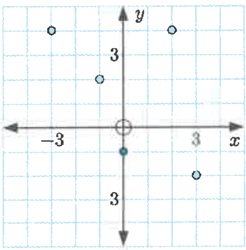
3 **a**



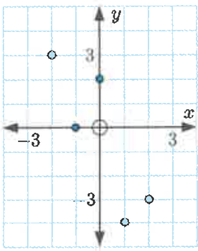
b



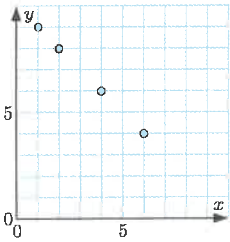
c



d

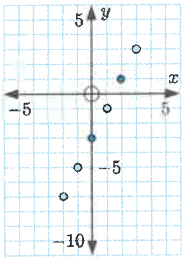


4 **a**



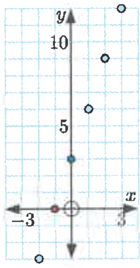
b Yes, the points lie in a straight line.
c 10 L

5 **a**



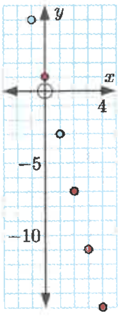
x	1
y	-1

b



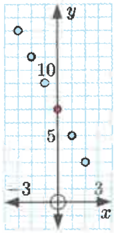
x	-1	2
y	0	9

c



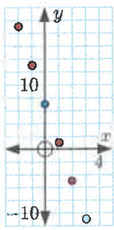
x	0	1	2	4
y	1	-3	-7	-15

d



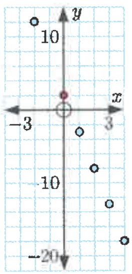
x	-2	0	1
y	11	7	5

e



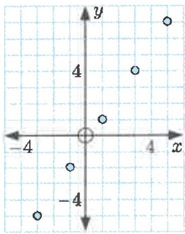
x	-2	-1	1	2
y	19	13	1	-5

f



x	-2	0	2	4
y	12	2	-8	-18

6 **a**



b **i** (7, 10)
ii $(\frac{1}{3}, 0)$
iii $(0, -\frac{1}{2})$
iv $(-2, -3\frac{1}{2})$
v $(1\frac{2}{3}, 2)$

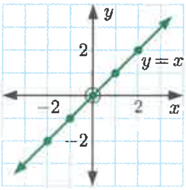
EXERCISE 15D

1 **a** $y = x + 3$ **b** $y = x + 5$ **c** $y = x - 4$
d $y = x - 7$ **e** $y = 3x$ **f** $y = \frac{1}{2}x$
g $y = 2x + 1$ **h** $y = 1 - 3x$

2 **a** The y-coordinate is the x-coordinate plus 2.
b The y-coordinate is the x-coordinate minus 5.
c The y-coordinate is twice the x-coordinate.
d The y-coordinate is a quarter of the x-coordinate.
e The y-coordinate is 4 minus the x-coordinate.
f The y-coordinate is 3 times the x-coordinate, minus 1.
g The y-coordinate is 1 plus half the x-coordinate.
h The y-coordinate is 2 minus 4 times the x-coordinate.

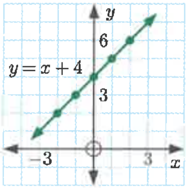
3 **a**

x	-2	-1	0	1	2
y	-2	-1	0	1	2



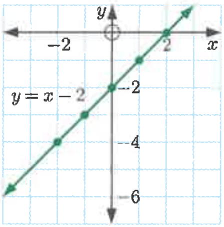
b

x	-2	-1	0	1	2
y	2	3	4	5	6



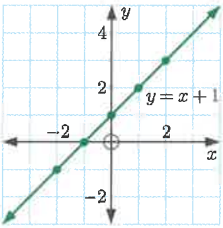
c

x	-2	-1	0	1	2
y	-4	-3	-2	-1	0



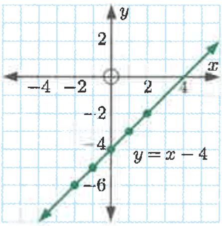
d

x	-2	-1	0	1	2
y	-1	0	1	2	3



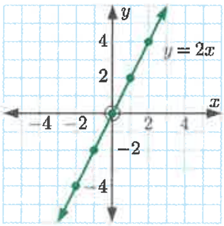
e

x	-2	-1	0	1	2
y	-6	-5	-4	-3	-2



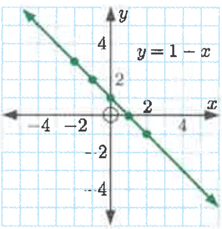
f

x	-2	-1	0	1	2
y	-4	-2	0	2	4



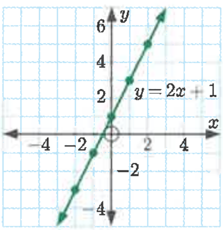
g

x	-2	-1	0	1	2
y	3	2	1	0	-1



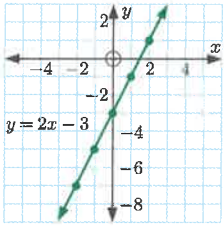
h

x	-2	-1	0	1	2
y	-3	-1	1	3	5



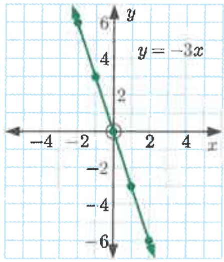
i

x	-2	-1	0	1	2
y	-7	-5	-3	-1	1



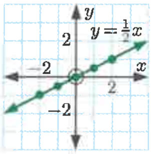
j

x	-2	-1	0	1	2
y	6	3	0	-3	-6



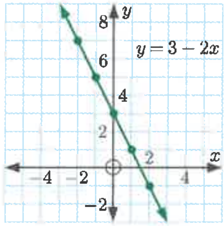
k

x	-2	-1	0	1	2
y	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1



l

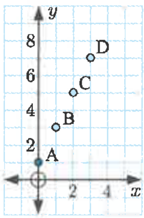
x	-2	-1	0	1	2
y	7	5	3	1	-1



REVIEW SET 15A

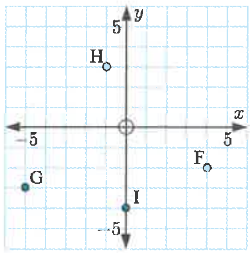
1 **a** **i** 3 **ii** -1 **b** **i** -2 **ii** 4
c **i** A(3, -2) **ii** B(0, -2) **iii** E(3, 0) **iv** F(-2, 1)

2 **a**



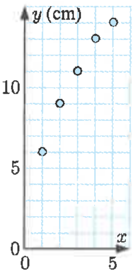
b E(4, 9)

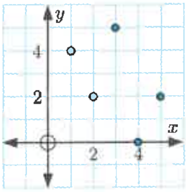
3

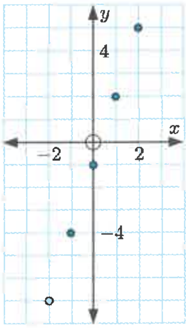


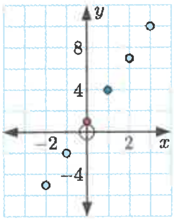
4 **a** second
b third
c fourth
d first

5

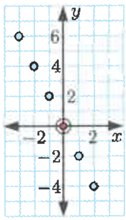


6 **a** 

b 

7 **a** 

x	0	2
y	1	7

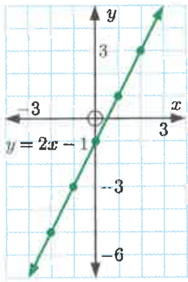
b 

x	-2	-1	0	2
y	4	2	0	-4

8 **a** $y = x - 3$ **b** $y = 2x$

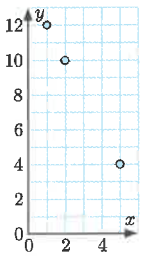
9

x	-2	-1	0	1	2
y	-5	-3	-1	1	3



10 **a**

Week number (x)	1	2	5
Sessions per day (y)	12	10	4

b 

c Yes, the points lie in a straight line.

d **i** 8 session times per day
ii 6 session times per day

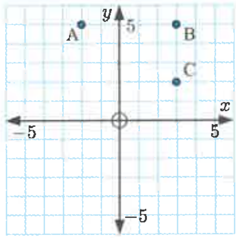
e It is unreasonable to continue to use the pattern at 8 weeks as it gives a negative number of session times (-2), which is impossible.

REVIEW SET 15B

1 **a** **i** A(0, 4) **ii** B(7, 1) **iii** C(3, 2) **iv** D(4, 0) **v** E(6, 6)
b E

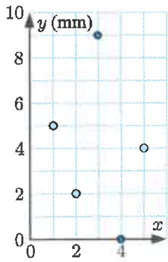
2 third quadrant

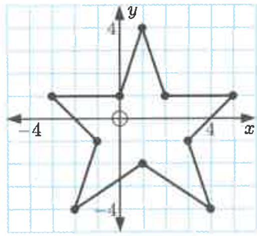
3 **a** E **b** F **c** A **d** B **e** C **f** D

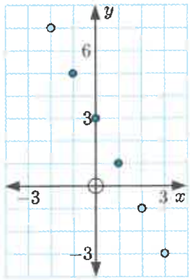
4 **a** 

b D(-2, 2)

5 No, they are different points because their x-coordinates and y-coordinates are different.

6 

7 

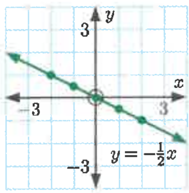
8 **a** 

b (4, -5)

9 $y = x - 2$

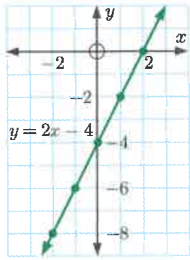
10 **a**

x	-2	-1	0	1	2
y	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1



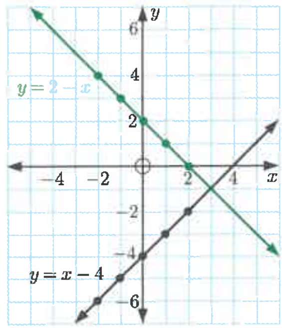
b

x	-2	-1	0	1	2
y	-8	-6	-4	-2	0



11 **a** $y = x - 4$ **b**

x	-2	-1	0	1	2
y	-6	-5	-4	-3	-2

c, d 

e (3, −1) **f** quadrants 1 and 4

EXERCISE 16A

1 **a** 4 : 5 **b** 15 : 8 **c** 1 : 4 **d** 8 : 7 **e** 9 : 5 **f** 2 : 11
2 **a** 3 : 4 **b** 1 : 5 **c** 2 : 3 **d** 3 : 5
3 **a** 17 : 100 **b** 50 : 60 **c** 1000 : 150 **d** 9 : 24 **e** 12 : 180 **f** 400 : 1000 **g** 14 : 9 **h** 780 : 3100
4 **a** 152 : 164 **b** 2 : 5 **c** 3 : 500 **d** 20 : 12 **e** 8 : 5 **f** 200 : 800 **g** 2000 : 700 **h** 850 : 1000

EXERCISE 16B

1 **a** $\frac{3}{8}$ **b** $\frac{5}{8}$ **2** **a** $\frac{2}{5}$ **b** $\frac{3}{5}$
3 **a** **i** 1 : 3 **ii** $\frac{1}{4}$ **iii** 25%
b **i** 2 : 3 **ii** $\frac{2}{5}$ **iii** 40%
c **i** 3 : 7 **ii** $\frac{3}{10}$ **iii** 30%
d **i** 2 : 1 **ii** $\frac{2}{3}$ **iii** $66\frac{2}{3}\% \approx 66.7\%$
e **i** 3 : 1 **ii** $\frac{3}{4}$ **iii** 75%
f **i** 6 : 2 **ii** $\frac{3}{4}$ (or $\frac{6}{8}$) **iii** 75%
4 **a** 3 : 1 **b** $\frac{1}{4}$ **5** **a** $\frac{7}{16}$ **b** 7 : 9
6 1 : 39 **7** **a** 8 : 4 **b** 8 : 3 **c** 4 : 3 **d** 8 : 4 : 3

EXERCISE 16C

1 **a** 12 : 30 **b** 2 : 5 **2** **a** 24 : 60 **b** 4 : 10
3 **a** 4 : 100 **b** 40 : 1000 **4** **B** and **E**
5 **a** equal **b** equal **c** not equal **d** equal **e** not equal **f** equal
6 No, we cannot multiply both sides of 5 : 3 by the same number to obtain 3 : 5.
7 **a** equal **b** not equal **c** equal

EXERCISE 16D

1 **a** 1 : 2 **b** 3 : 1 **c** 1 : 5 **d** 3 : 1 **e** 4 : 5 **f** 7 : 4 **g** 2 : 5 **h** 3 : 2 **i** 2 : 3 **j** 2 : 7 **k** 3 : 2 **l** 1 : 3
2 **a** no **b** yes **c** no **d** no **e** no **f** yes **g** yes **h** no
3 **a** 3 : 1 **b** 3 : 2 **4** **a** 1 : 10 **b** 1 : 2 **c** 3 : 2
5 **a** 1 : 2 **b** 5 : 4
6 **a** 1 : 3 **b** 1 : 2 **c** 1 : 2 **d** 1 : 1 **e** 3 : 5 **f** 2 : 1
7 **a** 3 : 10 **b** 1 : 5 **c** 1 : 4 **d** 1 : 10 **e** 1 : 20 **f** 4 : 1 **g** 5 : 7 **h** 5 : 1 **i** 3 : 5 **j** 2 : 5 **k** 1 : 15 **l** 7 : 8
8 **a** equal **b** not equal **c** not equal **d** not equal **e** equal **f** not equal **g** equal **h** not equal

EXERCISE 16E

1 **a** □ = 6 **b** □ = 3 **c** □ = 33 **d** □ = 15 **e** □ = 18 **f** □ = 8 **g** □ = 4 **h** □ = 2 **i** □ = 1 **j** □ = 10 **k** □ = 28 **l** □ = 20
2 **a** 36 doctors **b** 150 nurses **3** 20 teachers
4 21 necklaces **5** 56 station wagons
6 **a** 1 : 10 **b** 10 mL of chocolate topping
7 120 g (45 g raisins and 75 g nuts)

EXERCISE 16F

1 **a** **i** $\frac{2}{3}$ **ii** $\frac{1}{3}$ **b** **i** 12 chocolates **ii** 6 chocolates
2 **a** 125 g beetroot, 75 g yoghurt **b** 375 g beetroot, 225 g yoghurt
3 **a** 320 g butter, 480 g flour **b** 800 g butter, 1200 g flour
4 **a** £5, £25 **b** €20, €8 **c** \$480, \$120
5 ¥48 000 **6** 3 : 8

EXERCISE 16G

1 **a** 5 km in 1 hour **b** 15 dollars in 1 hour **c** 7 litres in 1 second **d** 99 cents for 1 litre **e** 30 kg in 1 hour **f** 14 grams in 1 minute **g** 96 dollars in 1 day **h** 66 metres in 1 second
2 **a** 16 km per litre **b** 52 km per hour **c** 3.5 L per s **d** £17 per hour **e** \$2.49 per L **f** 40 cents per banana
3 **a** Annie: ≈ 0.556 km per min, Victoria: 0.5 km per min **b** train
4 **a** Xinsong: \$21 per hour, Jay: \$22 per hour **b** Jay
5 the horse is faster (17.6 m per s compared with ≈ 17.3 m per s)

EXERCISE 16H

1 **a** \$1.20 per bar **b** RM3.80 per ball **c** \$1.69 per kg **d** 0.96 cents per g **e** 79.2 cents per L **f** €2 per m **g** £165.40 per night **h** 124 cents per L **i** \$106.52 per m^2
2 **a** 400 g at \$1.275 per 100 g **b** 200 mL at £0.895 per 100 mL **c** 4 boxes at \$0.5125 per box **d** 36 tablets at ≈ \$0.247 per tablet **e** 50 m at €0.73 per 10 m **f** 250 g at 8.6 cents per 10 g
3 **a** 110 g tube: \$0.29 per 10 g
160 g tube: ≈ \$0.249 per 10 g
b 160 g tube **c** Yes, rate is ≈ \$0.193 per 10 g.

REVIEW SET 16A

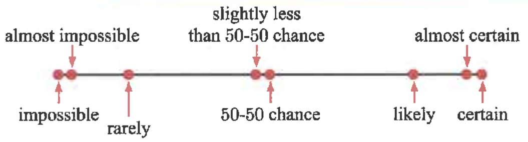
1 **a** 9 : 4 **b** 11 : 5 **c** 7 : 30 **2** **a** $\frac{13}{15}$ **b** $\frac{2}{15}$
3 9 : 2 **4** **a** equal **b** not equal **c** equal
5 **a** 1 : 4 **b** 8 : 5 **c** 4 : 11
6 **a** □ = 6 **b** □ = 9 **c** □ = 45 **7** 21 trucks
8 48 sit-ups **9** **a** $\frac{3}{10}$ **b** €90 **10** 14 litres per minute
11 **a** 1.3 kg packet at ≈ \$0.423 per 100 g **b** 1.3 kg packet at ≈ \$0.423 per 100 g
12 **a** LED: 25 kilojoules per hour
CFL: 30 kilojoules per hour
b LED
13 **a** **i** 6 lessons **ii** 10 lessons **b** **i** 40 minutes **ii** 18 minutes **iii** 45 minutes
14 **a** **i** 5 : 8 **ii** 5 : 4 **b** 48 triangular tiles **c** 65 tiles **d** 16 m^2

REVIEW SET 16B

1 7 : 2 **2** **a** 5 : 7 **b** $\frac{5}{12}$ **3** **a** 64 : 24 **b** 8 : 3
4 **a** 53 : 120 **b** 10 : 3 **c** 3 : 20 **5** 27 cm
6 16 matches **7** **a** 1 : 4 **b** 1 : 8
8 400 mL **9** 720 apple, 600 pear **10** \$27.50 per h
11 Steve at 28 km per h **12** 80 g bar at ≈ \$0.274 per 10 g
13 **a** **i** \$0.9375 per 100 g **ii** \$0.75 per 100 g **b** 1 kg bag **c** **i** ≈ \$0.703 per 100 g **ii** yes
14 **a** 12 L **b** 8 L cordial concentrate, 48 L water **c** **i** 4 L **ii** 52 L **iii** 1 : 13

EXERCISE 17A

1



2 **a** highly unlikely **b** almost impossible **c** certain **d** highly unlikely **e** 50-50 chance **f** likely **g** highly likely

3 **a** possible **b** possible **c** impossible **d** possible **e** possible **f** **i** possible **ii** impossible **iii** certain

4 **a** no **b** almost certain

5 **a** It is more likely that the marble is red, as there are more red marbles than blue marbles.
b **i** unlikely **ii** impossible **iii** likely

6 **a** **i** likely **ii** 50-50 chance
b No, we are not told what colour(s) the female kittens are.

EXERCISE 17B

1 **a** highly likely **b** unlikely **c** 50-50 chance **d** certain **e** slightly more than 50-50 chance

2 **a** 0.2 **b** unlikely

3 **a** **i** slightly less than 50-50 chance **ii** likely
b Sunday

4 **a** Terry will not go to school tomorrow.
b Jennifer has less than 3 pets.
c When selecting a ball from a bag, the result is neither red nor blue.

5 **a** E' = Evelyn will forget to get her diary signed tonight
b $\mathrm{P}(E') = 0.37$ **c** E

6 0.13

7 **a** $\frac{1}{10} = 0.1$, $\frac{1}{2} = 0.5$, $\frac{2}{5} = 0.4$
b The sum of the probabilities is 1. It is certain that one of these events will occur.
c the 8:10 am train **d** 0.6

8 **a** **i** 50-50 chance **ii** 0.5
b B' = a randomly selected disc is white, $\mathrm{P}(B') = 0.5$

9 **a** $\mathrm{P}(R) = 0$, there are no red tokens in the bag.
b $\mathrm{P}(R') = 1$, all the tokens in the bag are *not* red.

10 **a** likely **b** Annabeth
c **i** A' = Annabeth fails the test **ii** $\mathrm{P}(A') = 0.08$
d **i** $\mathrm{P}(B) + \mathrm{P}(C) = 1$
ii No, both B and C could occur at the same time.

11 **a** **i** 0.1 **ii** 0.2
b **i** Lily (0.45) **ii** Ralph (0.5)
c $\mathrm{P}(L) = 0.35$, $\mathrm{P}(L') = 0.65$
d Ralph has greater probability of knocking over 2 or 3 pins, while Lily is more likely to knock over 0 or 1 pins. So, Ralph appears to be better at the game.

EXERCISE 17C

1 **a** {smile, frown}, 2 outcomes
b {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}, 7 outcomes
c {10, 11, 12, 13, 14, 15, 16, 17, 18, 19}, 10 outcomes
d {January, February, March, April, May, June, July, August, September, October, November, December}, 12 outcomes
e **i** {1, 2, 3, 4}, 4 outcomes
ii {A, B, C, D, E}, 5 outcomes
iii {blue, red, yellow}, 3 outcomes

2 **a** {11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97}
b 21 outcomes **c** 10 outcomes

3 **a** {HH, HT, TH, TT}, 4 outcomes
b {ABC, ACB, BAC, BCA, CAB, CBA}, 6 outcomes
c {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}, 8 outcomes
d {FFF, FFM, FMF, MFF, FMM, MFM, MMF, MMM}, 8 outcomes
e {PP, PQ, PR, QP, QQ, QR, RP, RQ, RR}, 9 outcomes
f {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}, 36 outcomes
g {WXYZ, WXZY, WYXZ, WYZX, WZXY, WZYX, XWYZ, XWZY, XYWZ, XYZW, XZWY, XZYW, YWXZ, YWZX, YXWZ, YXZW, YZWX, YZXW, ZWXY, ZWYX, ZXWY, ZXYW, ZYWX, ZYXW}, 24 outcomes

4 **a** {(1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 3), (2, 4), (2, 5), (3, 1), (3, 2), (3, 4), (3, 5), (4, 1), (4, 2), (4, 3), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4)}
b 20 outcomes **c** 6 outcomes

EXERCISE 17D

1 **a** {1, 2, 3, 4, 5, 6} **b** 6 outcomes
c **i** $\frac{1}{6}$ **ii** $\frac{3}{6} = \frac{1}{2}$ **iii** $\frac{5}{6}$

2 **a** 26 outcomes **b** **i** $\frac{1}{26}$ **ii** $\frac{4}{26} = \frac{2}{13}$ **iii** $\frac{7}{26}$

3 **a** $\frac{1}{2}$ **b** 1 **c** $\frac{1}{4}$ **d** $\frac{2}{3}$

4 **a** $\frac{1}{12}$ **b** $\frac{1}{6}$ **c** $\frac{2}{3}$ **d** $\frac{1}{3}$ **e** $\frac{7}{12}$

5 $\frac{24}{29}$

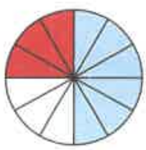
6 **a** $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$ **d** 0 **e** $\frac{5}{8}$

7 **a** $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** $\frac{5}{8}$ **d** 0 **e** $\frac{7}{8}$ **f** $\frac{3}{4}$ **g** $\frac{5}{8}$ **h** 1 **i** 0

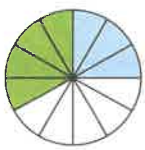
8 **B**

9 **a** **i** $\frac{2}{15}$ **ii** $\frac{1}{5}$ **iii** $\frac{2}{3}$
b **i** $\mathrm{P}(E) = \frac{8}{15}$
ii E' = the selected card is not horizontally or vertically adjacent to exactly three other cards.
iii $\mathrm{P}(E') = \frac{7}{15}$

10 **a**



b



c

11 **a** $P(A) = \frac{4}{10} = \frac{2}{5}$ **b** $P(B) = \frac{6}{10} = \frac{3}{5}$
c $P(A) + P(B) = \frac{2}{5} + \frac{3}{5} = \frac{5}{5} = 1$
d No, as A and B can both occur at the same time.

12 **a** $\frac{1}{52}$ **b** $\frac{1}{4}$ **c** $\frac{1}{13}$ **d** $\frac{12}{13}$ **e** $\frac{3}{13}$
f $\frac{1}{2}$ **g** $\frac{1}{26}$ **h** $\frac{3}{26}$ **i** $\frac{2}{13}$

13 **a** {HH, HT, TH, TT} **b** 4 outcomes
c **i** $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{3}{4}$
d **c ii** and **c iv**

14 **a** $\frac{1}{3}$ **b** $\frac{1}{6}$ **c** $\frac{1}{3}$ **d** $\frac{2}{3}$

15 **a** $\frac{1}{9}$ **b** $\frac{4}{9}$ **c** $\frac{2}{9}$ **d** $\frac{5}{9}$

16 **a** $\frac{1}{15}$ **b** $\frac{1}{10}$ **c** $\frac{2}{5}$ **d** $\frac{4}{15}$ **e** $\frac{11}{15}$

EXERCISE 17E

1 ≈ 0.731 **2** ≈ 0.36 **3** **a** ≈ 0.712 **b** ≈ 0.288

4 **a** ≈ 0.667 **b** ≈ 0.333

5 **a** ≈ 0.125 **b** ≈ 0.8

EXERCISE 17F

1 **a** **i** ≈ 0.761 **ii** ≈ 0.613
b The probability in **a i** is likely to be more accurate as the first experiment involves a higher number of throws.

2 **a** ≈ 0.963 **b** ≈ 0.925
c The estimate in **b** as it involves a higher number of shots.

3 Kelly's results are likely to be **B**, Lucy's results are likely to be **A**. The experimental probabilities in **B** are closer to the theoretical probabilities, suggesting they probably came from Kelly's larger sample.

4 **a** 0 heads: ≈ 0.267, 1 head: ≈ 0.475, 2 heads: ≈ 0.258
b 0 heads: 0.25, 1 head: 0.5, 2 heads: 0.25

5 **a** **i** ≈ 0.24 **ii** ≈ 0.3
b Keith's estimate, since he surveyed more people.
c $\frac{54}{210} \approx 0.257$

REVIEW SET 17A

1 **a** unlikely **b** almost certain

2 **a** unlikely **b** extremely likely **c** impossible
d "50-50" chance **e** certain **f** probable

3 **a** $P(A') = 0.06$ **b** **i** highly likely **ii** highly unlikely

4 **a** {23, 29, 31, 37}
b {(1, 0), (1, 2), (1, 4), (1, 6), (1, 8), (3, 0), (3, 2), (3, 4), (3, 6), (3, 8), (5, 0), (5, 2), (5, 4), (5, 6), (5, 8), (7, 0), (7, 2), (7, 4), (7, 6), (7, 8), (9, 0), (9, 2), (9, 4), (9, 6), (9, 8)}

5 **a** $\frac{4}{9}$ **b** $\frac{1}{3}$ **c** $\frac{7}{9}$ **d** $\frac{2}{3}$ **e** $\frac{1}{3}$

6 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{5}{8}$ **d** $\frac{1}{3}$

7 **a** $\frac{1}{100}$ **b** $\frac{1}{2}$ **c** $\frac{9}{20}$

8 **a** $\frac{1}{24}$ **b** $\frac{1}{6}$ **c** $\frac{1}{2}$ **d** $\frac{1}{3}$

9 **a** 240 seats **b** **i** $\frac{1}{6}$ **ii** $\frac{1}{240}$ **iii** $\frac{1}{20}$ **iv** $\frac{1}{2}$

10 ≈ 0.422

11 **a** **i** ≈ 0.532 **ii** ≈ 0.576
b Emily's, the sample size is larger.

12 **a** **i** ≈ 0.12 **ii** ≈ 0.08 **b** Nyree's **c** ≈ 0.0857

REVIEW SET 17B

1 **a** possible **b** certain **c** impossible

2 **a** highly likely **b** almost impossible
c slightly more than 50-50 chance

3 **a** E' = Jacqueline will not win **b** $P(E') = 0.4$

4 **a** {Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune}, 8 outcomes
b {L2, L7, L4, C2, C7, C4, N2, N7, N4}, 9 outcomes

5 $\frac{1}{8}$ **6** $\frac{7}{10}$ **7** No, as the probabilities add to $1\frac{1}{8} > 1$.

8 **a** $\frac{1}{16}$ **b** $\frac{1}{4}$ **c** $\frac{7}{16}$

9 The theoretical probabilities for obtaining each colour are:

Red	Yellow	Green
0.25	0.5	0.25

Since Michelle used a larger sample size, her results are set **B**, as they are closer to the theoretical values.
Jiao's results are therefore set **A**.

10 **a** {Teegan, Daniel, Casey, Ben, Trish, Deb, Skye, Tim, Wendy, Donna}
b **i** $\frac{3}{10}$ **ii** $\frac{2}{5}$ **c** $\frac{1}{3}$

11 **a** ≈ 0.414 **b** ≈ 0.207

12 **a** 36 tiles **b** **i** A **ii** E, unlikely **iii** $\frac{7}{12}$ **c** 0.15

EXERCISE 18A

1 **a** census **b** census **c** sample **d** sample
e sample **f** census **g** sample

2 census

EXERCISE 18B

1 **a**

<table>
<tr><th>Grade</th><th>Tally</th><th>Frequency</th></tr>
<tr><td>A</td><td>|||</td><td>3</td></tr>
<tr><td>B</td><td>$\cancel{||||}$</td><td>5</td></tr>
<tr><td>C</td><td>$\cancel{||||}$ $\cancel{||||}$ $\cancel{||||}$ |</td><td>16</td></tr>
<tr><td>D</td><td>|||</td><td>3</td></tr>
<tr><td>E</td><td>|</td><td>1</td></tr>
<tr><td></td><td>Total</td><td>28</td></tr>
</table>

b 16 students **c** $\frac{5}{28}$
d C, it was the most common grade obtained.

2 **a**

<table>
<tr><th>Sport</th><th>Tally</th><th>Frequency</th></tr>
<tr><td>Tennis</td><td>$\cancel{||||}$ |</td><td>6</td></tr>
<tr><td>Swimming</td><td>||||</td><td>4</td></tr>
<tr><td>Cricket</td><td>$\cancel{||||}$</td><td>5</td></tr>
<tr><td>Basketball</td><td>||</td><td>2</td></tr>
<tr><td>Athletics</td><td>$\cancel{||||}$ |||</td><td>8</td></tr>
<tr><td></td><td>Total</td><td>25</td></tr>
</table>

b athletics
c $\frac{1}{5}$

3 **a**

<table>
<tr><th>Attraction</th><th>Tally</th><th>Frequency</th></tr>
<tr><td>Side shows</td><td>$\cancel{||||}$ $\cancel{||||}$</td><td>10</td></tr>
<tr><td>Animals</td><td>|||</td><td>3</td></tr>
<tr><td>Main ring events</td><td>$\cancel{||||}$ |</td><td>6</td></tr>
<tr><td>Rides</td><td>||</td><td>2</td></tr>
<tr><td>Food</td><td>||||</td><td>4</td></tr>
<tr><td></td><td>Total</td><td>25</td></tr>
</table>

b side shows **c** 24%

4 **a**

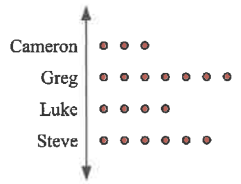
Exhibit	*Tally*	*Frequency*
Aerodynamics	卌 卌 |	11
Nanocircuits	卌 ||	7
Robotics	卌 ||	7
Space travel	卌 ||||	9
Telecommunications	||	2
	Total	36

b $\frac{1}{4}$ **c** $\approx 30.6\%$

EXERCISE 18C

1 **a** 5 cats **b** 25 animals **c** dog **d** 16%

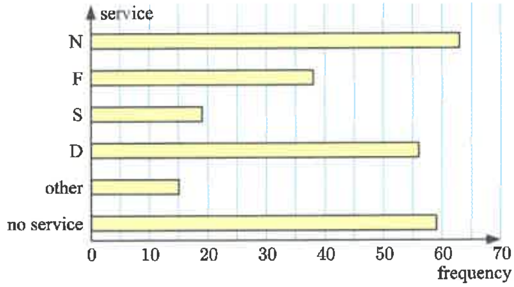
2 **a** **Votes for team captain**



b **i** Greg **ii** Cameron
c **i** 20% **ii** 65%

3 **a** 250 people **b** N **c** **i** 25.2% **ii** 23.6%

d **Streamed television services**

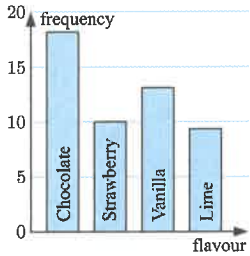


4 **a**

Ice cream flavour	*Tally*	*Frequency*
Chocolate	卌 卌 卌 |||	18
Strawberry	卌 卌	10
Vanilla	卌 卌 |||	13
Lime	卌 ||||	9
	Total	50

b 13 students
c 18%
d chocolate

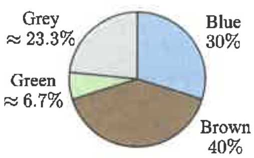
e **Favourite ice cream flavours**



5 **a** false **b** true **c** true **d** false

6 **a** Action/Adventure
b **i** 84 teenagers **ii** 156 teenagers **c** 82.8°

7 **a** **Eye colour**



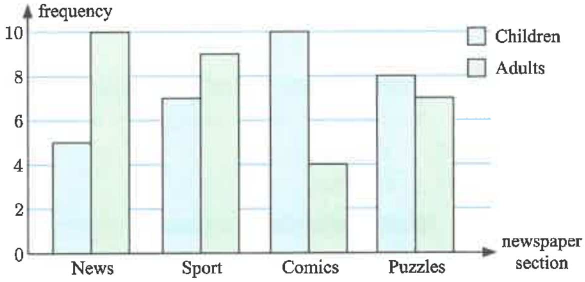
b **i** $\approx 6.67\%$ **ii** $\approx 53.3\%$
c $\frac{7}{10}$

EXERCISE 18D

1 **a** 10 students **b** Redstone School
c **i** Redstone School **ii** Hillsvale School
d The mode of Hillsvale's data is Malaysia, whereas the mode of Redstone's data is Japan.

2 **a** 250 g bag: orange, 500 g bag: yellow
b 250 g bag **c** 500 g bag
d We would expect the 500 g bag to have twice as many lollies as the 250 g bag. So, comparing the frequency of each colour would not be very meaningful as the 500 g bag would have more of each colour.

3 **a** **Most enjoyed newspaper section**



b children: comics; adults: news
c News and comics, as adults are more interested in the news whilst children are more interested in comics.

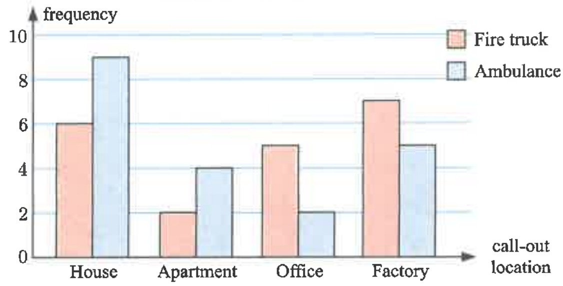
4 **a** **Call-out location for fire truck**

Location	*Tally*	*Frequency*
House (H)	卌 |	6
Apartment (A)	||	2
Office (O)	卌	5
Factory (F)	卌 ||	7
	Total	20

Call-out location for ambulance

Location	*Tally*	*Frequency*
House (H)	卌 ||||	9
Apartment (A)	||||	4
Office (O)	||	2
Factory (F)	卌	5
	Total	20

b **Location of call-outs**

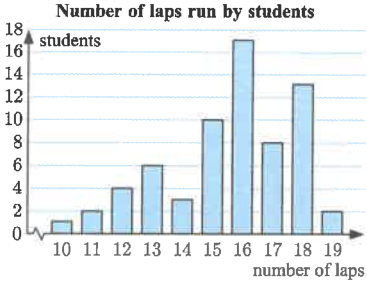


c fire truck: factory; ambulance: house **d** fire truck

EXERCISE 18E

1 **a** 5 workers **b** 30 workers **c** 30%
d 8 times is the outlier

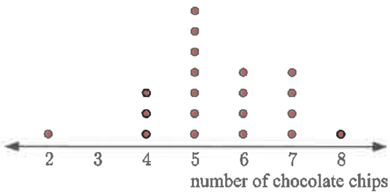
2 **a** Number of laps run by students



b 16 laps **c** 7 students **d** $\frac{53}{66}$ **e** no

3 **a** 16 people **b** 2 slices **c** 4 people **d** yes, 10 slices

4 **a** Number of chocolate chips in biscuits



b 5 chocolate chips **c** 8 chocolate chips
d 7 biscuits **e** 20% **f** yes, 2 chocolate chips

EXERCISE 18F

1 **a** 20 schools **b** 2 schools **c** 111 students **d** 8 schools

2 **a** Unordered

0	7 2 5
1	2 1 9 4 2 3 6
2	7 9 2 2 8 1
3	0 5 7 2
4	2 9 3 0
5	1 9
6	2 2 9
7	4

b Ordered

0	2 5 7
1	1 2 2 3 4 6 9
2	1 2 2 7 8 9
3	0 2 5 7
4	0 2 3 9
5	1 9
6	2 2 9
7	4

Scale: 1 | 2 means 12 runs

c 17 times **d** **i** 2 runs **ii** 74 runs

3 **a** Unordered

7	8 8
8	6 8 0 8 6 2
9	4 6 2 6 2 2 4
10	8 0 0 4 6
11	2 2 6 4 6
12	2 4
13	
14	
15	
16	0

Ordered

7	8 8
8	0 2 6 6 8 8
9	2 2 2 4 4 6 6
10	0 0 4 6 8
11	2 2 4 6 6
12	2 4
13	
14	
15	
16	0

Scale: 12 | 2 means 122 pages

b 13 newspapers **c** $\approx 46.4\%$ **d** yes, 160 pages

EXERCISE 18G

1 **a** 8 **b** 11 **c** 4 **d** 3.5 **e** 5.64 **f** 5

2 **a** 8 **b** 12 **c** 3 **d** 6 **e** 2.2 **f** 4.1

3 **a** **i** 4 **ii** 3 **iii** 0

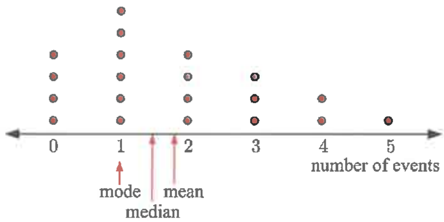
b No, as the mode is 0 which is also the minimum. It does not give an accurate measure of the "centre".

4 **a** 204.8 **b** 200.5

5 **a** 6 texts **b** 4 texts **c** 3 texts

6 **a** **i** 5 **ii** 4.7 **iii** 5
b **i** 3.6 **ii** ≈ 2.94 **iii** 3.05
c **i** 4 **ii** ≈ 2.54 **iii** 3
d **i** 4 **ii** 4.6 **iii** 4

7 **a** 4 students
b **i** 1 event **ii** 1.5 events **iii** 1.8 events
c



8 **a** *Zach*: mean = 7.2 fish, median = 7.5 fish
Ed: mean = 6.5 fish, median = 6.5 fish
b Zach, as he has a higher mean and median.

9 **a** *Group X*: 6.5 marks, *Group Y*: ≈ 7.64 marks
b No. It is fair as the mean calculates the average score per student.
c Group Y

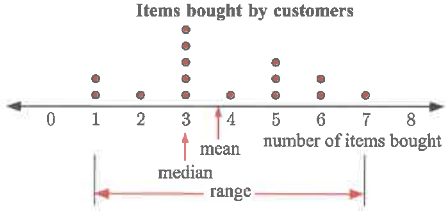
10 **a** 47 hot dogs
b *Josh*: mean ≈ 34.4 hot dogs, median = 34 hot dogs
Eugene: mean ≈ 49.4 hot dogs, median = 50 hot dogs
c Eugene, as he has a higher mean and median.

EXERCISE 18H

1 **a** 11 **b** 2 **c** 8 **d** 14 **e** 29 **f** 5.1

2 **a** 5 **b** 8 **c** 33 **d** 2.7

3 **a** Items bought by customers



b mean = 3.8 items, median = 3 items, range = 6 items

4 **a**

City	Range
Adelaide	8°C
Brisbane	2°C
Canberra	6°C
Darwin	3°C
Hobart	5°C
Melbourne	10°C
Perth	9°C
Sydney	5°C

b **i** Melbourne **ii** Brisbane

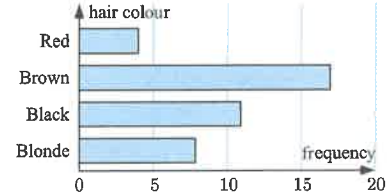
REVIEW SET 18A

1 **a** sample **b** census

2 **a** 8 times **b** 26% **c** **i** 3 **ii** 3

3 **a** True, as type O blood has the largest sector.
b False, as the angle for the type B blood sector is less than $\frac{1}{4}$ of $360° = 90°$.
c True, as more than half of the pie chart represents blood types other than type O.

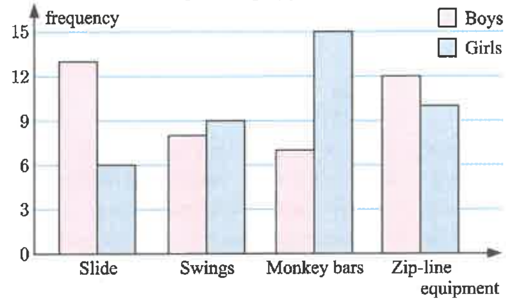
4 **a** Hair colour of students



b red **c** 20%

5 **a** 12 boys **b** 6 girls

c Favourite piece of playground equipment



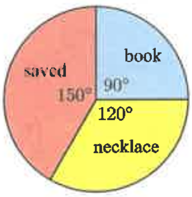
d **i** slide **ii** monkey bars **e** boys

6 **a** 10 **b** 9 **c** 6 **7** Alyssa

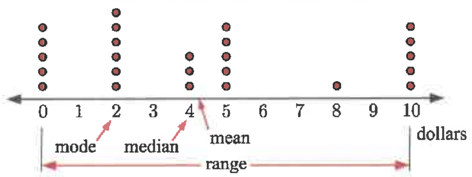
8 **a** *Group 1*: mean = 7.6, median = 7.5
Group 2: mean ≈ 8.27, median = 9

b Group 2 performed better because it had the higher mean and median.

9 Birthday money



10 **a, c** Pocket money received each week



b **i** \$2 **ii** \$4.28 **iii** \$4 **iv** \$10

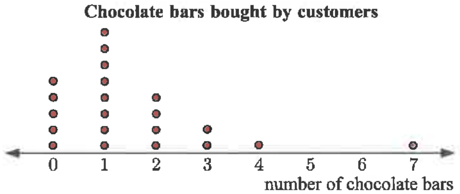
REVIEW SET 18B

1 **a** sample **b** sample **c** census **d** census **e** sample

2 categorical **3** **a** 1 drink **b** 30%

4 **a** **i** 5 knee injuries **ii** 7 knee injuries **b** Panthers

5 **a** Chocolate bars bought by customers

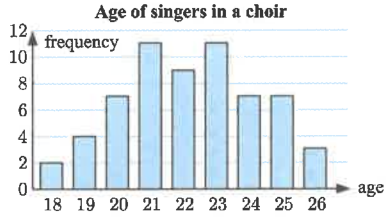


b yes, 7 chocolate bars

6 **a**

Age	*Tally*	*Frequency*
18	$\vert\vert$	2
19	$\vert\vert\vert\vert$	4
20	$\cancel{\vert\vert\vert\vert}\ \vert\vert$	7
21	$\cancel{\vert\vert\vert\vert}\ \cancel{\vert\vert\vert\vert}\ \vert$	11
22	$\cancel{\vert\vert\vert\vert}\ \vert\vert\vert\vert$	9
23	$\cancel{\vert\vert\vert\vert}\ \cancel{\vert\vert\vert\vert}\ \vert$	11
24	$\cancel{\vert\vert\vert\vert}\ \vert\vert$	7
25	$\cancel{\vert\vert\vert\vert}\ \vert\vert$	7
26	$\vert\vert\vert$	3
	Total	61

b Age of singers in a choir



c 21 and 23 years old **d** ≈ 22.2 years old **e** ≈ 16.4%

7 **a** mean = 11.75, median = 11.5, mode = 11, range = 4
b mean = 3.4, median = 4, mode = 4, range = 4

8 **a** ≈ 26.3% **b** 51 pages

9 **a**

Unordered		Ordered	
2	6 8 3	2	3 6 8
3	2 5 9 9 0 2	3	0 2 2 5 9 9
4	3 1 6 5	4	1 3 5 6
5	9 3 4	5	3 4 9
6	0 7 5 5	6	0 5 5 7

Scale: 2 | 3 means 23

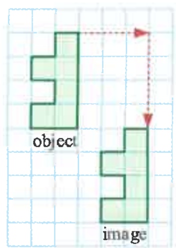
b 9 children **c** 25% **d** **i** 44.1 **ii** 42 **iii** 44

EXERCISE 19A

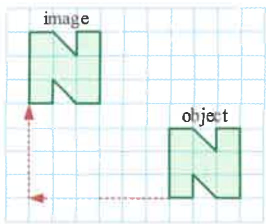
1 **a** 3 units right, 2 units down **b** 4 units right, 3 units up
c 4 units down **d** 4 units left
e 3 units left, 2 units down **f** 3 units right, 4 units down

2 **a** 3 units right, 1 unit up **b** 3 units left, 1 unit down
c 3 units right, 3 units down **d** 3 units left, 3 units up
e 5 units right, 1 unit up **f** 5 units left, 1 unit down
g 2 units left, 4 units down **h** 2 units right, 4 units up

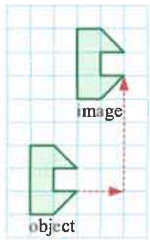
3 **a**



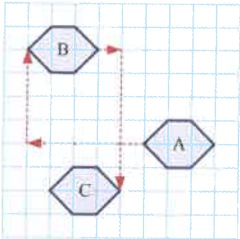
b



c



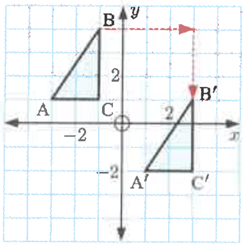
4 **a, b**



c **i** 4 units left, 2 units down **ii** 4 units right, 2 units up

5 **a** **i** A$(-3, 1)$, B$(-1, 4)$, C$(-1, 1)$

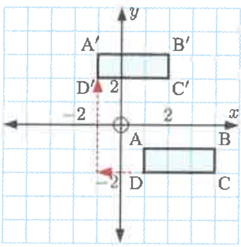
ii



iii A′$(1, -2)$
B′$(3, 1)$
C′$(3, -2)$

b **i** A$(1, -1)$, B$(4, -1)$, C$(4, -2)$, D$(1, -2)$

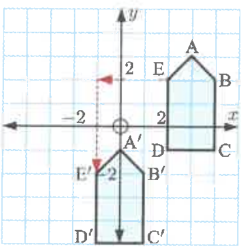
ii



iii A′$(-1, 3)$
B′$(2, 3)$
C′$(2, 2)$
D′$(-1, 2)$

c **i** A$(3, 3)$, B$(4, 2)$, C$(4, -1)$, D$(2, -1)$, E$(2, 2)$

ii



iii A′$(0, -1)$
B′$(1, -2)$
C′$(1, -5)$
D′$(-1, -5)$
E′$(-1, -2)$

6 **a** **i** $(2, 5)$ **ii** $(3, 8)$ **iii** $(-1, 0)$ **iv** $(4, 4)$

b When P(a, b) is translated 2 units right and 5 units up, the image is P′$(a+2,\ b+5)$.

7 **a** **i** $(-3, -1)$ **ii** $(0, 1)$ **iii** $(-4, 3)$ **iv** $(-2, -3)$

b When P(a, b) is translated 3 units left and 1 unit down, the image is P′$(a-3,\ b-1)$.

8 **a** **i** P′$(4, 2)$ **ii** P′$(-3, -5)$

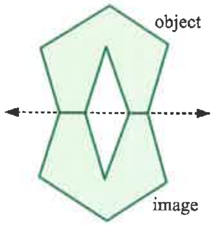
b **i** P′$(4, 4)$ **ii** P′$(1, 0)$ **iii** P′$(0, 6)$

c **i** When P(a, b) is translated m units right and n units up, the image is P′$(a+m,\ b+n)$.

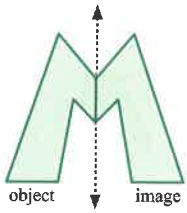
ii When P(a, b) is translated m units left and n units down, the image is P′$(a-m,\ b-n)$.

EXERCISE 19B

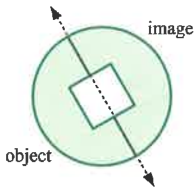
1 **a**



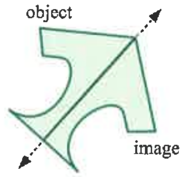
b



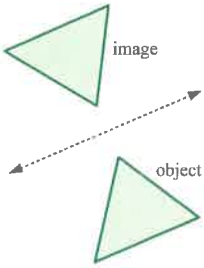
c



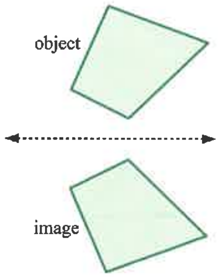
d



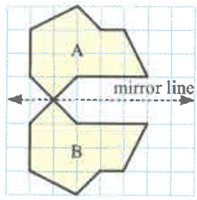
e



f

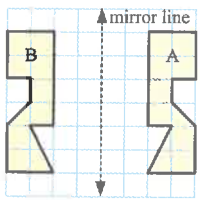


2 **a** no

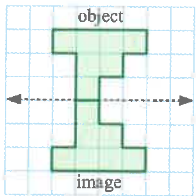


b yes

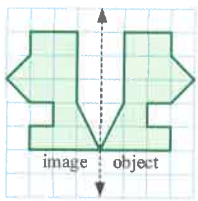
c no



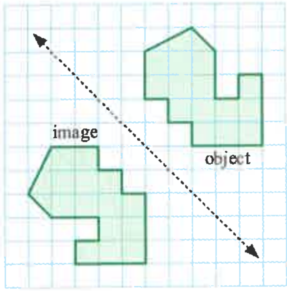
3 **a**



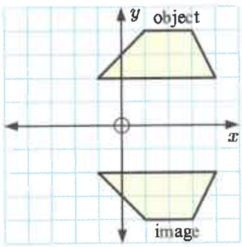
b



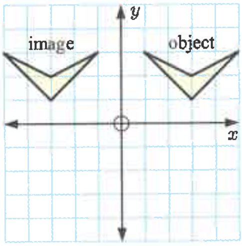
c



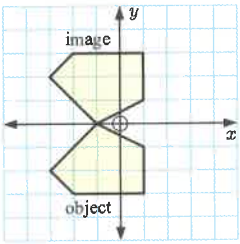
4 **a**



b

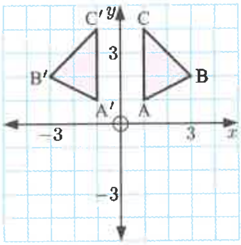


c



5 **a** **i** A(1, 1)
B(3, 2)
C(1, 4)

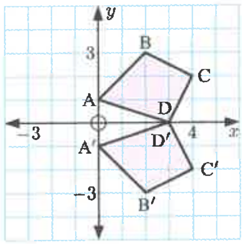
ii



iii A′(−1, 1)
B′(−3, 2)
C′(−1, 4)

b **i** A(0, 1)
B(2, 3)
C(4, 2)
D(3, 0)

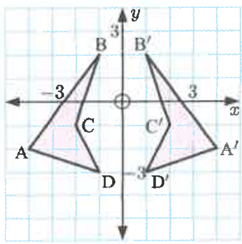
ii



iii A′(0, −1)
B′(2, −3)
C′(4, −2)
D′(3, 0)

c **i** A(−4, −2)
B(−1, 2)
C(−2, −1)
D(−1, −3)

ii



iii A′(4, −2)
B′(1, 2)
C′(2, −1)
D′(1, −3)

6 **a** 4th quadrant **b** 1st quadrant **c** 2nd quadrant

7 **a** **i** (3, −2) **ii** (1, −3) **iii** (−4, −5)
iv (2, 4) **v** (0, 3)

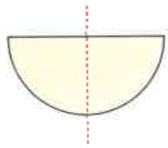
b **i** (−2, 1) **ii** (−4, 3) **iii** (−5, −1)
iv (3, 4) **v** (5, 0)

c **i** When P(a, b) is reflected in the x-axis, the image is P′(a, $-b$).

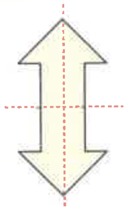
ii When P(a, b) is reflected in the y-axis, the image is P′($-a$, b).

EXERCISE 19C

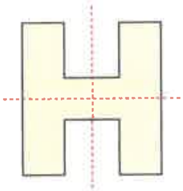
1 **a**



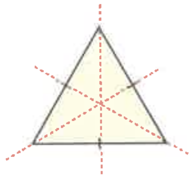
b



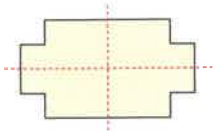
c



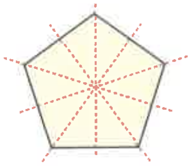
d



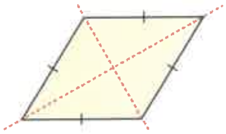
e



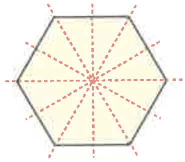
f



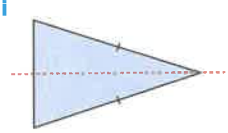
g



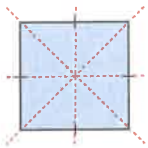
h



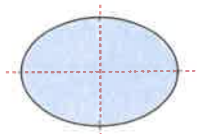
2 **a** **i**



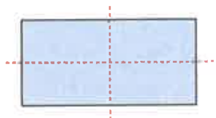
ii



iii



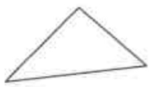
iv



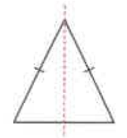
b the square in **ii**

3 **a** one line of symmetry **b** one line of symmetry

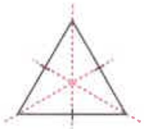
4 **a**



A scalene triangle has no lines of symmetry.

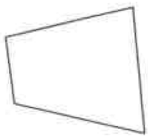


An isosceles triangle has one line of symmetry.

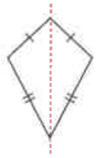


An equilateral triangle has three lines of symmetry.

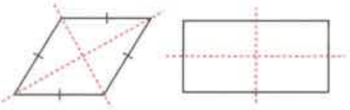
b



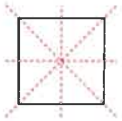
An irregular quadrilateral has no lines of symmetry.



A kite has one line of symmetry.



A rhombus and a rectangle have two lines of symmetry.



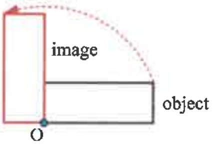
A square has four lines of symmetry.

c A circle has infinitely many lines of symmetry.

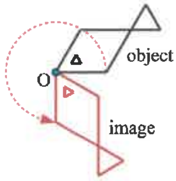
EXERCISE 19D

1 **a** **B** **b** **D** **c** **A** **d** **C**

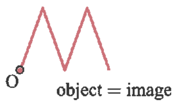
2 **a**



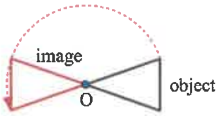
b



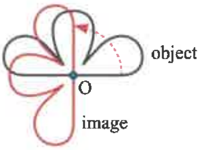
c



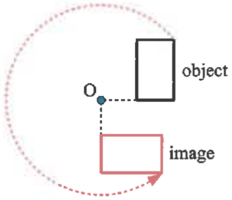
d



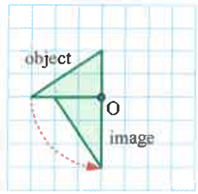
e



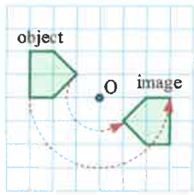
f



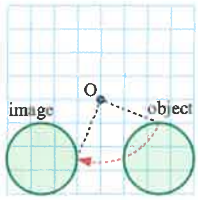
3 **a**



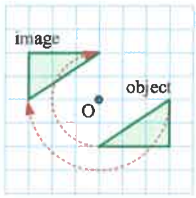
b



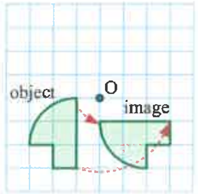
c



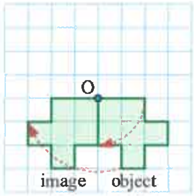
d



e



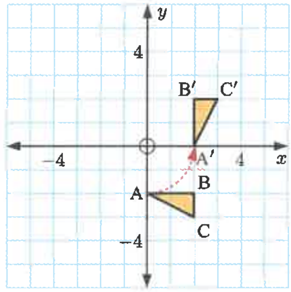
f



4 **a** **i** A(0, −2)
B(2, −2)
C(2, −3)

iii A′(2, 0)
B′(2, 2)
C′(3, 2)

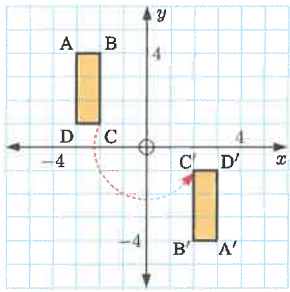
ii



b **i** A(−3, 4)
B(−2, 4)
C(−2, 1)
D(−3, 1)

iii A′(3, −4)
B′(2, −4)
C′(2, −1)
D′(3, −1)

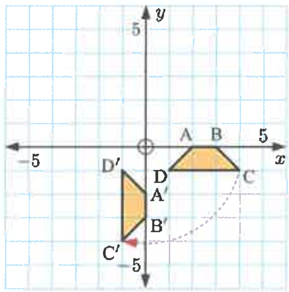
ii



c **i** A(2, 0)
B(3, 0)
C(4, −1)
D(1, −1)

iii A′(0, −2)
B′(0, −3)
C′(−1, −4)
D′(−1, −1)

ii



5 **a** 2nd quadrant **b** 3rd quadrant **c** 4th quadrant

6 **a** (−5, 2) **b** (−2, −5) **c** (5, −2)

7 **a** When $P(a, b)$ is rotated 90° about the origin, the image is $P'(-b, a)$.

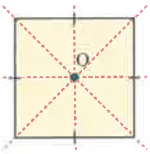
b When $P(a, b)$ is rotated 180° about the origin, the image is $P'(-a, -b)$.

c When $P(a, b)$ is rotated 270° about the origin, the image is $P'(b, -a)$.

EXERCISE 19E

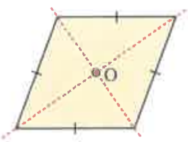
1 **a** no **b** yes **c** no **d** yes **e** no **f** yes

2 **a** **i**



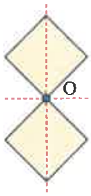
ii 4

b **i**



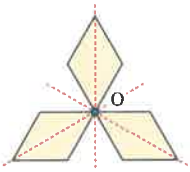
ii 2

c **i**



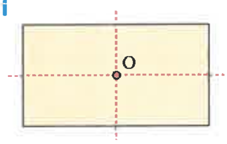
ii 2

d **i**



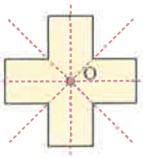
ii 3

e **i**



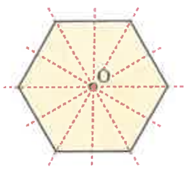
ii 2

f **i**



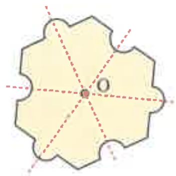
ii 4

g **i**



ii 6

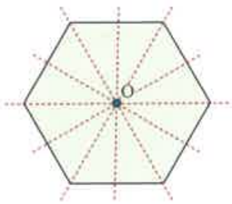
h **i**



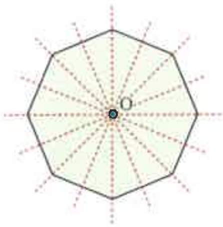
ii 3

3 **Note:** Other answers are possible.

a

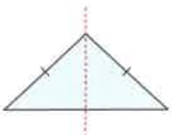


b



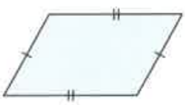
4 **Note:** Other answers are possible.

a



an isosceles triangle

b



a parallelogram

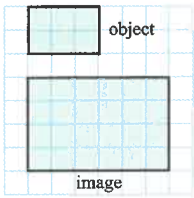
EXERCISE 19F

1 **a** 4 **b** 2 **c** 3 **2** **a** $\frac{1}{3}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$

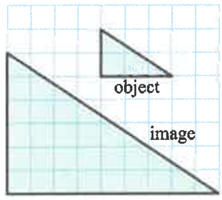
3 **a** The lengths on the object are multiplied by 5.

b The lengths on the object are divided by 3.

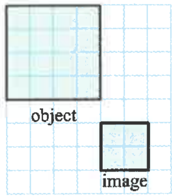
4 **a**



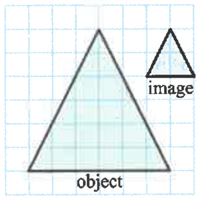
b



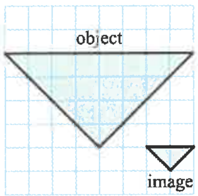
c



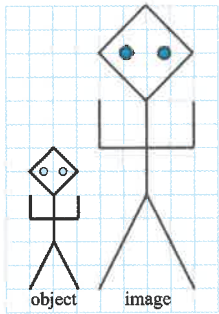
d



e



f



5 **a** **i** 3 **ii** $\frac{1}{3}$ **b** **i** 2 **ii** $\frac{1}{2}$

c **i** $\frac{1}{2}$ **ii** 2

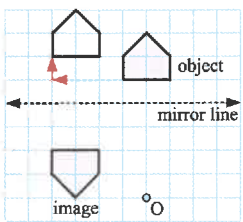
6 Reduce the object with scale factor $\dfrac{1}{k}$.

7 **a** Figure I is obtained by enlarging figure A with scale factor $\frac{3}{2}$.

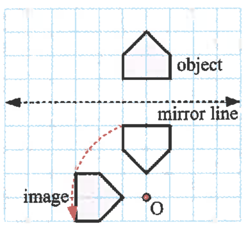
b Figure H is obtained by reducing figure D with scale factor $\frac{3}{4}$.

EXERCISE 19G

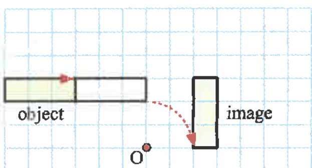
1 **a**



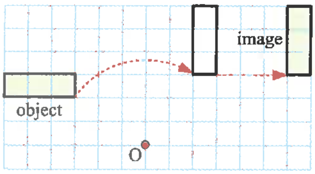
b



2 **a**



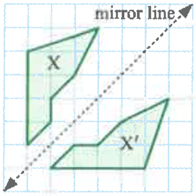
b



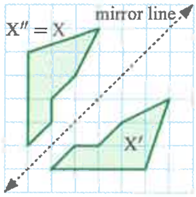
c yes

3 The original figure X is obtained.

Reflecting X to get X′:



Reflecting X′ in the same mirror line:

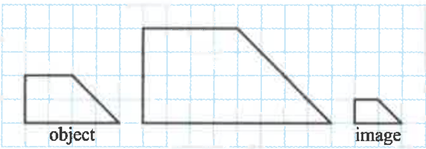


Note: Other illustrations are possible.

4 **a** Translate A 1 unit left and 4 units up, then rotate 90° anticlockwise about O.

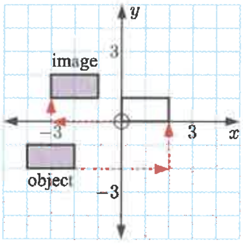
b Rotate B 90° clockwise about O, then translate 1 unit right and 4 units down.

5 **a**



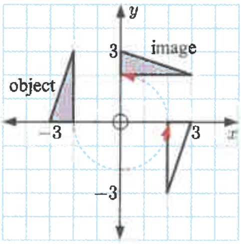
b A reduction with scale factor $\frac{1}{2}$.

6 **a** **i**



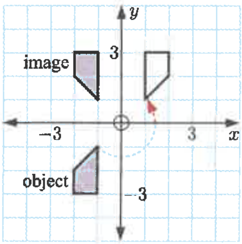
ii A translation of 1 unit right and 3 units up.

b **i**



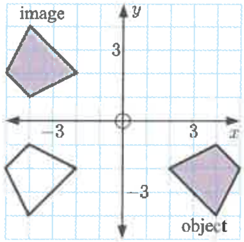
ii A rotation of 90° clockwise about O.

c **i**



ii A reflection in the x-axis.

d **i**



ii A rotation of 180° about O.

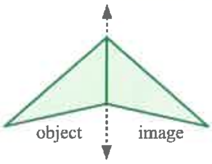
7 **a** Translate A 6 units right and 6 units up.

b Reflect A in the y-axis, then rotate 90° anticlockwise about O.

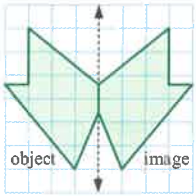
c Reflect A in the x-axis, then rotate 90° clockwise about O.

REVIEW SET 19A

1 **a**



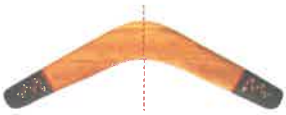
b



2 **a** 2 units right, 3 units up **b** 2 units left, 3 units down

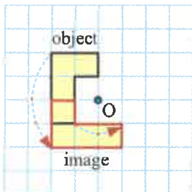
c 4 units right, 2 units down **d** 6 units left, 1 unit down

3 **a** Yes, one line of symmetry.

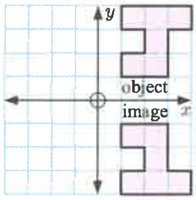


b no

4

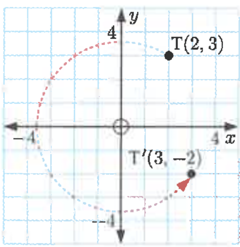


5



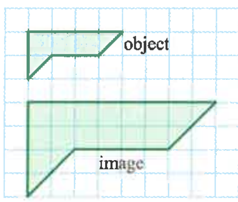
6 **a** 4 **b** 3 **c** 2

7 **a**

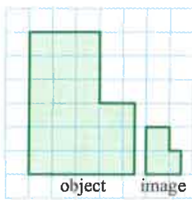


b T′(3, −2)

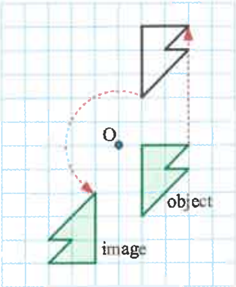
8 **a**



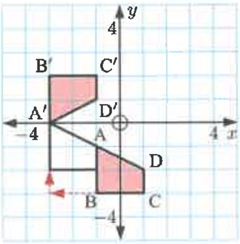
b



9

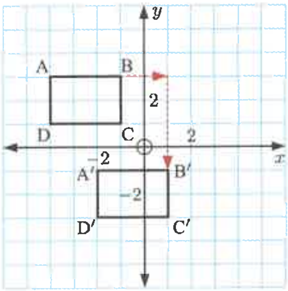


10 **a**



b A′(−3, 0)
B′(−3, 2)
C′(−1, 2)
D′(−1, 1)

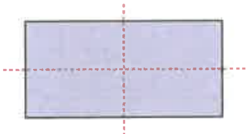
11 **a** 2 **b**



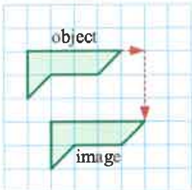
c 54 units^2

REVIEW SET 19B

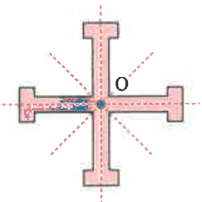
1



2

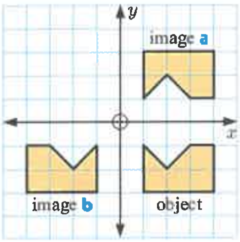


3 **a**



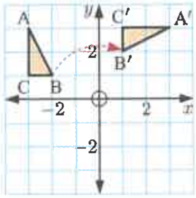
b 4

4



5 $\frac{1}{2}$

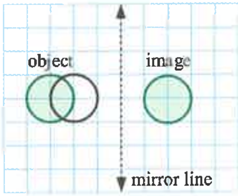
6 **a**



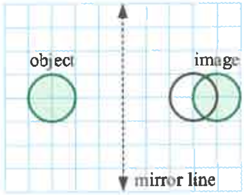
b A′(3, 3)
B′(1, 2)
C′(1, 3)

7 (5, 3) **8** 4th quadrant

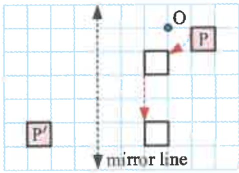
9 **a**



b



10



11 **a** **i** D **ii** 2
b **i** F **ii** Translate B 4 units left and 3 units down.
c **i** Rotate C 90° clockwise about O, then translate 2 units left and 1 unit down.
ii Translate C 1 unit right and 2 units down, then rotate 90° clockwise about O.

12 **a** **i** A translation of 4 units right and 4 units down.
ii A rotation of 90° anticlockwise about O.
iii A reflection in the x-axis.
b (−1, 1), (−3, 1), (−3, −3), (−1, −3)